Anisotropy Creases Delineate White Matter Structure in Diffusion Tensor MRI

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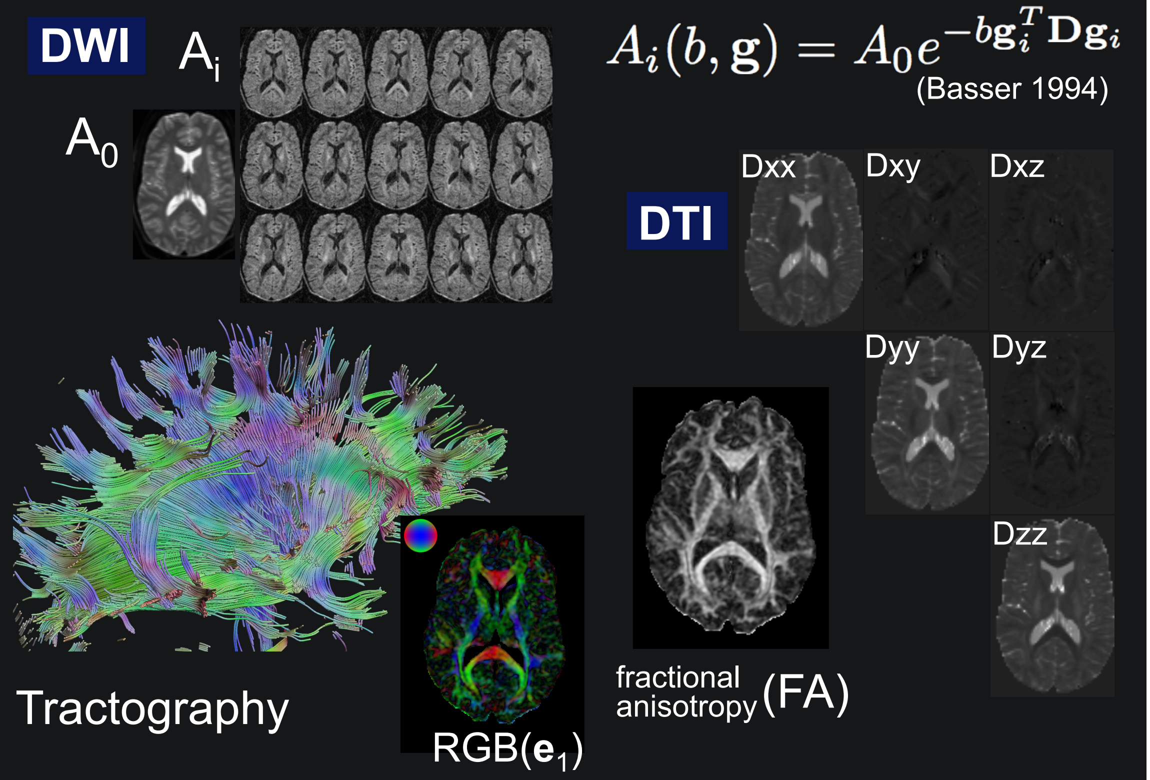
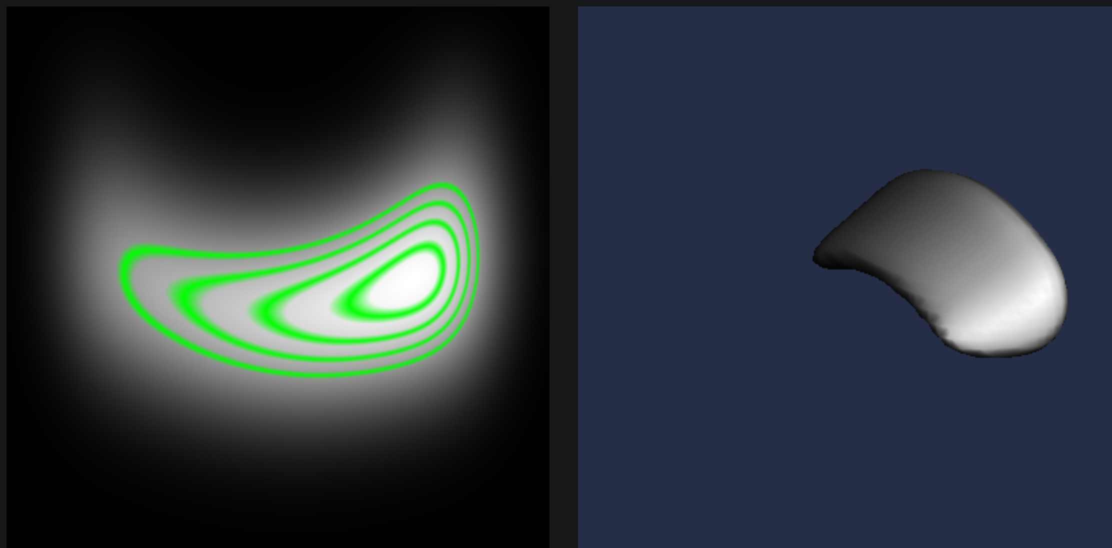
Diffusion weighted imaging, tensor imaging

$$A_i(b, \mathbf{g}) = A_0 e^{-b\mathbf{g}_i^T \mathbf{D} \mathbf{g}_i}$$

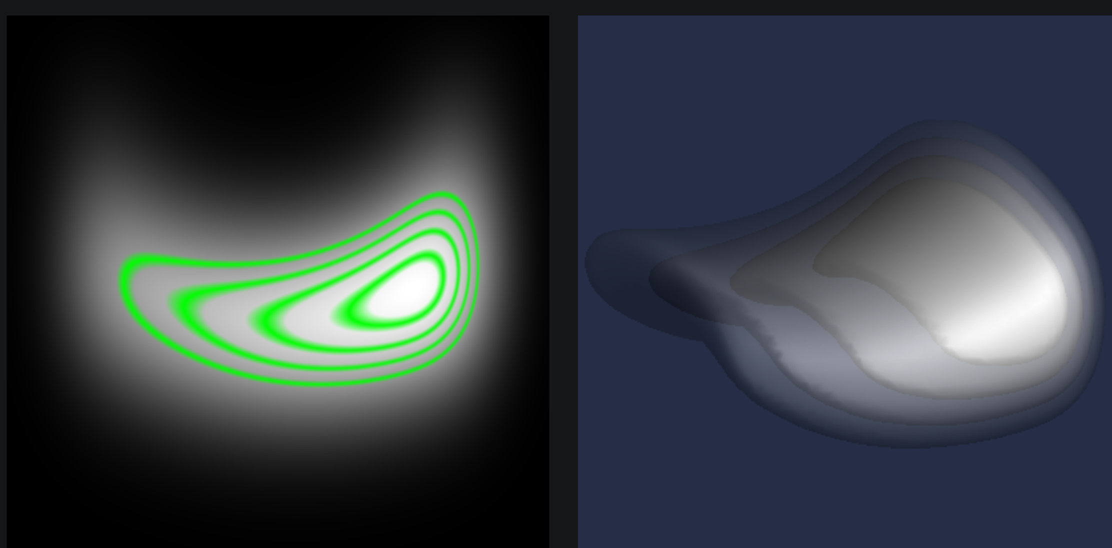
(Basser 1994)



