Sampling and Visualizing Creases with Scale-Space Particles

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Outline

- **Introduction**: motivation, example, contributions
- **Method**: interpolation, features, energy, visualization, computation
- **Results**: lung CT, brain DTI, more
- **Discussion**: scale, particles, analysis, future

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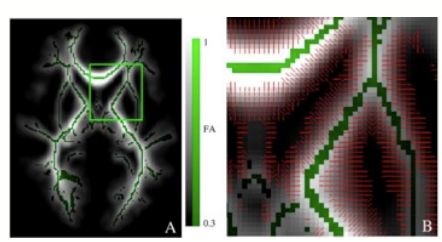
Context & Motivation

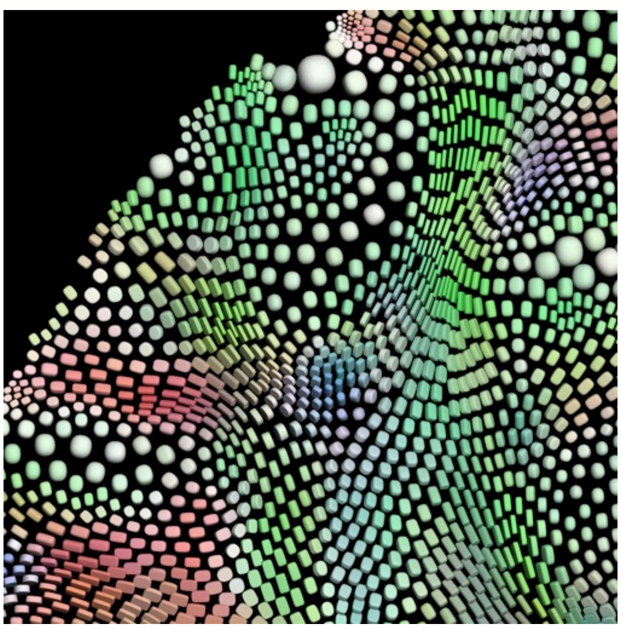
- Medical imaging is for measuring
- Often need "skeletons" of structures:
  - Ridges & valleys = creases
- Lung CT: find airways, measure radii, study COPD
- Brain DTI: find major WM tracts, measure FA, study psychiatric disorders
  - DTI not just for tractography
  - Much of it for FA studies



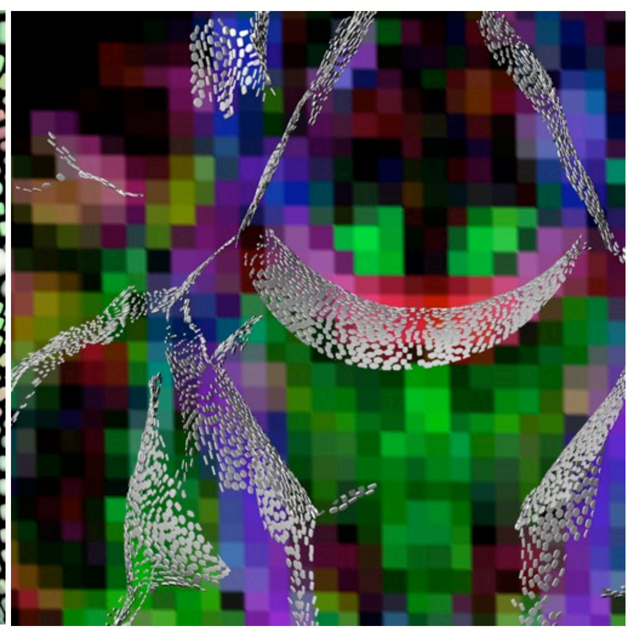
- Our initial work strives to be a general approach to detecting and sampling crease features

Take-aways

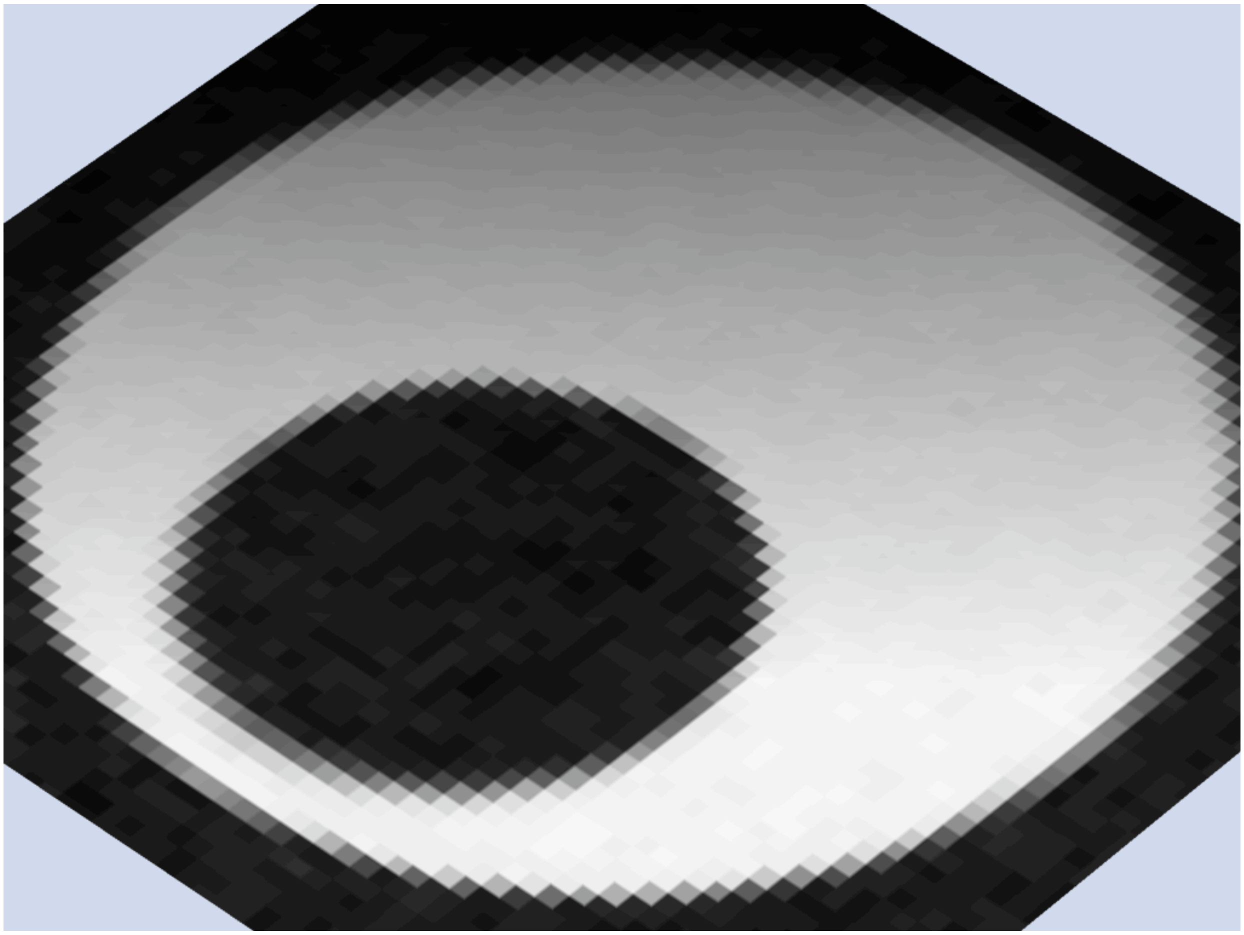
- Scale-space is useful, now more efficient in 3D
- Particles are natural for scale-space features
- Visualization as tool for design of image analysis

