Anisotropy Creases And Extremal Surfaces

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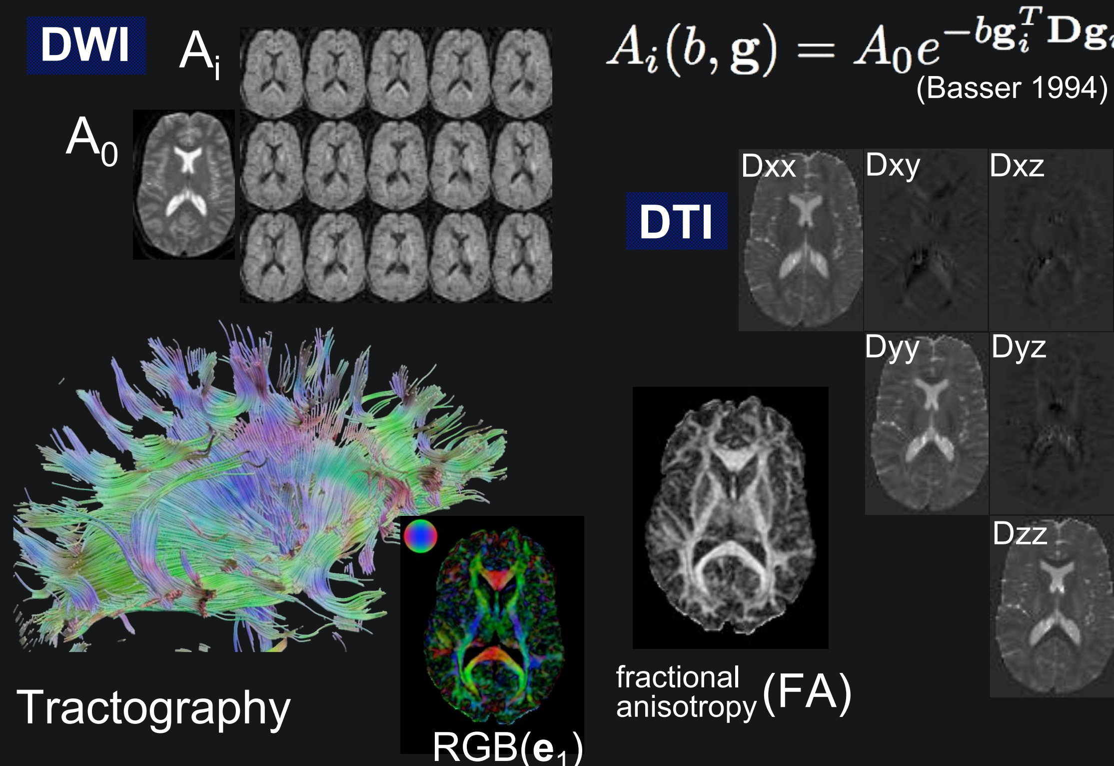
Diffusion weighted imaging, tensor imaging