Isosurfaces show global structure …

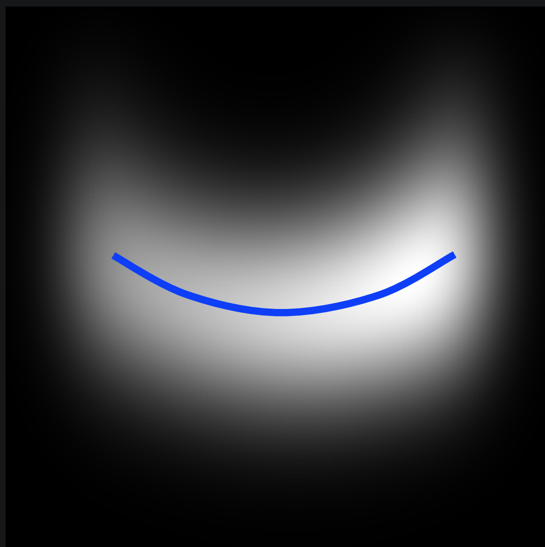


Isosurfaces show global structure …



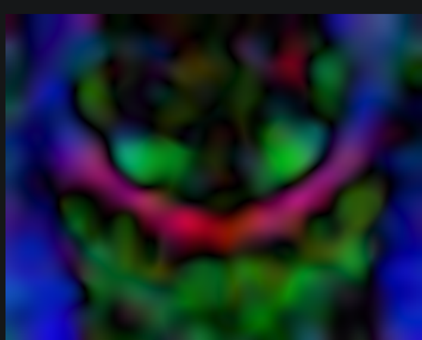
…. but don’t always show salient structure

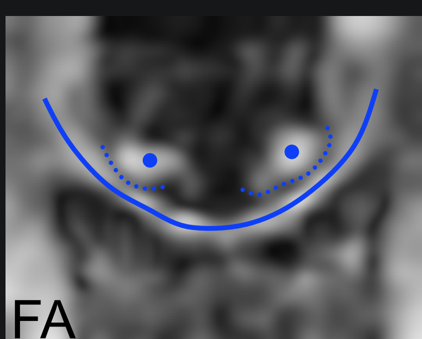
Creases!



Creases (ridges and valleys) do capture salient structure

Creases for DTI?



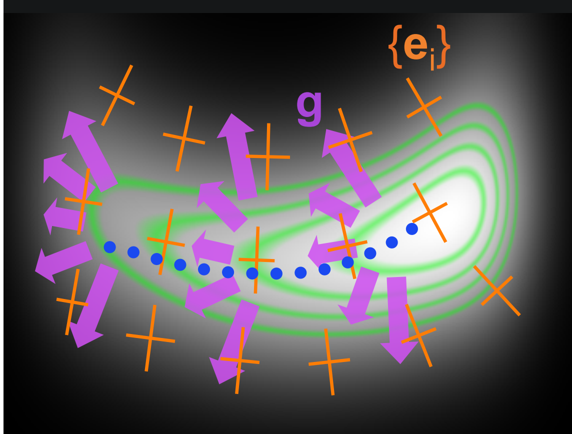


- Goal: model large-scale white matter structures
  - Robust + Repeatable
  - Few or Zero Parameters
  - “Sulci for white matter”
- Basic Idea: Creases of FA
  - Ridges: “cores”
  - Valleys: interfaces
  - Shape, not connectivity

Rest of talk

- Mathematical definition of creases
- Measurement by convolution
- Analytical differentiation of FA
- Slice inspection (2-D results)
- Modified Marching Cubes for surface
- 3-D results

Crease feature definition (Eberly 1994)



Constrained extremum

Gradient **g**

Hessian eigensystem $\mathbf{e}_i$, λ_i

Crease: **g** orthogonal to one or more $\mathbf{e}_i$

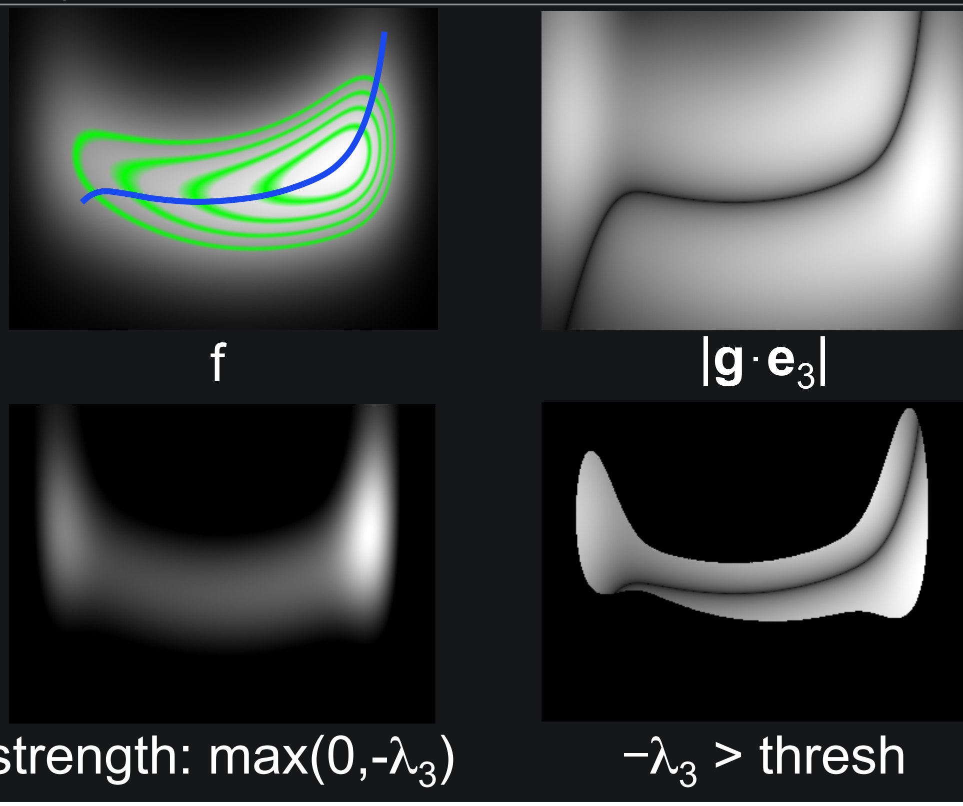
Eigenvalue gives strength

Ridge surface: $\mathbf{g} \cdot \mathbf{e}_3 = 0$; $\lambda_3 < \text{thresh}$

Ridge line: $\mathbf{g} \cdot \mathbf{e}_3 = \mathbf{g} \cdot \mathbf{e}_2 = 0$; $\lambda_3, \lambda_2 < \text{thresh}$

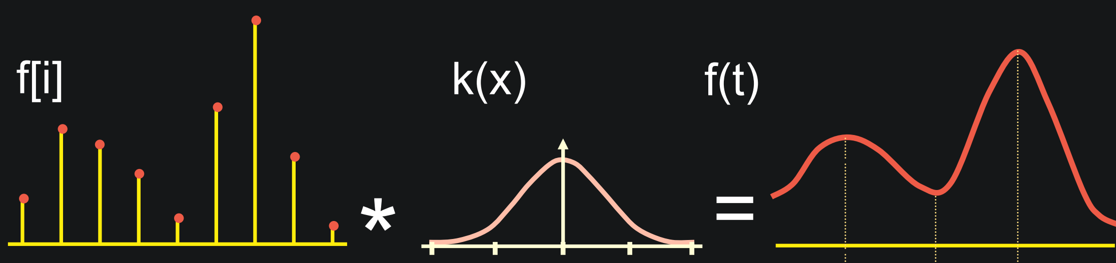
Valley surface: $\mathbf{g} \cdot \mathbf{e}_1 = 0$; $\lambda_1 > \text{thresh}$