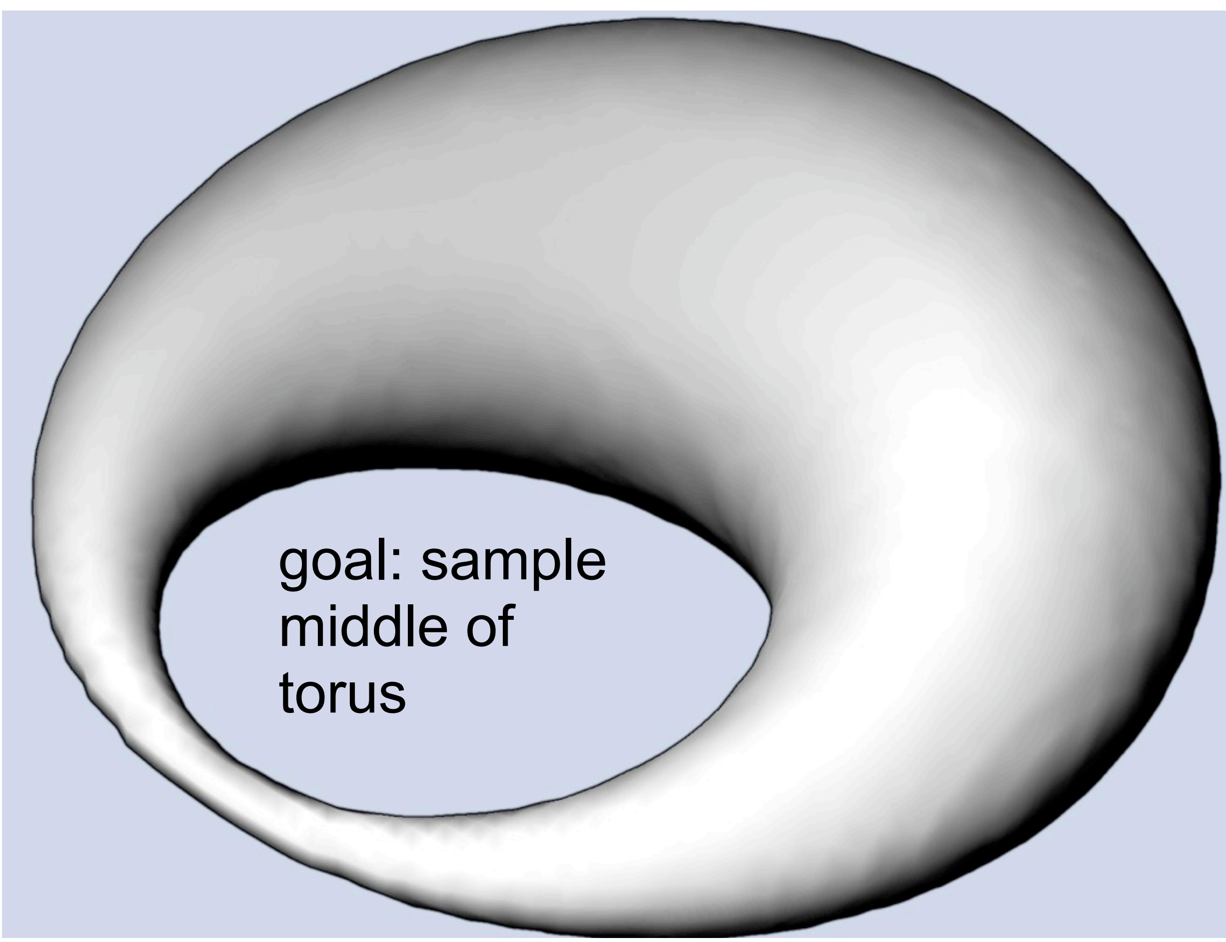


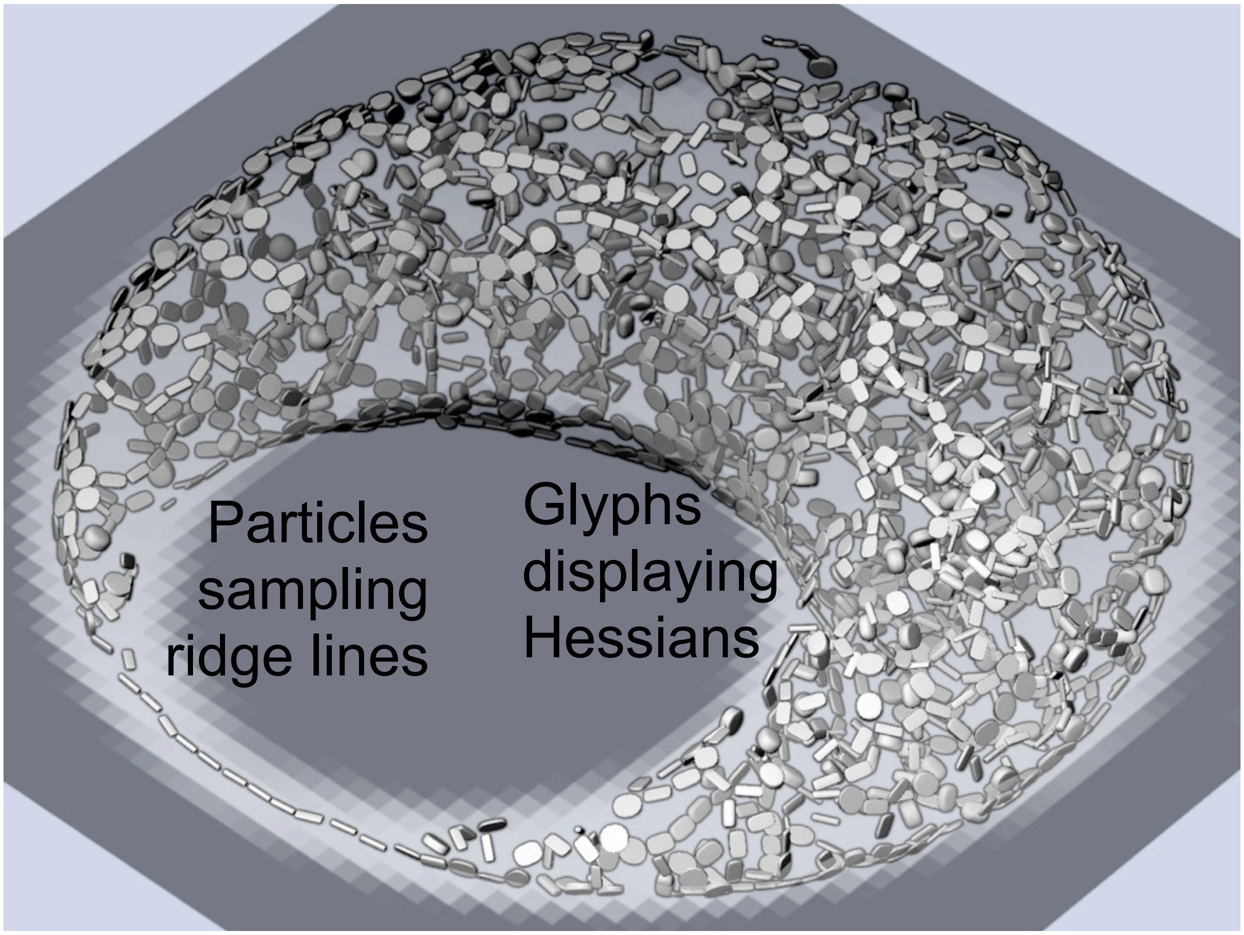
Particles for DTI <u>visualization</u>
(previous work, “Glyph Packing” Vis ’06)
Glyphs for diffusion tensors
Sampling whole field
Image for humans to look at