$$A_i(b, \mathbf{g}) = A_0 e^{-b\mathbf{g}_i^T \mathbf{D} \mathbf{g}_i}$$

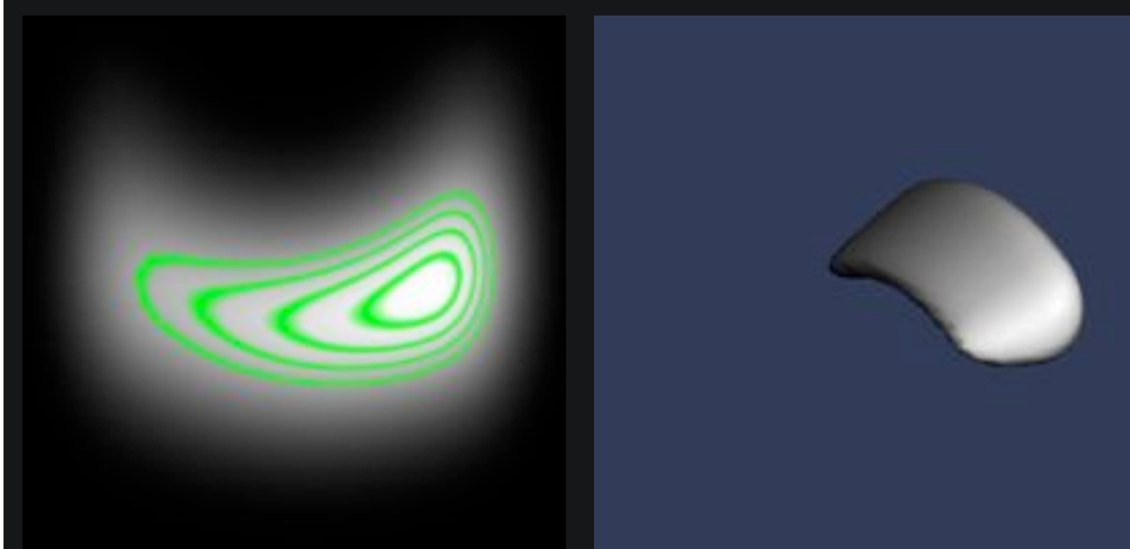
(Basser 1994)



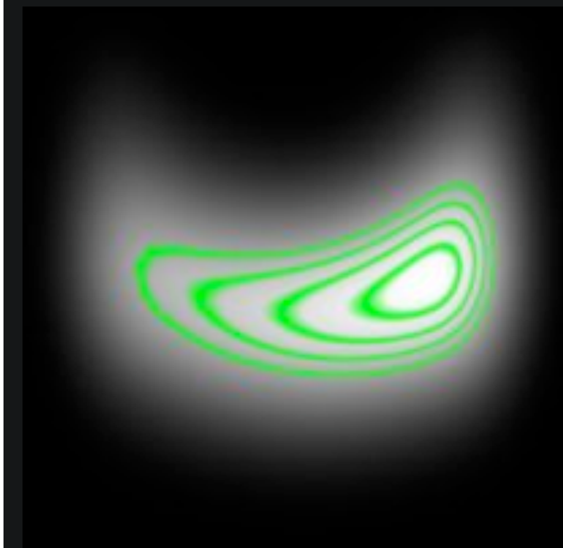
Is tractography/clustering the only way to get geometric models out of DTI?

Why not something global, like an isosurface?

Isosurfaces show global structure …

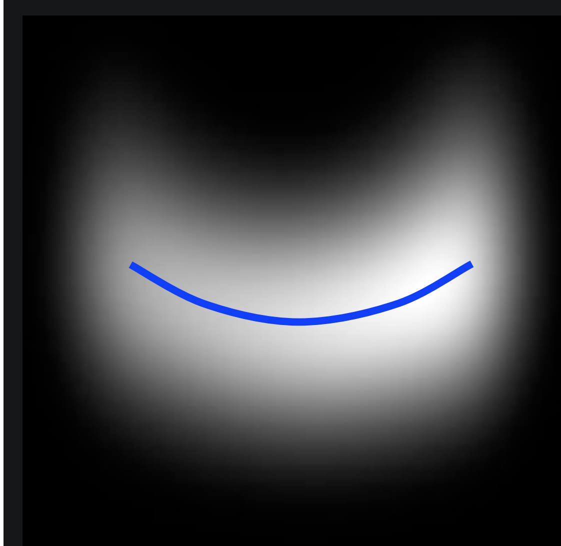


Isosurfaces show global structure …



…. but don’t always show salient structure

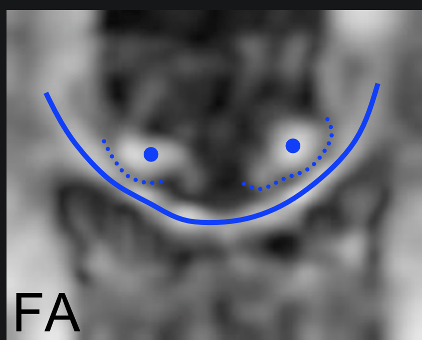
Creases!