2-D Synthetic Scalar Example

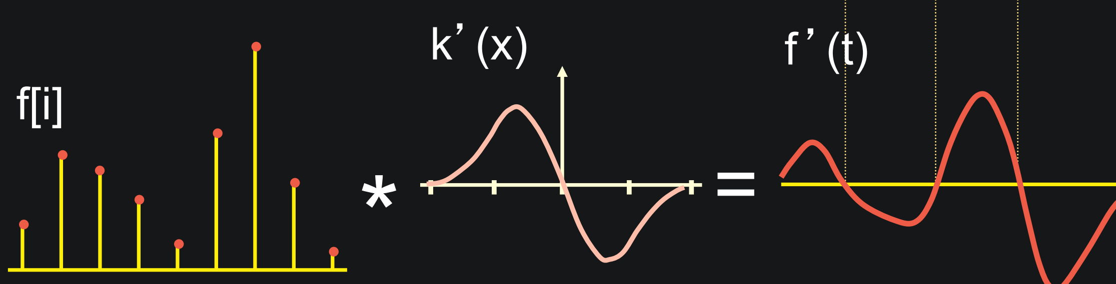


Measurement (of scalars) by convolution

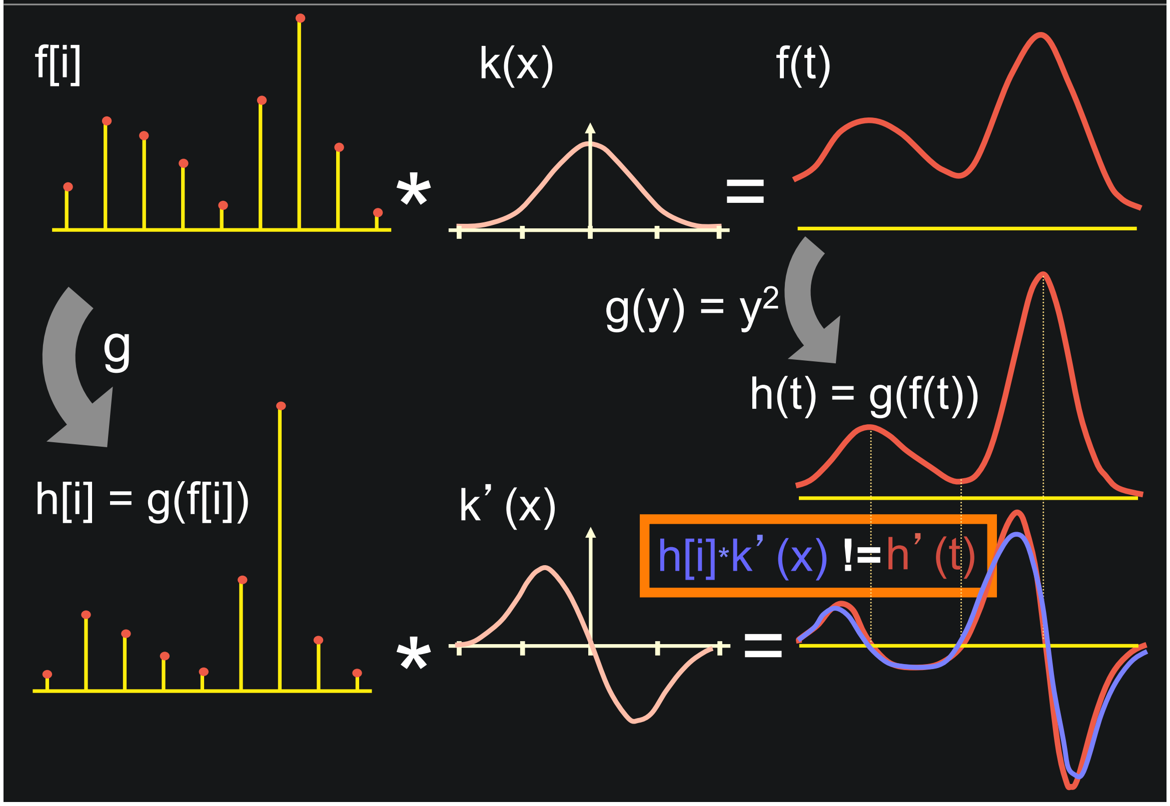
Continuous field: convolution of sampled coefficients with continuous reconstruction kernels



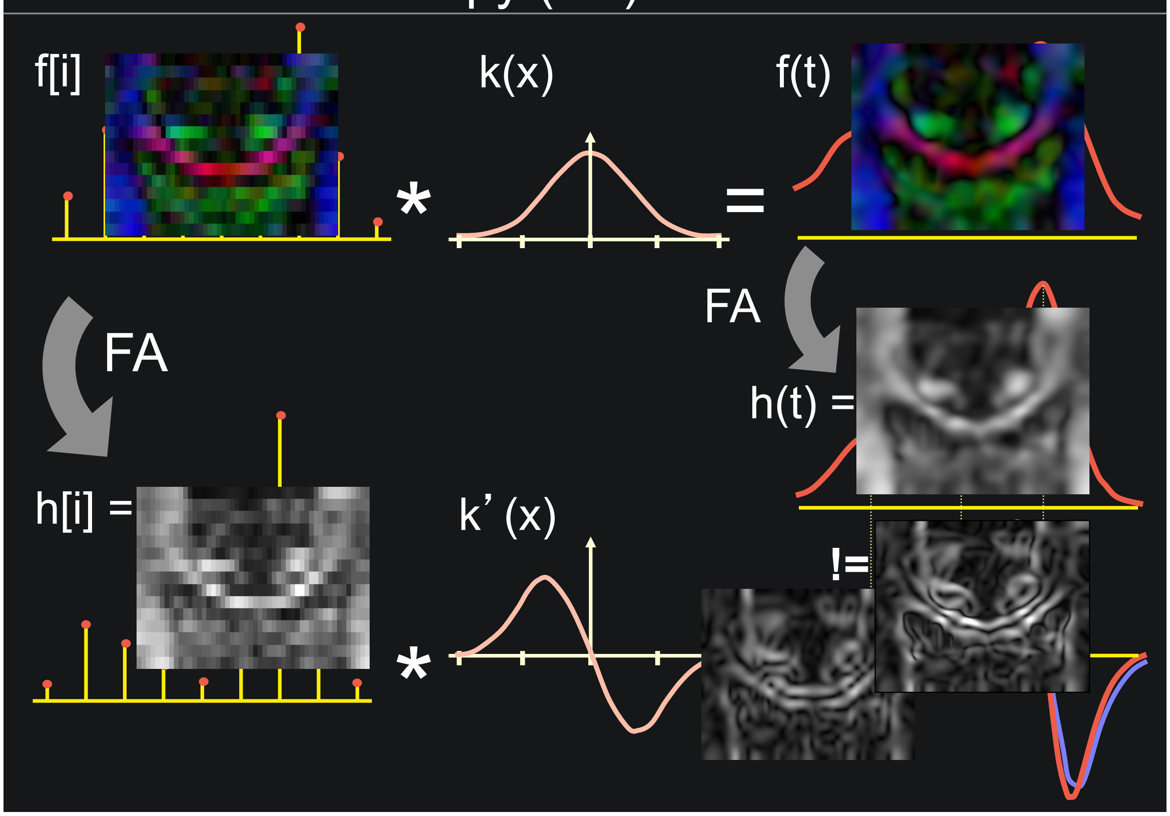
Differentiation: convolve w/ derivative of reconstruction kernel