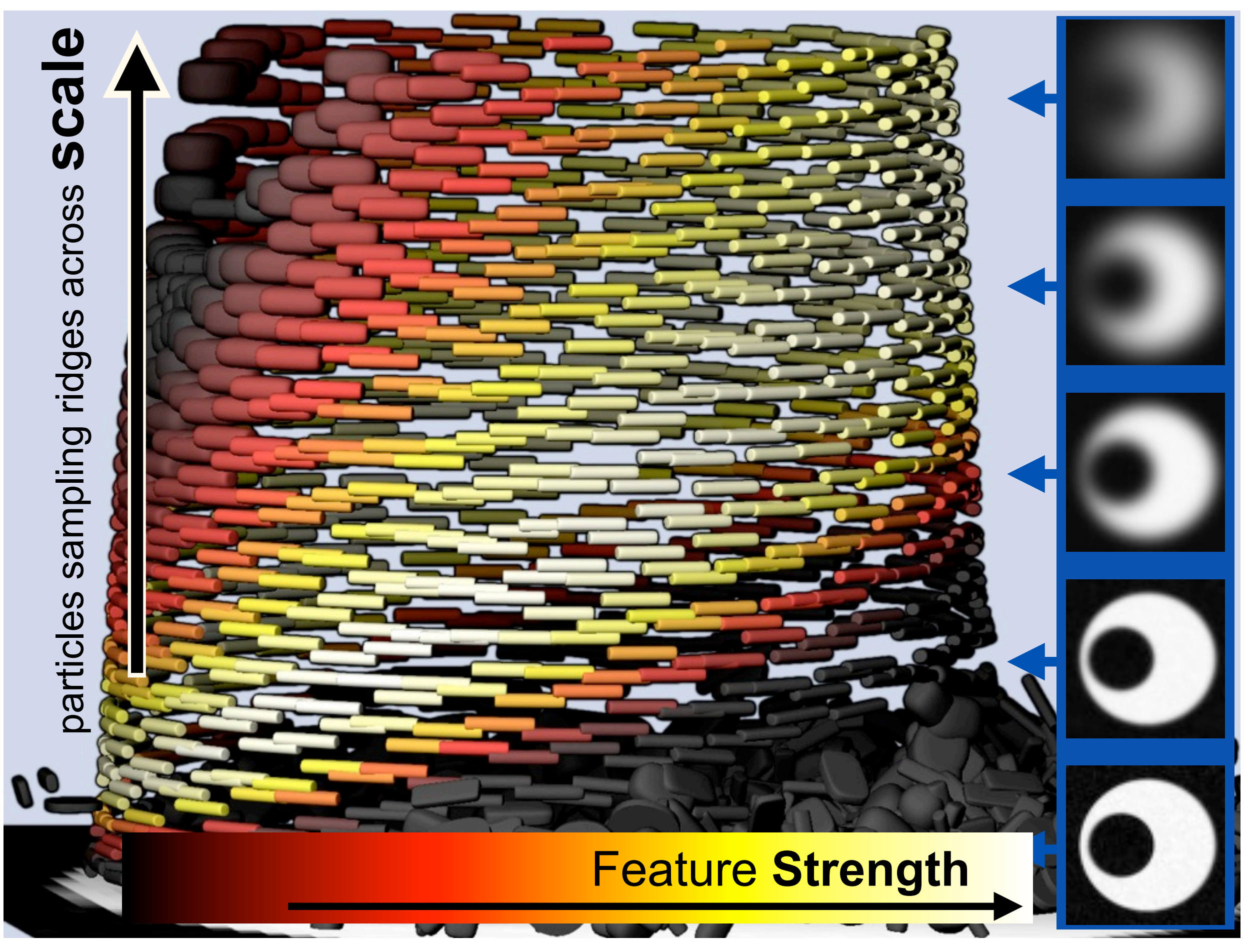


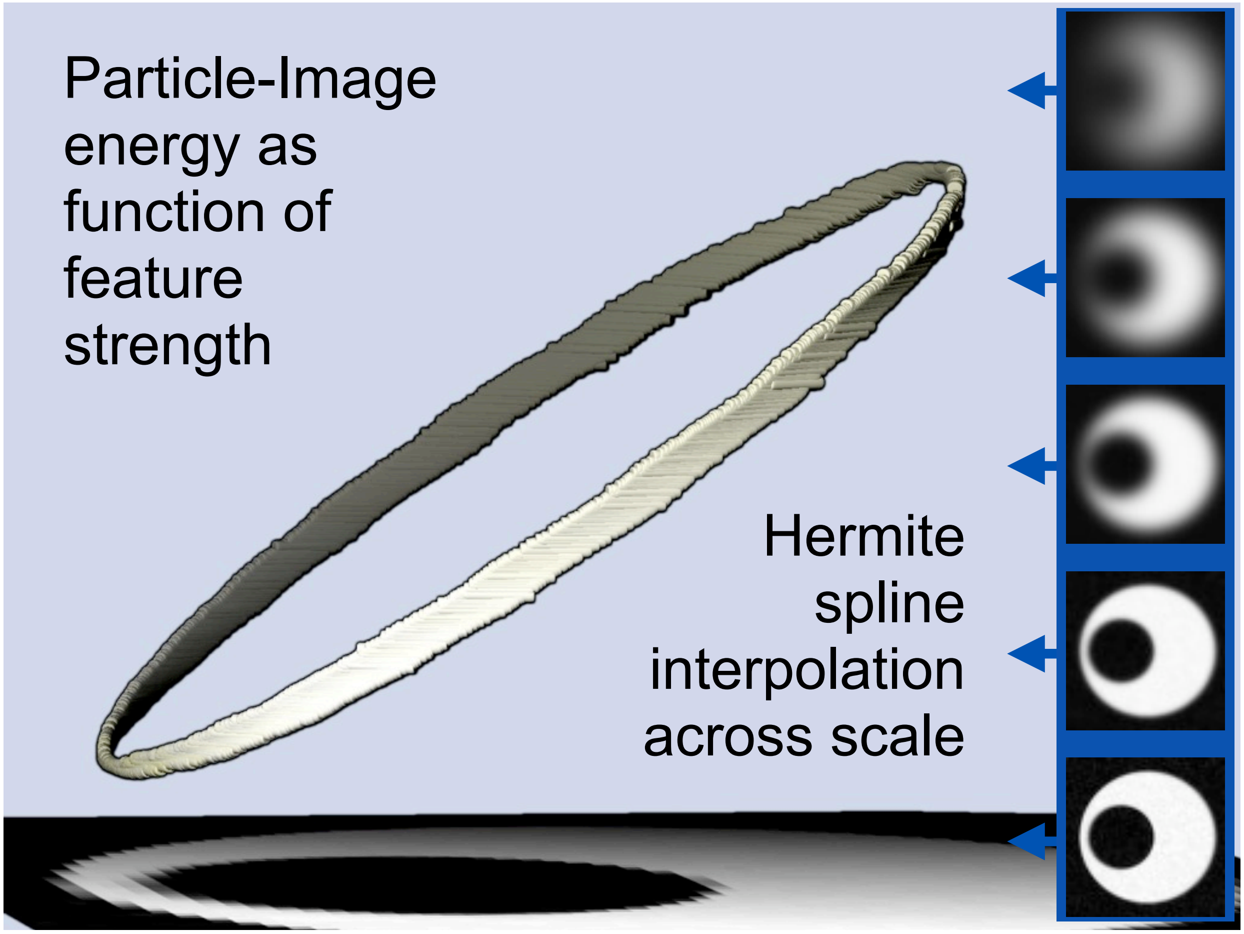
Particles for DTI <u>analysis</u>
(current work)
Glyphs for Hessians of FA
Sampling crease features
Geometry for measurement