Creases (ridges and valleys) do capture salient structure

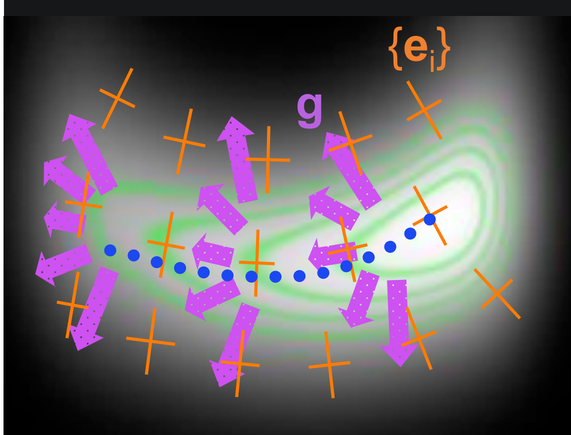
Creases for DTI?

- Goal: model large-scale white matter structures
  - Robust + Repeatable
  - Few or Zero Parameters
  - “Sulci for white matter”



- Basic Idea: Creases of FA
  - Ridges: “cores”
  - Valleys: interfaces
  - Shape, not connectivity

Crease feature definition (Eberly 1994)



Constrained extremum

Gradient $\mathbf{g}$

Hessian eigensystem $\mathbf{e}_i$, λ_i

Crease: $\mathbf{g}$ orthogonal to one or more $\mathbf{e}_i$

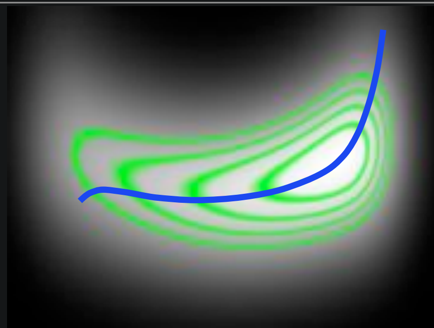
Eigenvalue gives strength

Ridge surface: $\mathbf{g} \cdot \mathbf{e}_3 = 0$; $\lambda_3 <$ thresh

Ridge line: $\mathbf{g} \cdot \mathbf{e}_3 = \mathbf{g} \cdot \mathbf{e}_2 = 0$; $\lambda_3, \lambda_2 <$ thresh

Valley surface: $\mathbf{g} \cdot \mathbf{e}_1 = 0$; $\lambda_1 >$ thresh

2-D Synthetic Scalar Example



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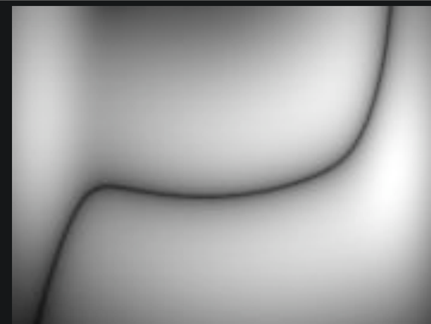
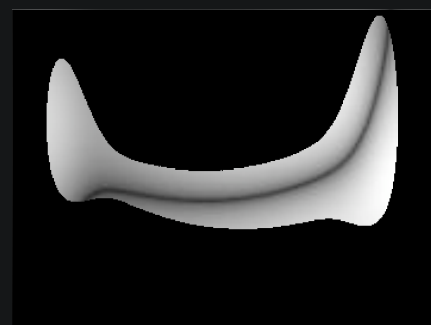