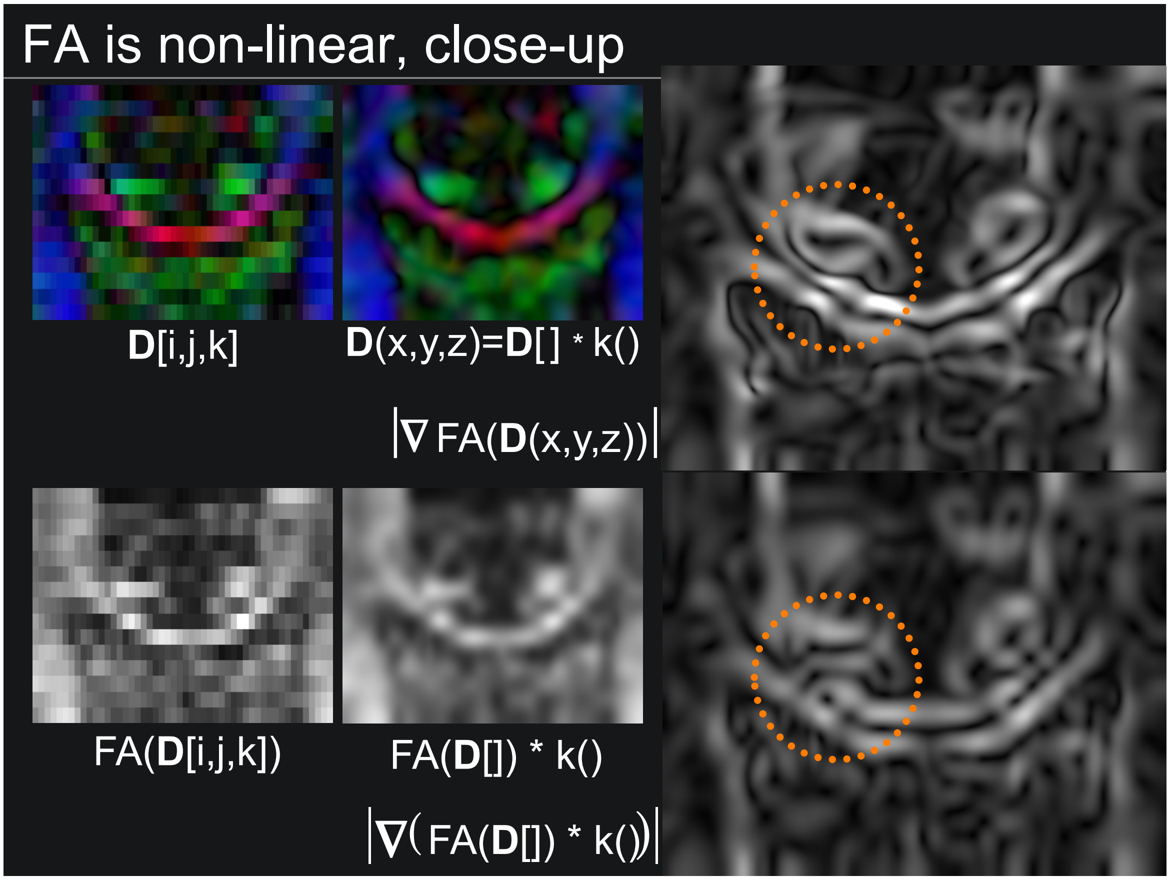


Non-linear transform of data



Fractional Anisotropy (FA) is non-linear





FA from invariants, from coefficients

$$\mathrm{FA} \equiv \sqrt{\frac{3\, D{:}D}{2\, \mathbf{D}{:}\mathbf{D}}} \qquad D = \mathbf{D} - \langle \mathbf{D} \rangle \mathbf{I} \qquad \mathbf{D}{:}\mathbf{D} = \mathrm{tr}(\mathbf{D}\mathbf{D}^T)$$

$$J_2 = \begin{array}{c} D_{xx}D_{yy} + D_{xx}D_{zz} + D_{yy}D_{zz} \\ -D_{xy}^2 - D_{xz}^2 - D_{yz}^2 \end{array}$$

$$Q = \frac{S - J_2}{9}$$

$$\mathrm{FA} = 3\sqrt{\frac{Q}{S}}$$

$$S = \mathbf{D}{:}\mathbf{D} = \begin{array}{c} D_{xx}^2 + D_{yy}^2 + D_{zz}^2 \\ +2D_{xy}^2 + 2D_{xz}^2 + 2D_{yz}^2 \end{array}$$

$$\nabla J_2 = \begin{array}{c} (D_{yy} + D_{zz})\nabla D_{xx} + (D_{xx} + D_{zz})\nabla D_{yy} + (D_{xx} + D_{yy})\nabla D_{zz} \\ -2D_{xy}\nabla D_{xy} - 2D_{xz}\nabla D_{xz} - 2D_{yz}\nabla D_{yz} \end{array}$$

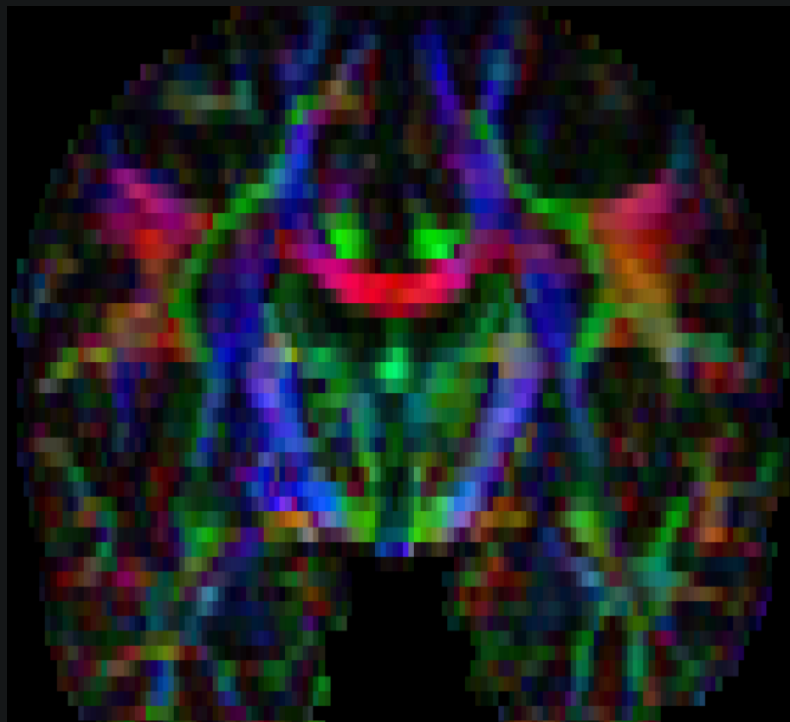
$$\nabla Q = \frac{\nabla S - \nabla J_2}{9}$$

$$\nabla \mathrm{FA} = \frac{3}{2}\left(\sqrt{\frac{1}{SQ}}\nabla Q - \sqrt{\frac{Q}{S^3}}\nabla S\right)$$