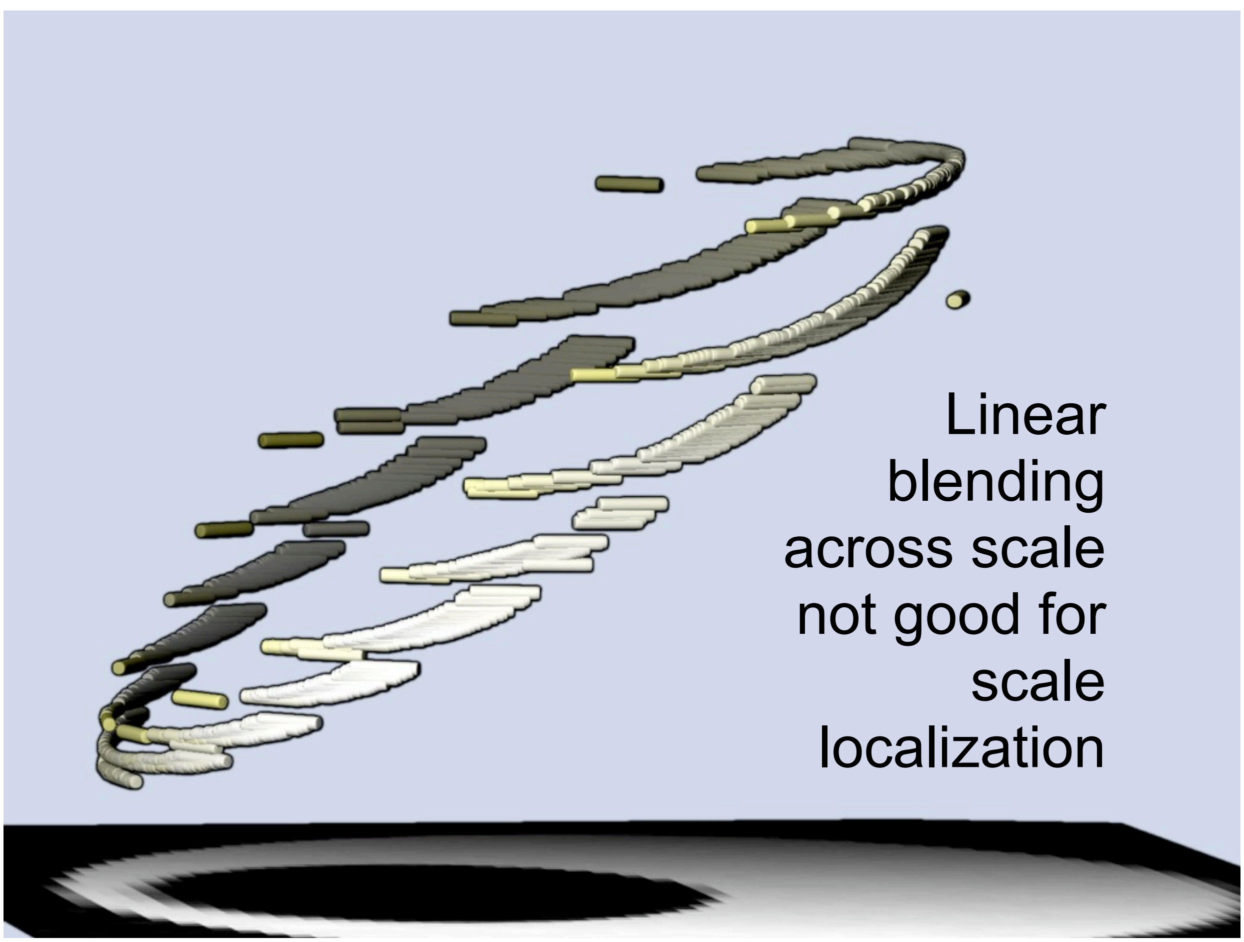


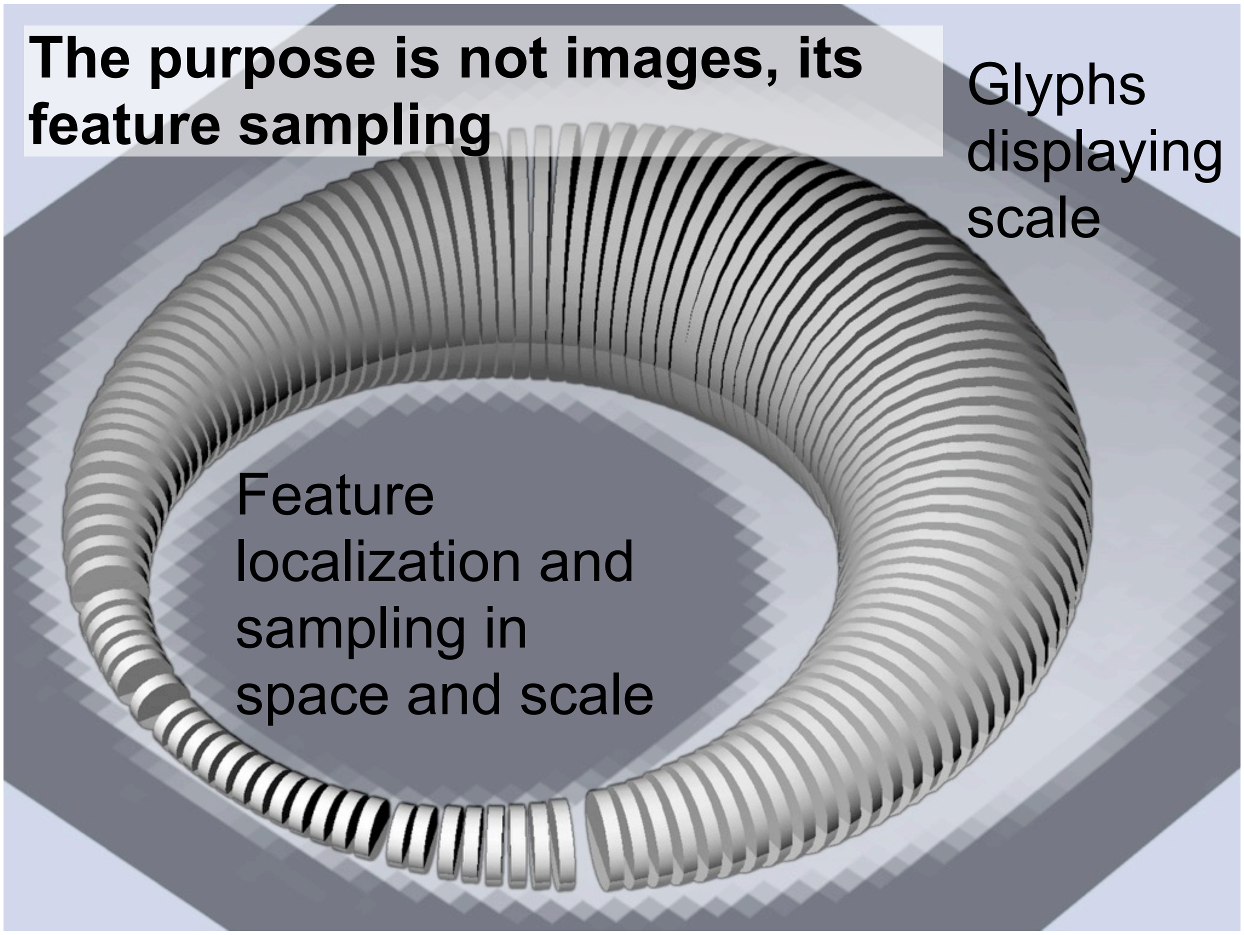

goal: sample
middle of
torus


Particles
sampling
ridge lines
Glyphs
displaying
Hessians


particles sampling ridges across scale
Feature Strength


Particle-Image energy as function of feature strength
Hermite spline interpolation across scale


Linear blending across scale not good for scale localization



Contributions

- Efficient interpolation of scale for 3D images
- Particle-based sampling of ridges and valleys, in scale-space (vs. implicit surfaces, at single scale)
    - Energies that implement scale-localization
- Glyphs for depicting Hessians and/or scale
- Scale-space feature extraction in DTI

Not doing

- Meshing
  - Particles separate locations from connectivity
  - Meyer, Whitaker et al. 2005-2008

- Detailed comparison to application-specific segmentation methods
  - More general features, in various domains

- Performance optimization
  - Reference implementation, needs GPUs

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Method Overview

- Governing equation
- Interpolation: discrete to continuous, scale & space
- Crease feature constraints and strengths
- Particle-Image energy
- Inter-particle energy
- System visualization and dynamics

Governing equation

$$\underbrace{\operatorname*{argmin}_{\{(\mathbf{x}_i, s_i)\}, N}}_{\text{Particle positions}} \mathcal{E} = \operatorname*{argmin}_{\{(\mathbf{x}_i, s_i)\}, N} (1-\alpha) \underbrace{\sum_{i=1}^{N} E_i}_{} + \frac{\alpha}{2} \underbrace{\sum_{i,j=1}^{N} E_{ij}}_{}$$

Particle positions

↑ Particle number

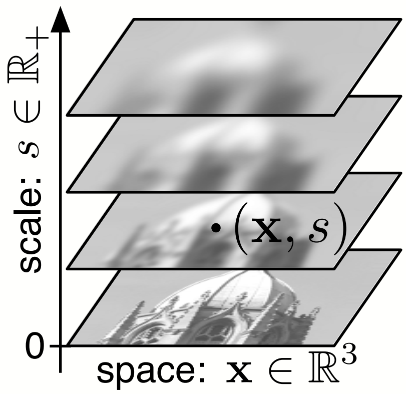
Particle-image energy: localizes feature **scale**

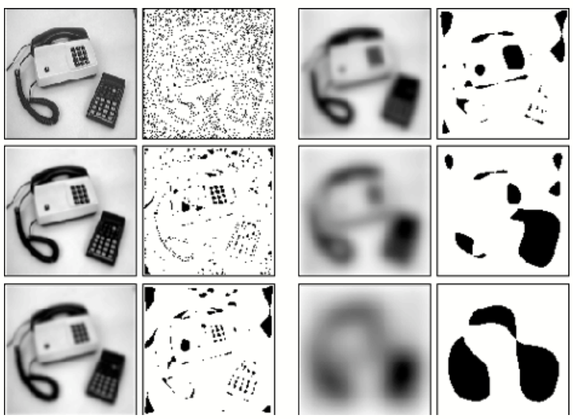
Inter-particle energy: induces uniform **spatial** sampling

- Feature constraints: enforced without contributing to energy
- Population control (finding N) becomes side-effect of energy minimization

Scale-Space

- From ‘90s computer vision
- Image and all possible **blurrings**
- More general than multi-resolution methods: scale is **continuous**





Structures of different sizes are naturally extracted at different scales

Tony Lindeberg ’96

Scale-Space And Diffusion

- Various considerations lead to blurring with a **Gaussian**
- Image L(*x*) diffuses for time *t* →continuum of images L(*x*;*t*)

$$L(x;t) = (L(\cdot) \star g(\cdot;t))(x) = \int g(\xi;t)L(x-\xi)d\xi$$

$$g(\xi;t) = \exp(-\xi^2/2t)/\sqrt{2\pi t}$$

- Heat equation: 1st deriv. in time → 2nd deriv. in space

$$\frac{\partial L(x;t)}{\partial t} \propto \frac{\partial^2 L(x;t)}{\partial x^2}$$

- What’s the analog in the **discrete** (implementation) domain?