$|\mathbf{g} \cdot \mathbf{e}_3|$

strength: $\max(0,-\lambda_3)$



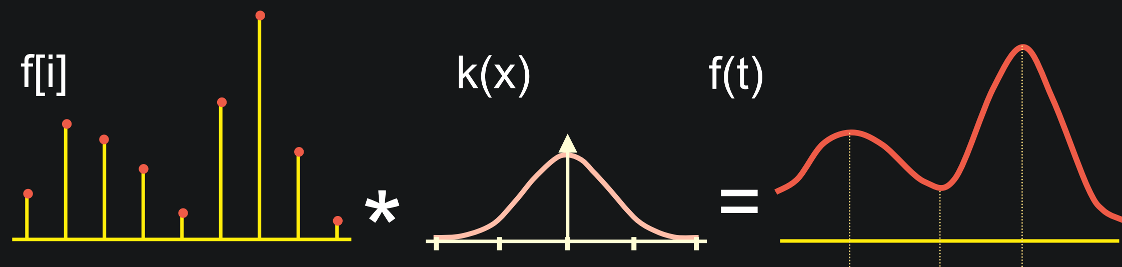
$-\lambda_3 >$ thresh

How do you measure derivatives

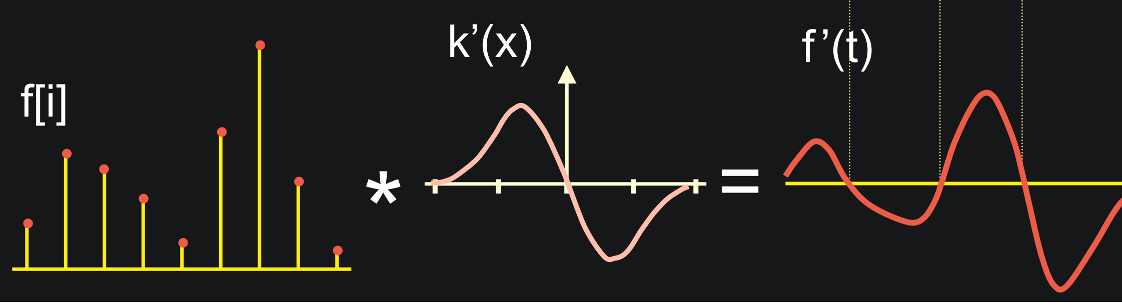
in the tensor field?

Measurement (of scalars) by convolution

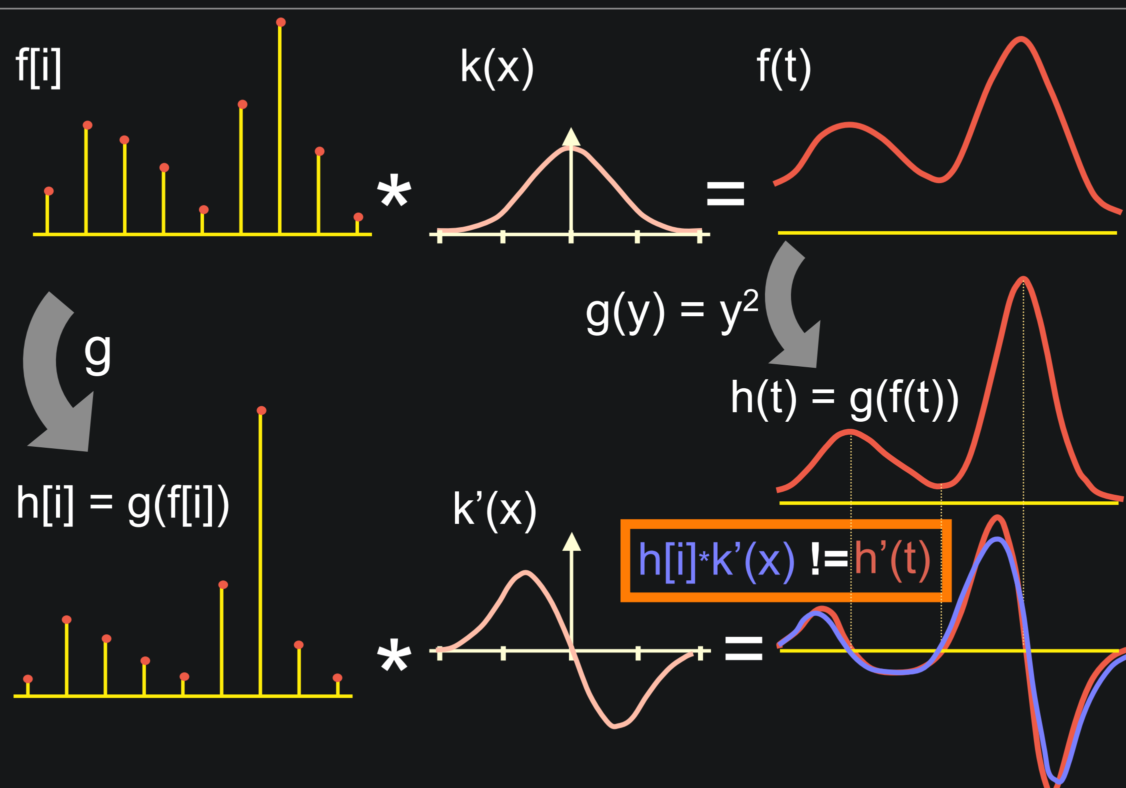
Continuous field: convolution of sampled coefficients with continuous reconstruction kernels