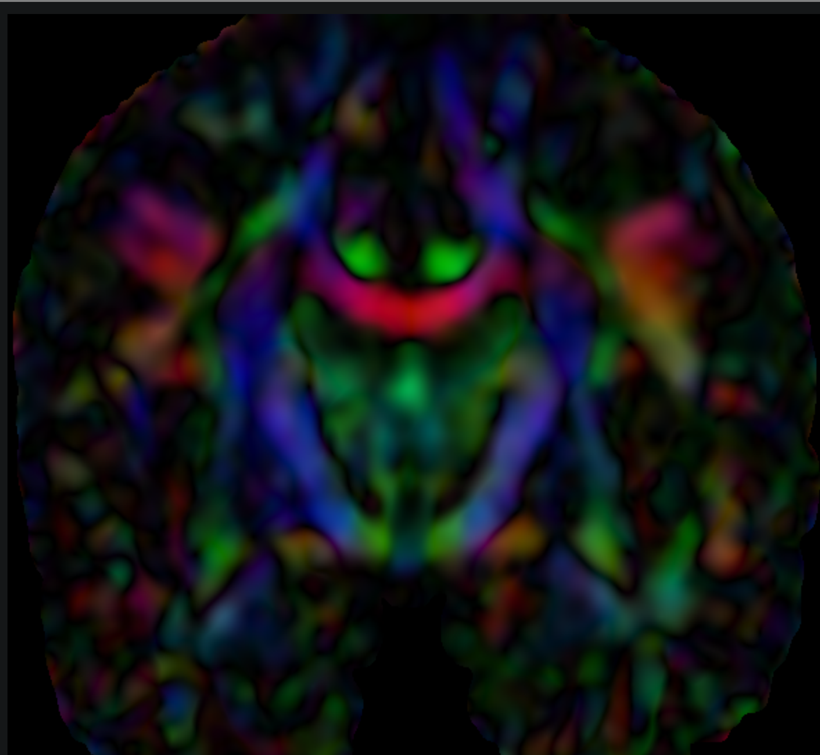
$$\nabla S = \begin{array}{c} 2D_{xx}\nabla D_{xx} + 2D_{yy}\nabla D_{yy} + 2D_{zz}\nabla D_{zz} \\ +4D_{xy}\nabla D_{xy} + 4D_{xz}\nabla D_{xz} + 4D_{yz}\nabla D_{yz} \end{array}$$

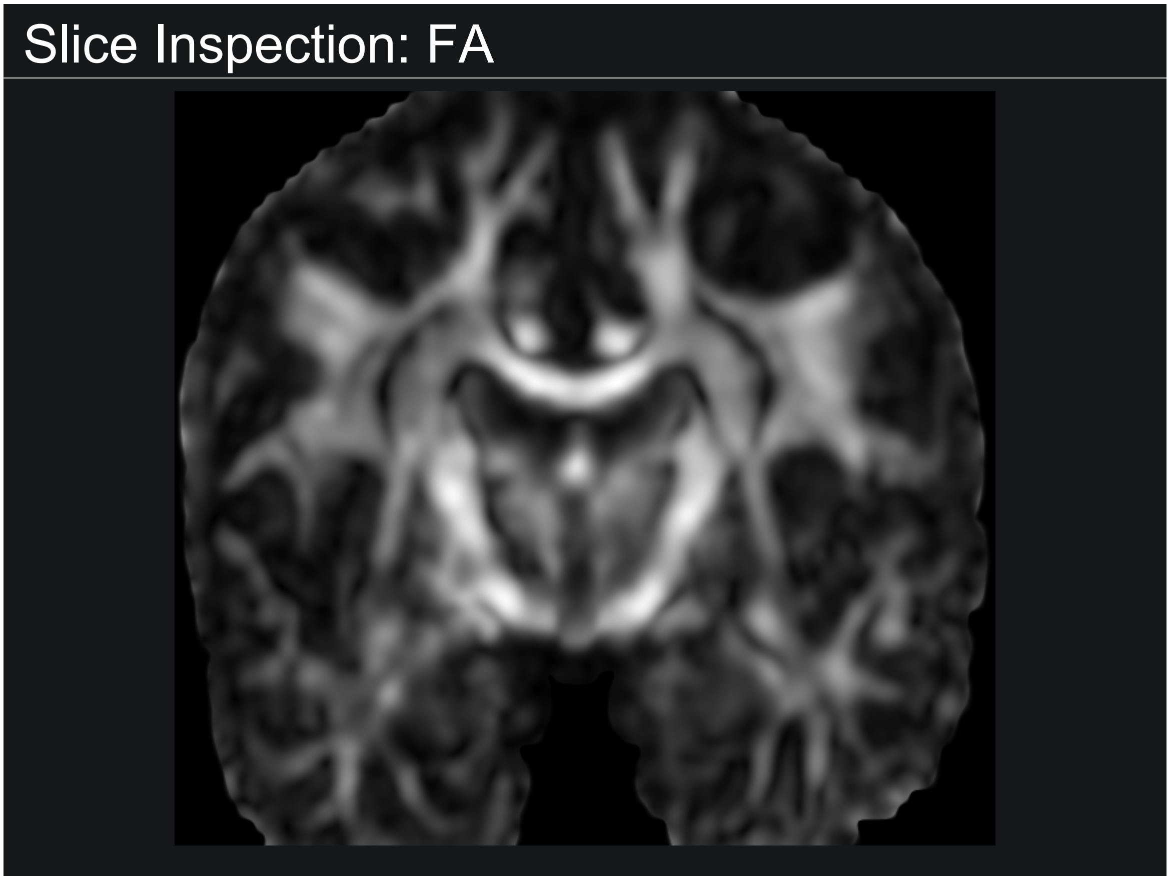
Hessian(FA) more complicated, but similarly derived

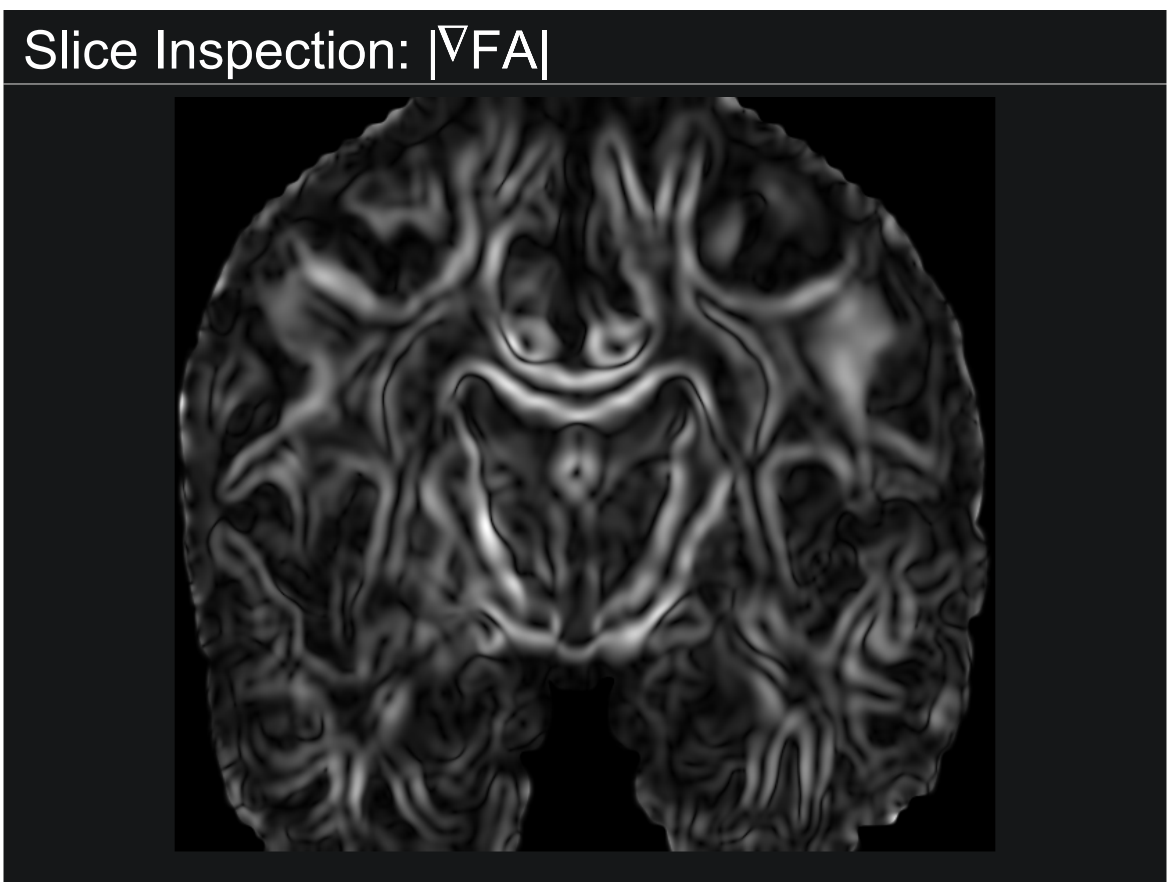
Slice Inspection: RGB($\mathbf{e}_1$) (original data)