Lindeberg's Discrete Gaussian

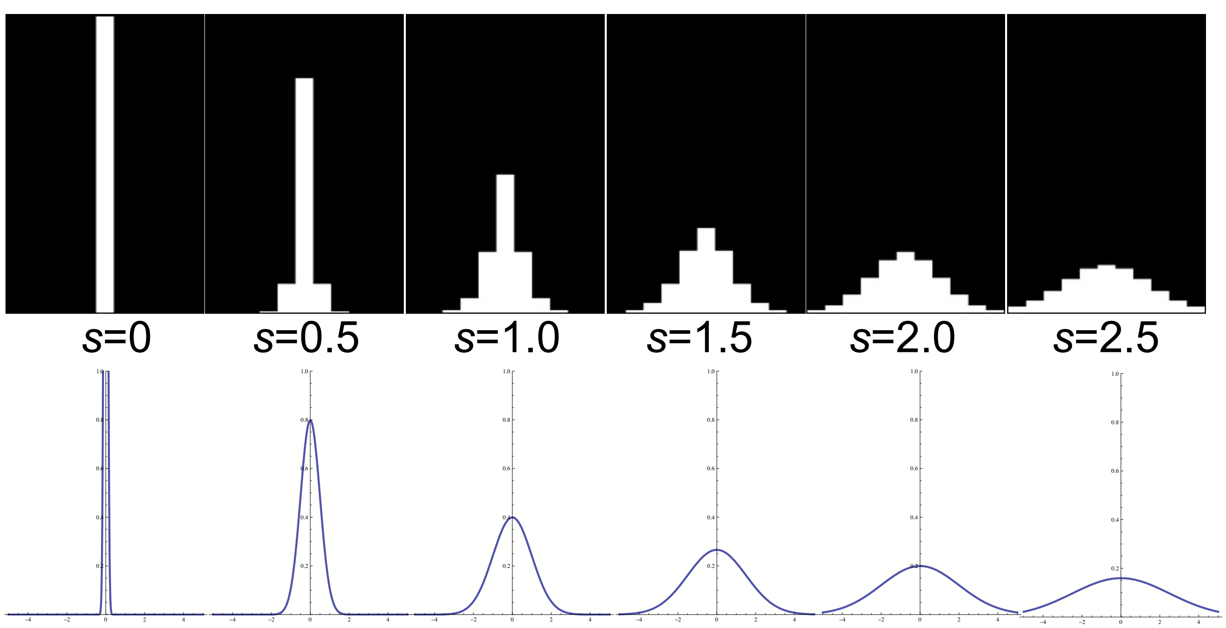
- "Scale-Space for Discrete Signals" IEEE PAMI 1990
- Beautiful analog to continuous Gaussian

$$\begin{aligned} L[i;t] &= (f \star K[\cdot;t])[i] = \sum_n K[n;t] f[i-n] \\ K[n;t] &= \exp(-t) I_n(t)\ ;\ s = \sqrt{t} = \text{“}\sigma\text{”} \end{aligned}$$

- $I_n(t)$ = modified Bessel function of order *n*

$$\begin{aligned} \frac{\partial K[i;t]}{\partial t} &= \frac{1}{2}\left(K[\cdot;t] \star \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}\right)[i] \\ \Rightarrow \frac{\partial L[i;t]}{\partial t} &= \frac{1}{2}\left(L[\cdot;t] \star \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}\right)[i] \\ \Rightarrow \frac{\partial L[i;s^2]}{\partial s} &= s\left(L[\cdot;s^2] \star \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}\right)[i] \end{aligned}$$

Lindeberg's Discrete Gaussian



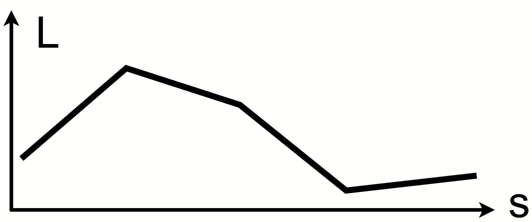
- Not the same as sampling a Gaussian
- Change **between** blurring levels important

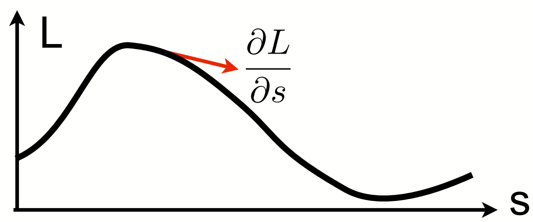
Interpolate along scale

- We leverage properties of Lindeberg's Gaussian

$$\frac{\partial L}{\partial s} = s\left(L[\cdot; s^2] \star \begin{bmatrix}1 & -2 & 1\end{bmatrix}\right)$$

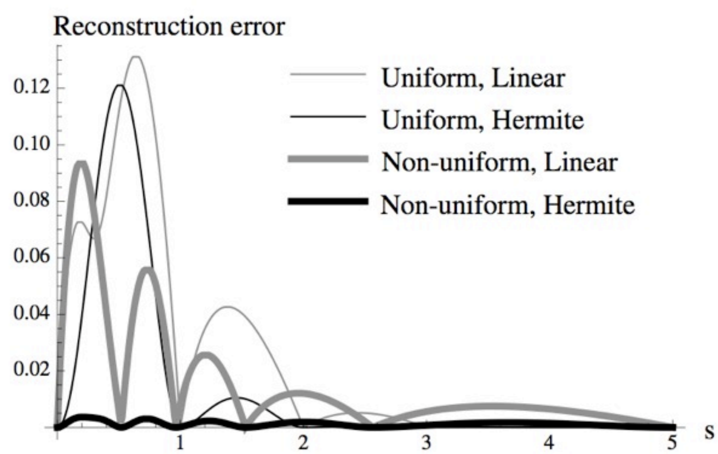
- Pre-compute different blurrings of image L for discrete set of blurring levels
- For intermediate scales, could linearly blend between, or
- With dL/ds at each scale, create cubic **Hermite spline**





Scale interpolation accuracy

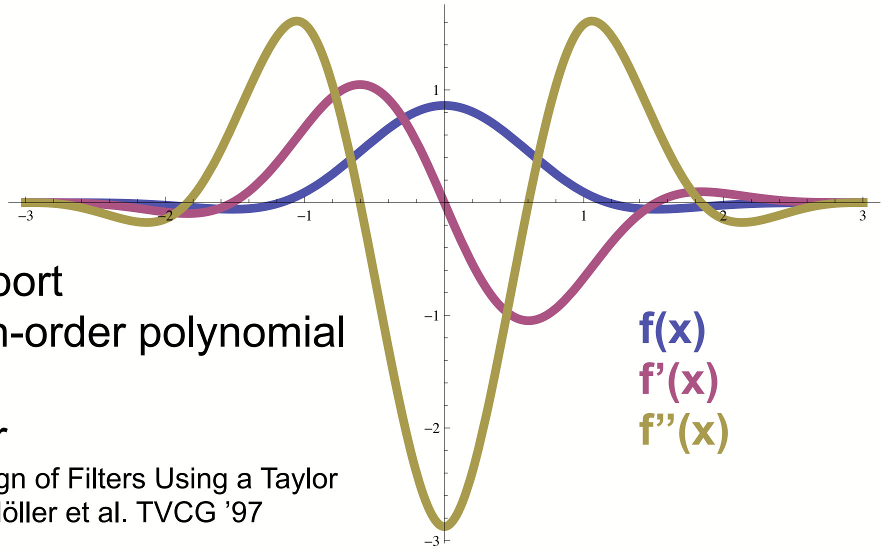
- Minimize number of pre-blurring volumes
- Measure error as squared difference between interpolated K[] and true K[], summed over support
- Optimize non-uniform scale sample locations by gradient descent on error



Hermite-spline scale interpolation makes scale-space practical for real-world 3D volumes

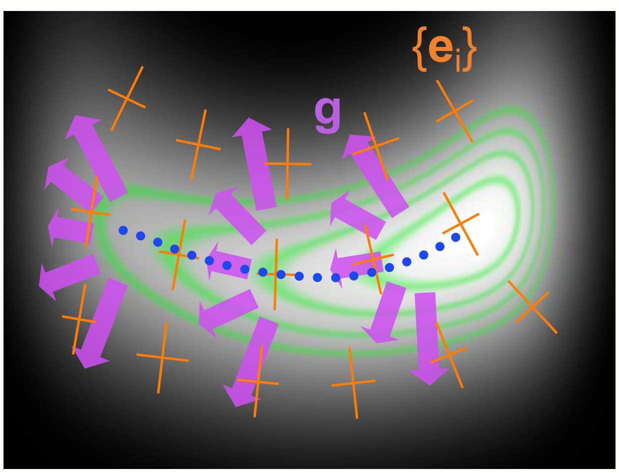
Spatial Interpolation

- At each particle location:
  - Scale interpolation → discrete spatial support
  - Separable convolution → values and derivatives



6-sample support
Piece-wise 6th-order polynomial
C^4 continuity
4th-order error
"Evaluation and Design of Filters Using a Taylor Series Expansion," Möller et al. TVCG '97

Ridges & Valleys = Creases



"Ridges in Image and Data Analysis" Eberly '96

Constrained extremum

Gradient **g**

Hessian eigensystem $\mathbf{e}_i, \lambda_i$

Crease: **g** orthogonal to one or more $\mathbf{e}_i$

Eigenvalue gives **strength**

Ridge surface: $\mathbf{g} \cdot \mathbf{e}_3 = 0$; $\lambda_3 <$ thresh

Ridge line: $\mathbf{g} \cdot \mathbf{e}_3 = \mathbf{g} \cdot \mathbf{e}_2 = 0$; $\lambda_3, \lambda_2 <$ thresh