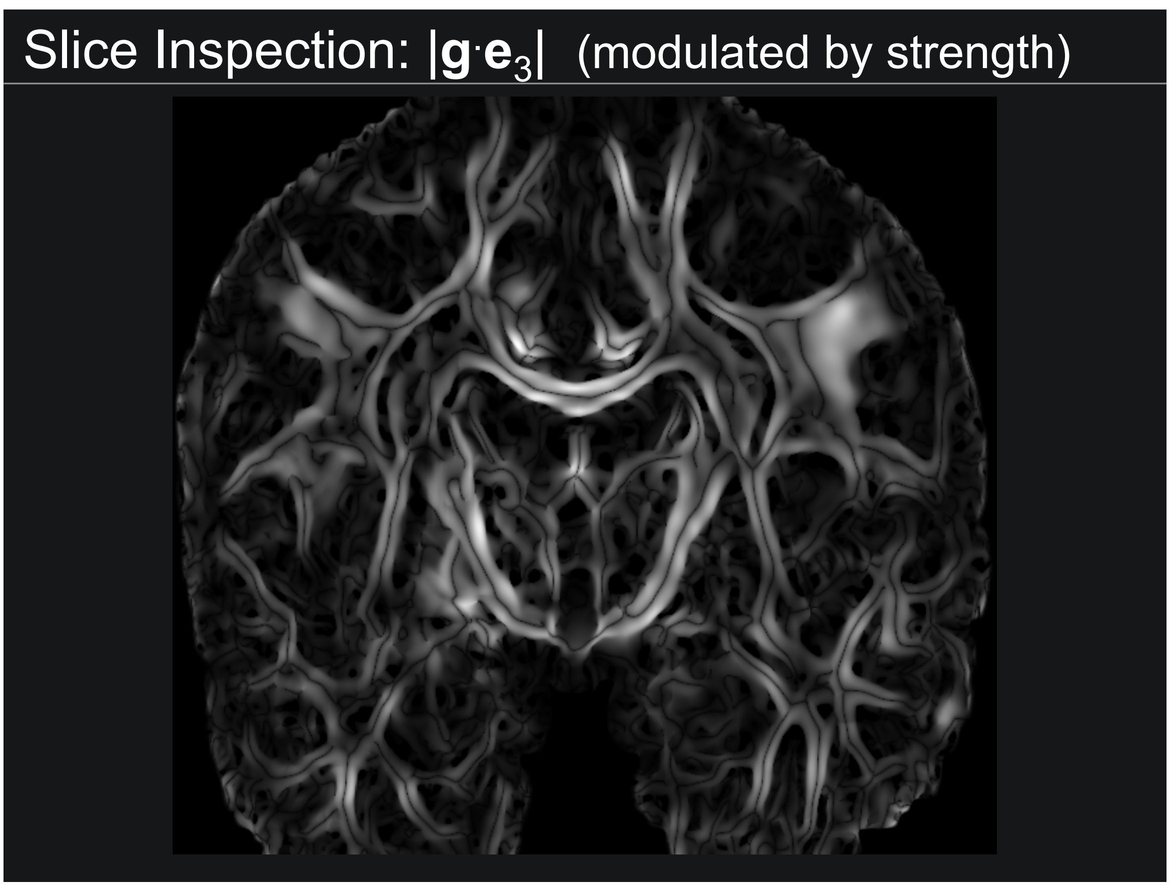
Slice Inspection: RGB($\mathbf{e}_1$)



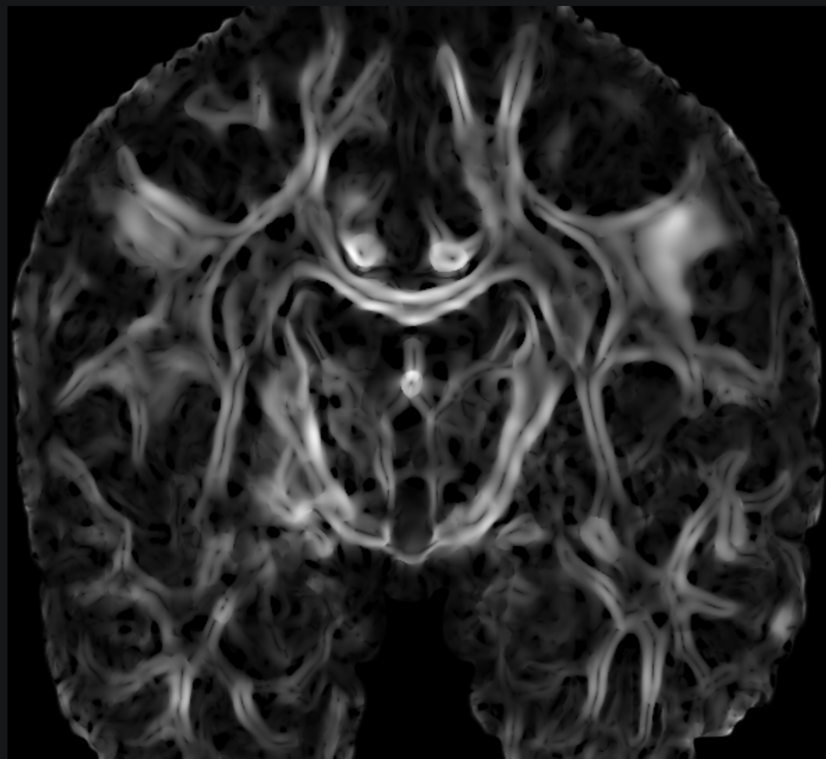

Slice Inspection: FA


Slice Inspection: |∇FA|

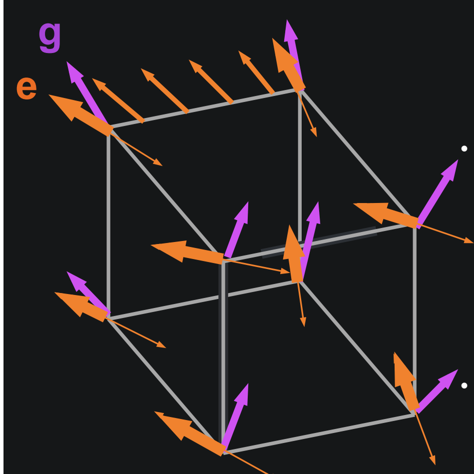

Slice Inspection: ridge strength: max(0, λ_3)


Slice Inspection: $|\mathbf{g}\cdot\mathbf{e}_3|$ (modulated by strength)

Slice Inspection: sqrt(($\mathbf{g} \cdot \mathbf{e}_3$)^2 + ($\mathbf{g} \cdot \mathbf{e}_2$)^2)

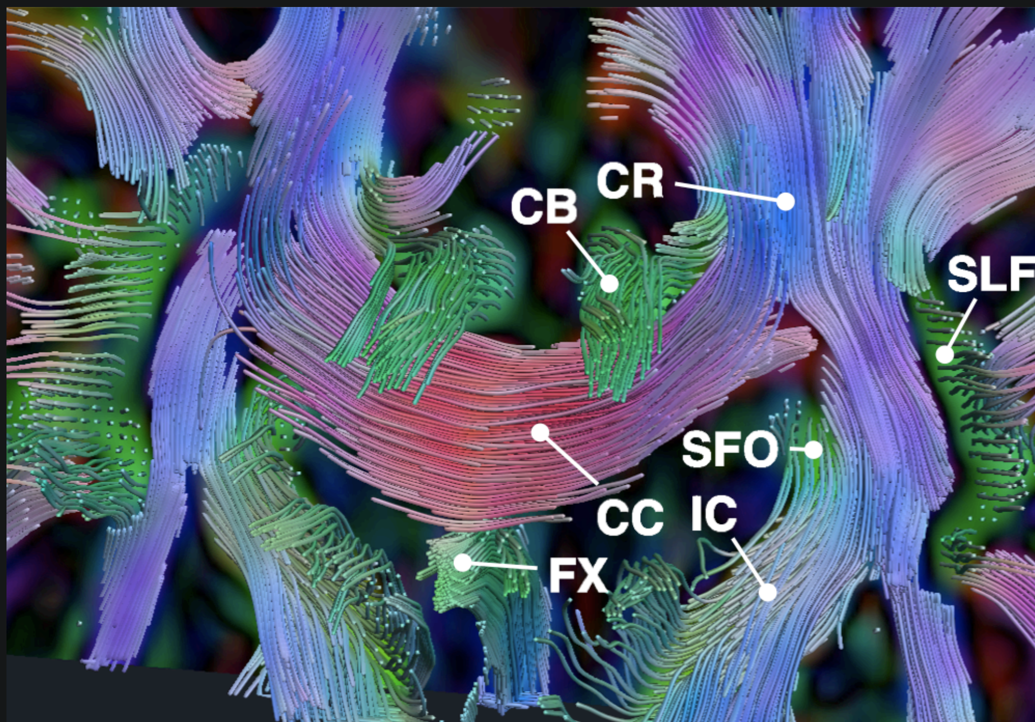


Modified Marching Cubes for Surfaces



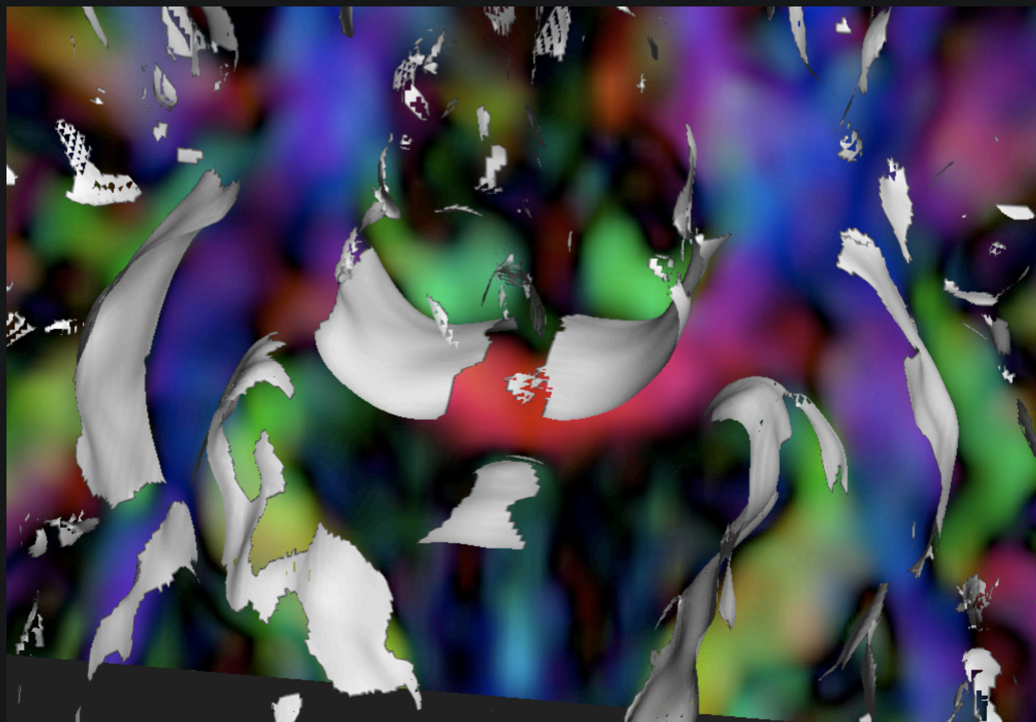
- Crease surface is isosurface (zero-crossing) of $\mathbf{g} \cdot \mathbf{e}_i$, but…
- Eigenvectors lack sign: enforce intra-voxel sign consistency
- Propagate eigenvector at one corner to all others
- $\mathbf{g} \cdot \mathbf{e}$ dot products, then MC case table