Valley surface: $\mathbf{g} \cdot \mathbf{e}_1 = 0$; $\lambda_1 >$ thresh

- Particles are **spatially** constrained to creases

Particle-Image Energy E_i

	RL	RS	VL	VS
Definition	$\mathbf{g}\cdot\mathbf{v}_2=0$ $\mathbf{g}\cdot\mathbf{v}_3=0$	$\mathbf{g}\cdot\mathbf{v}_3=0$	$\mathbf{g}\cdot\mathbf{v}_1=0$ $\mathbf{g}\cdot\mathbf{v}_2=0$	$\mathbf{g}\cdot\mathbf{v}_1=0$
λ sign	$\lambda_3 \le \lambda_2 < 0$	$\lambda_3 < 0$	$0 < \lambda_2 \le \lambda_1$	$0 < \lambda_1$
Strength h	$-\tilde{\lambda}_2$	$-\tilde{\lambda}_3$	$\tilde{\lambda}_2$	$\tilde{\lambda}_1$
Tangent $\mathbf{T}$	$\mathbf{v}_1\otimes\mathbf{v}_1$	$\mathbf{v}_1\otimes\mathbf{v}_1$ $+\mathbf{v}_2\otimes\mathbf{v}_2$	$\mathbf{v}_3\otimes\mathbf{v}_3$	$\mathbf{v}_2\otimes\mathbf{v}_2$ $+\mathbf{v}_3\otimes\mathbf{v}_3$

Ridge (R) and valley (V) surfaces (S) and lines (L)

$$E_i = e(f, \mathbf{x}_i, s_i) = -\gamma h(\mathbf{x}_i, s_i)$$

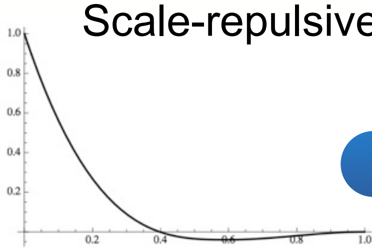
- Particles migrate to maximal feature strength as part of energy minimization (not a constriant)

Inter-particle Energy E_{ij}

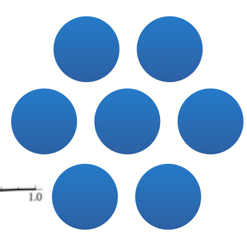
$$E_{ij} = \Phi(r_{ij}, s_{ij}) = \Phi\left(\frac{|\mathbf{x}_i - \mathbf{x}_j|}{\sigma_r}, \frac{s_i - s_j}{\sigma_s}\right)$$

- No intrinsic orientation to particles' potential
- Have user-set "radii" in space σ_r and scale σ_s

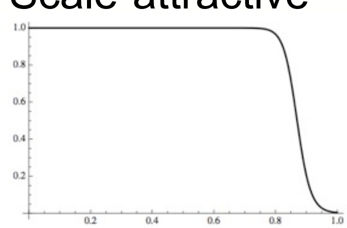
Scale-repulsive



$\phi_c(r)$



Scale-attractive

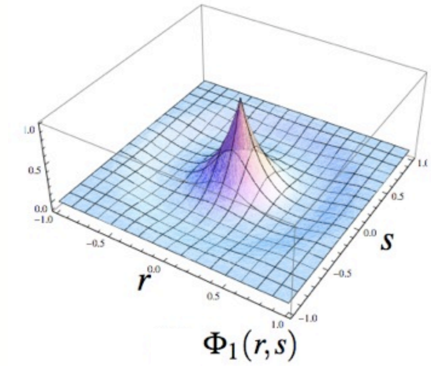


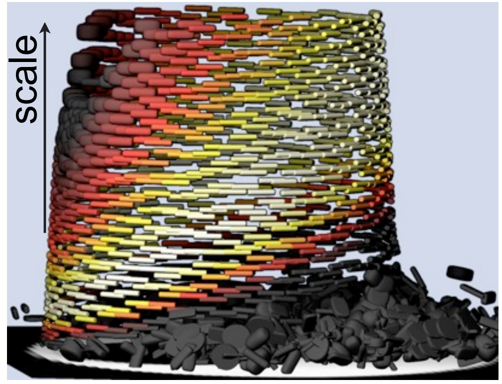
$b(\omega)$

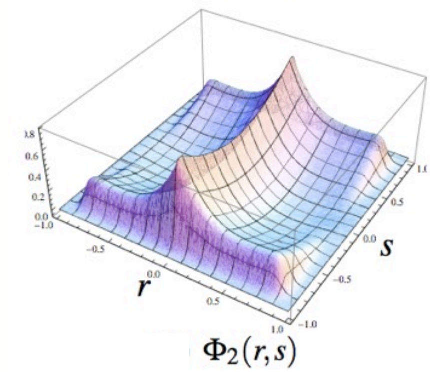
$$\Phi_1(r,s) = \phi_c(\sqrt{r^2+s^2})$$

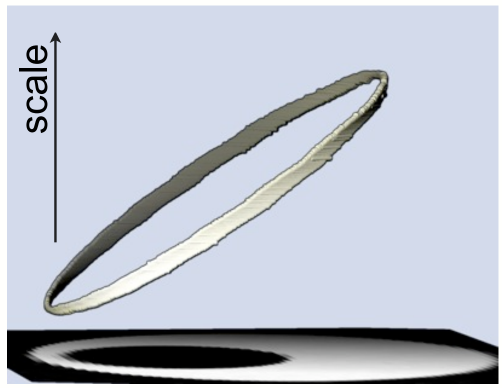
$$\Phi_2(r,s) = (1-\beta)\phi_c(r)b(s) + \beta b(r)b(s)s^2$$

Inter-particle Energy E_{ij}





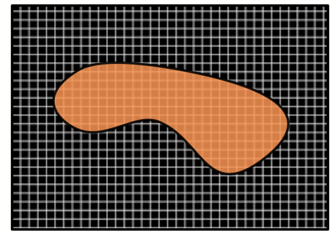


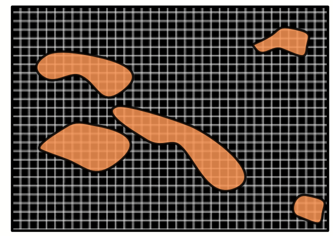


System Visualization

- Glyphs at particle locations, possibly colormapped
- Glyphs show tensors related to local Hessian
  - crease surfaces → discs, lines → rods
- Or, encode scale instead of Hessian eigenvalues

System Computation





- Initialize with particle at every Nth voxel
  - "CPM: A Deformable Model for Shape Recovery and Segmentation Based on Charged Particles" Jalba et al. IEEE PAMI '04
  - Sampling one vs. detecting all
  - Currently bottleneck
- Every iteration decreases energy
  - Move particles, with spatial constraint
  - Periodically try adding or nixing particles
- "Connected components"
  - Connected if non-zero inter-particle energy

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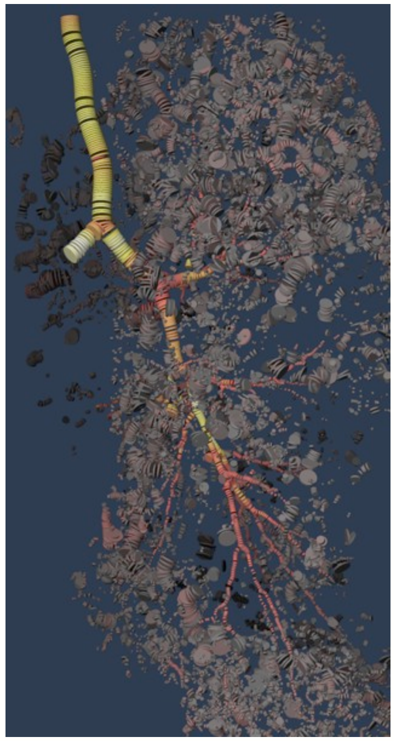
Lung CT Results

- Lung airway segmentation still not quite solved
- Particles captured 4-5 levels of branching, as well as size



All CCs

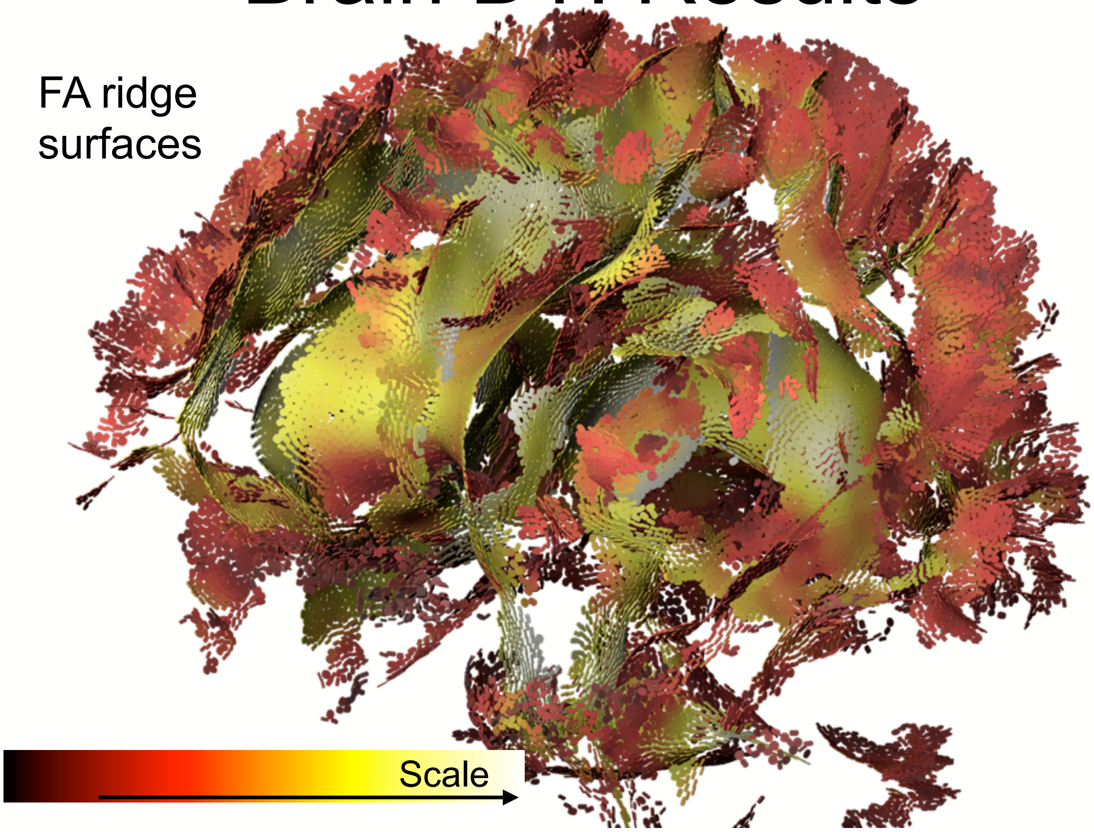
Biggest CC

Lung CT Results

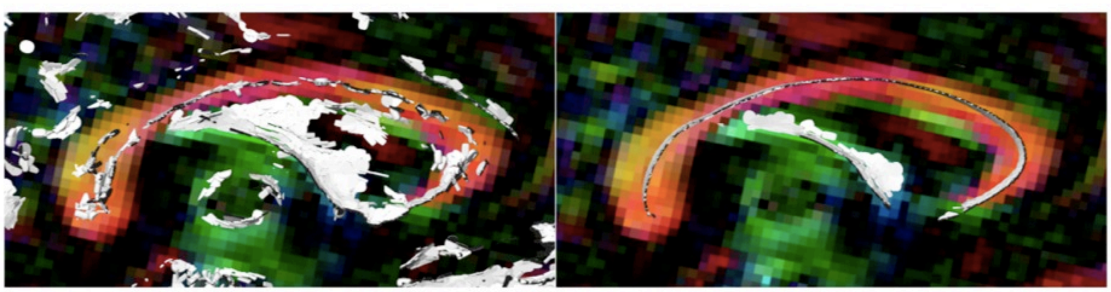
Smallest airways not much larger than voxels