3-D Results: coronal fibers

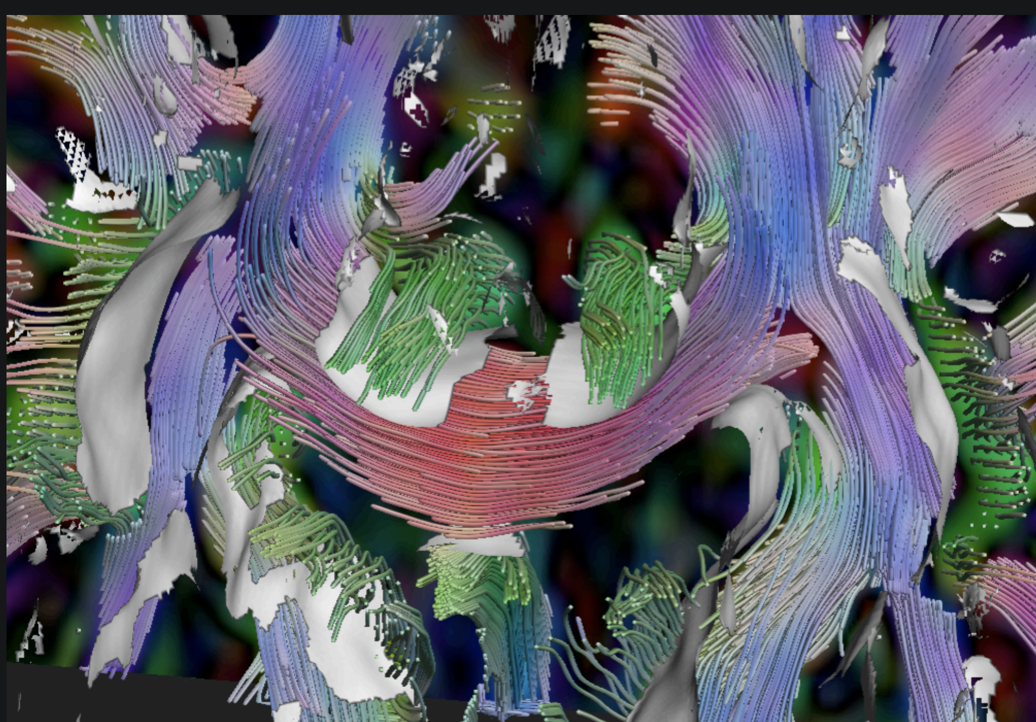


3-D Results: ridge surfaces

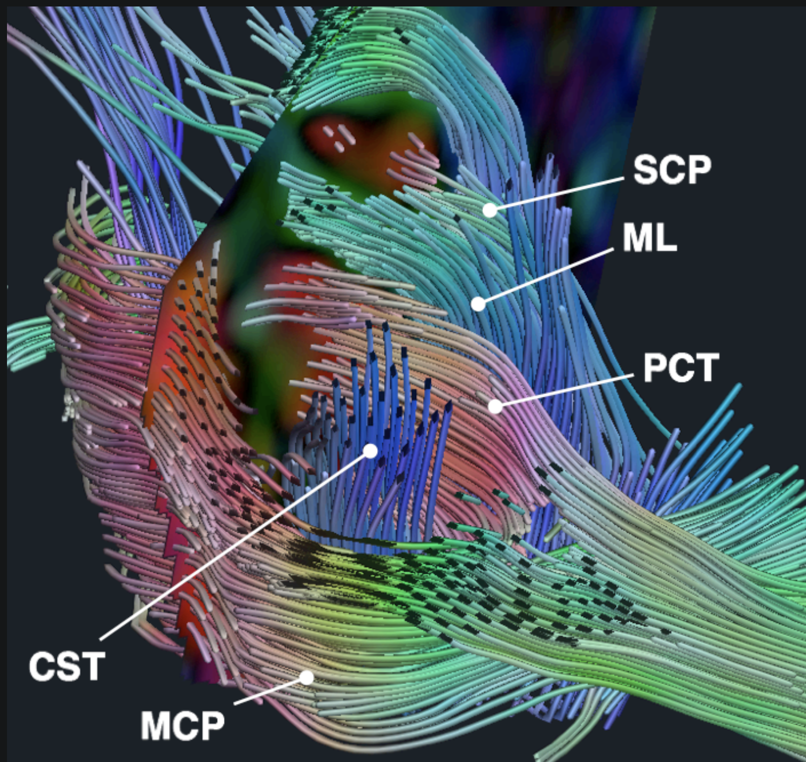
3-D Results: valley surfaces



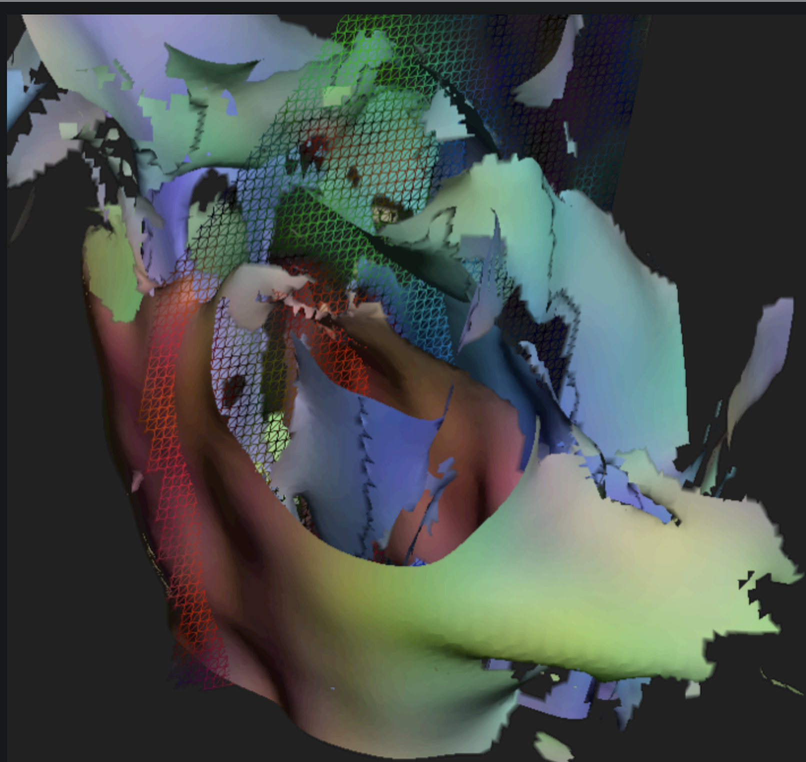
3-D Results: valley surfaces with fibers



3-D Results: brainstem fibers

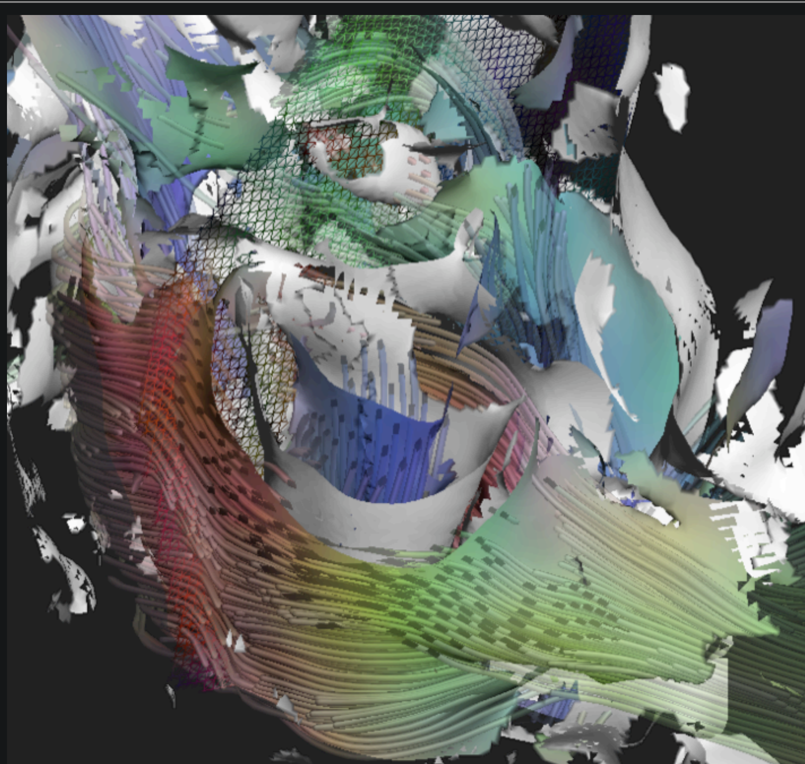


3-D Results: brainstem ridge surfaces



3-D Results: brainstem valley surfaces

3-D Results: combined results



Discussion & Ongoing Work

- Novel Aspects:
  - Application of computer vision to DTI
  - Extracting geometry from differential DTI structure
- Scale space: interfaces are easier than “cores”
- Tensor eigensystem orientation?
- Crease line extraction; line vs. surface decision
- Evaluation on more datasets
- Besides registration: Shape analysis, tracts as manifolds, assymetry measurement

Acknowledgements

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- URL for paper + software info:
- http://lmi.bwh.harvard.edu/~gk/miccai06/

thank you