Brain DTI Results

FA ridge surfaces

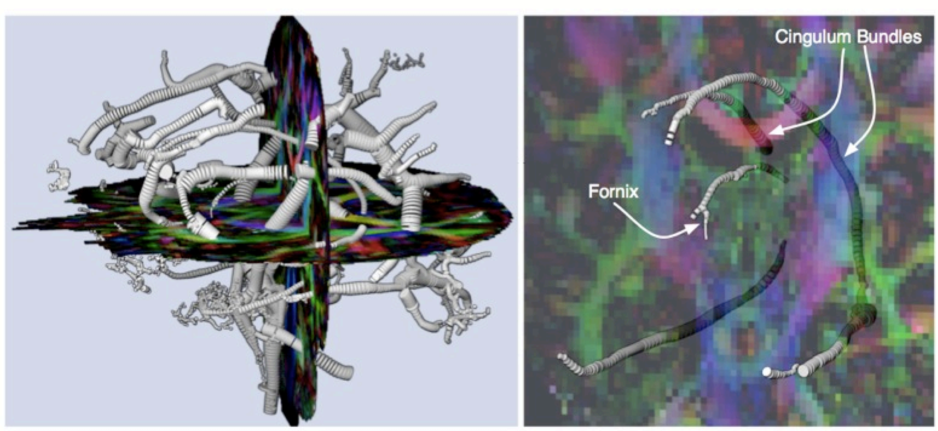


Brain DTI Results

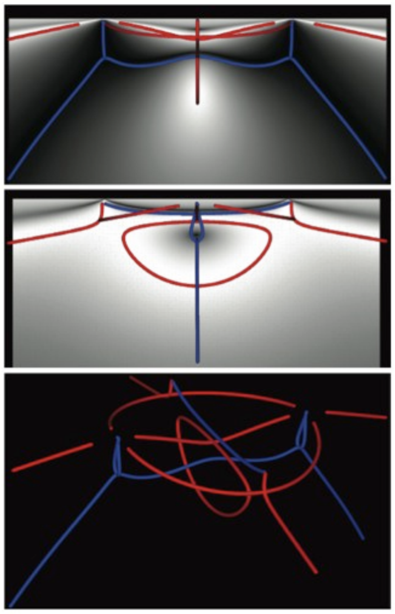


Without Scale-Space | With Scale-Space

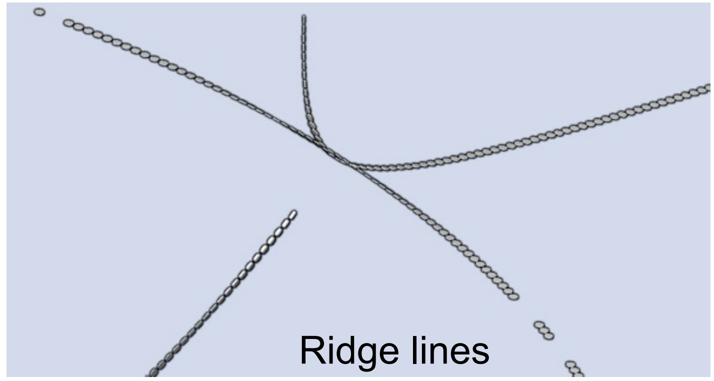
FA ridge lines

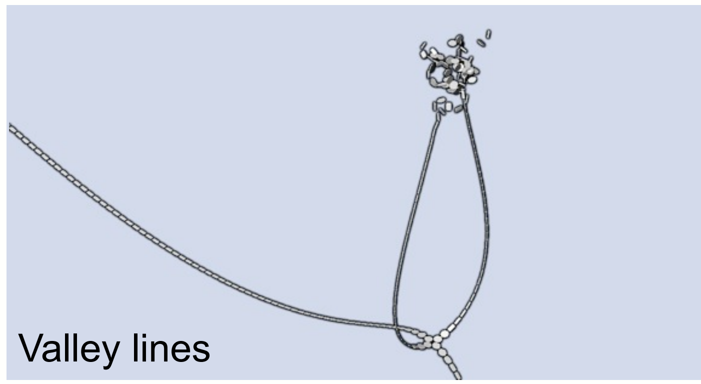


Double Point Load

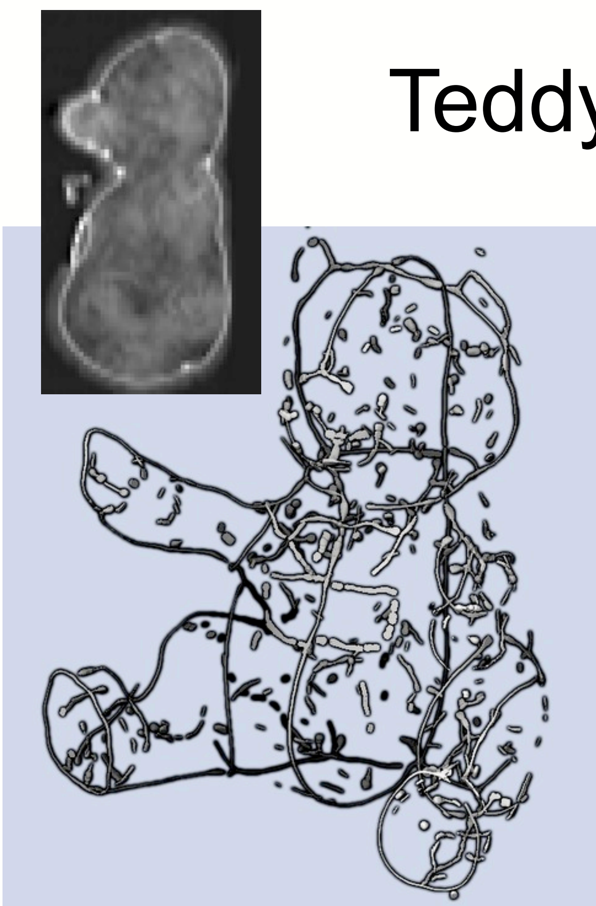


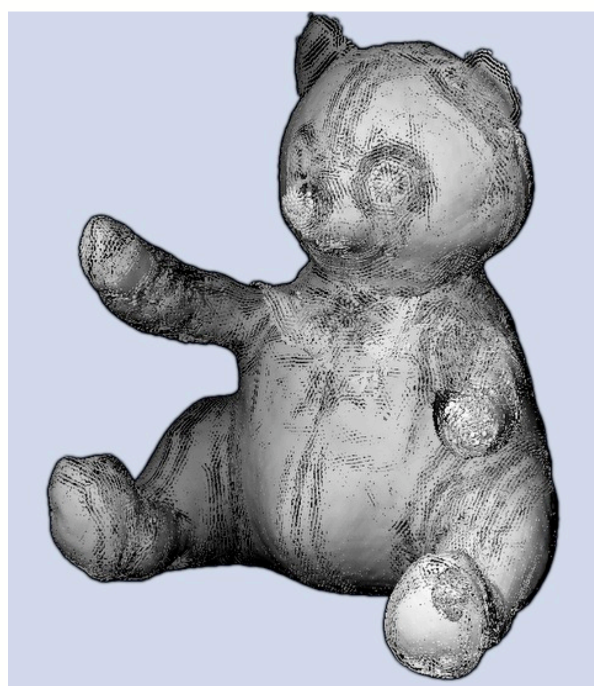
“Invariant Crease Lines for Topological and Structural Analysis of Tensor Fields” Tricoche et al. Vis ’08





Teddy Bear





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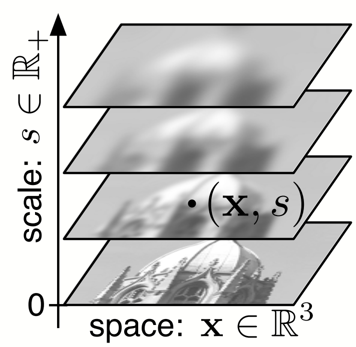
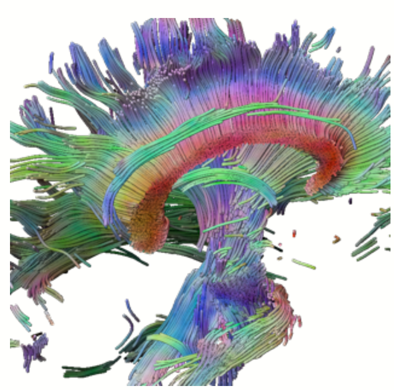
More Scale Space in Vis?

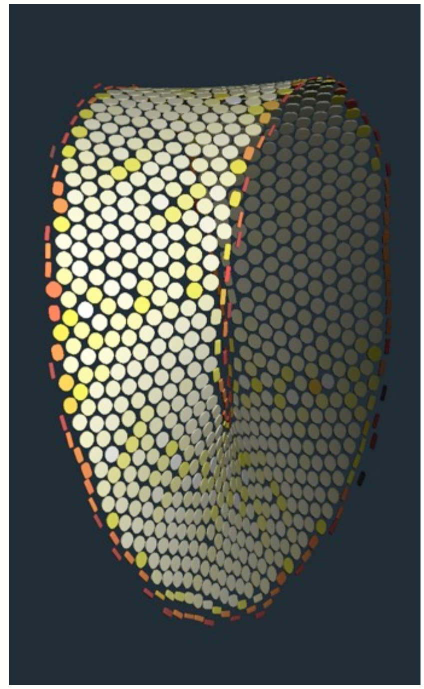
- Used some in visualization
  - “Surface Descriptors” session yesterday
  - Some feature tracking in flow fields
- Can always ask: but at what scale, and how stable WRT scale?





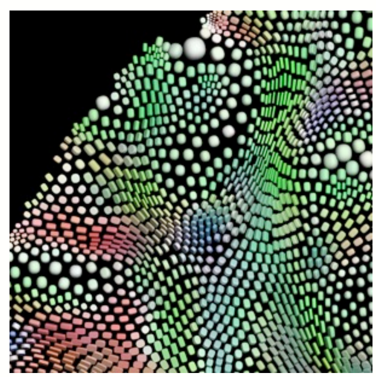
Joy of Particles

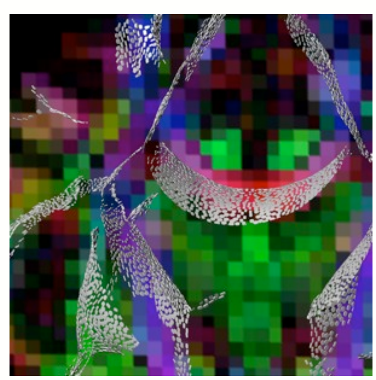
- Map from crease definition to particle motion
- Implement at level of individual particle behavior, watch group evolve
- Same system for curves or surfaces, with or without scale space



Visualization or Analysis?

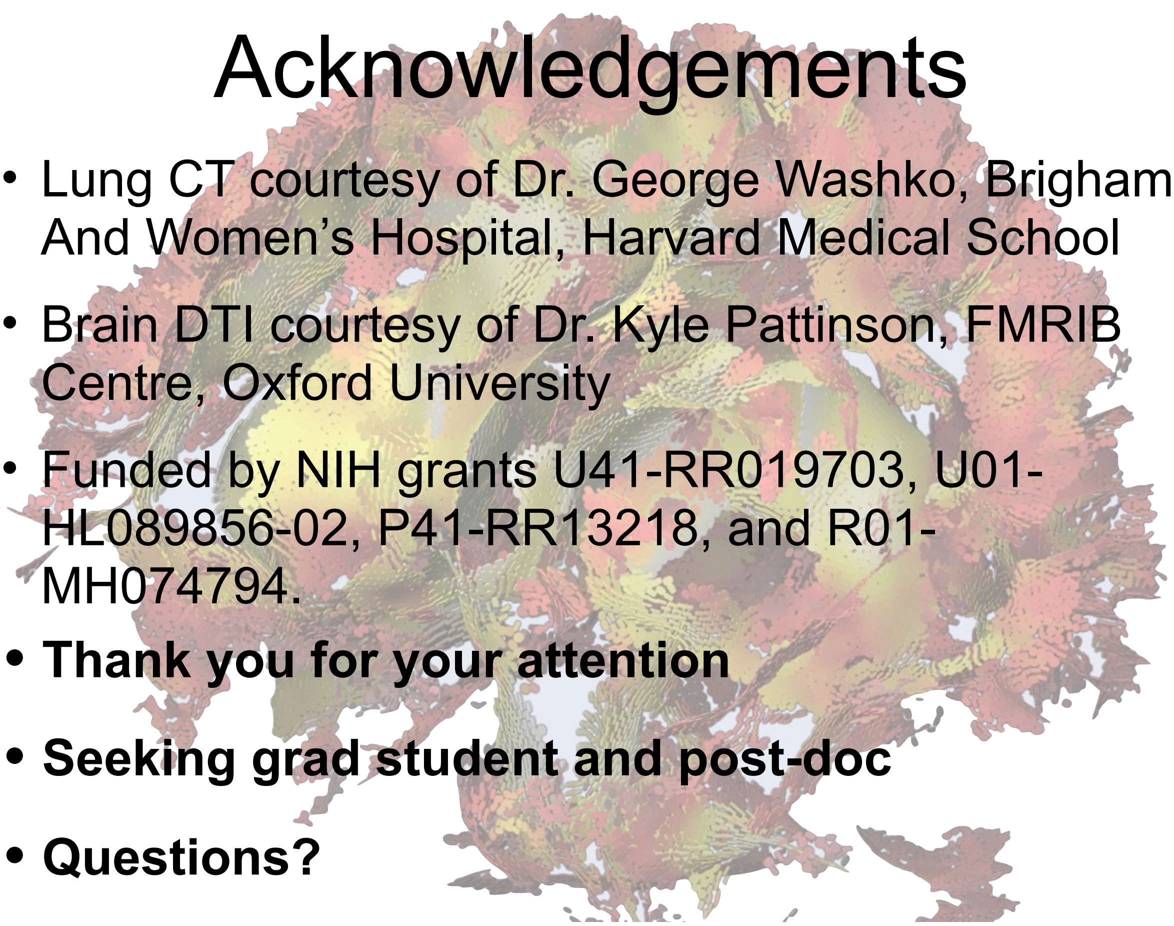
- Intersection of both
- Vis, by **methods** (particles, glyphs) and **modality** (CT, DT-MRI)
- Vis & Analysis, by **strategy**: inspecting computation, feature detection, optimizing sampling
- Analysis, by **goals**: quantitative studies of local properties, shape variation, registration





Future work

- Other kinds of features: Canny edges
- Varying sampling density (curvature, local size)
- More parameter automation (did scale)
- Meshing, where needed
- GPU-based optimization



Acknowledgements

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- Funded by NIH grants U41-RR019703, U01-HL089856-02, P41-RR13218, and R01-MH074794.
- **Thank you for your attention**
- **Seeking grad student and post-doc**
- **Questions?**