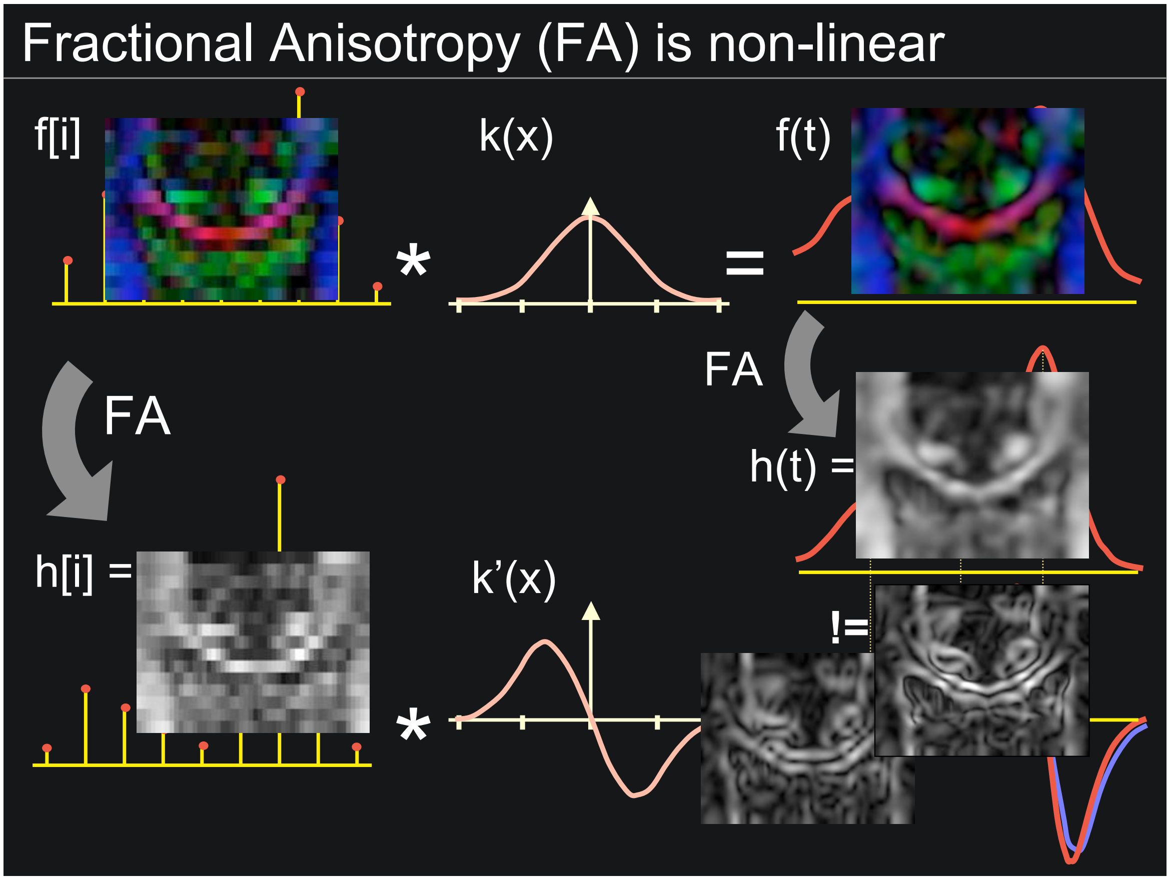


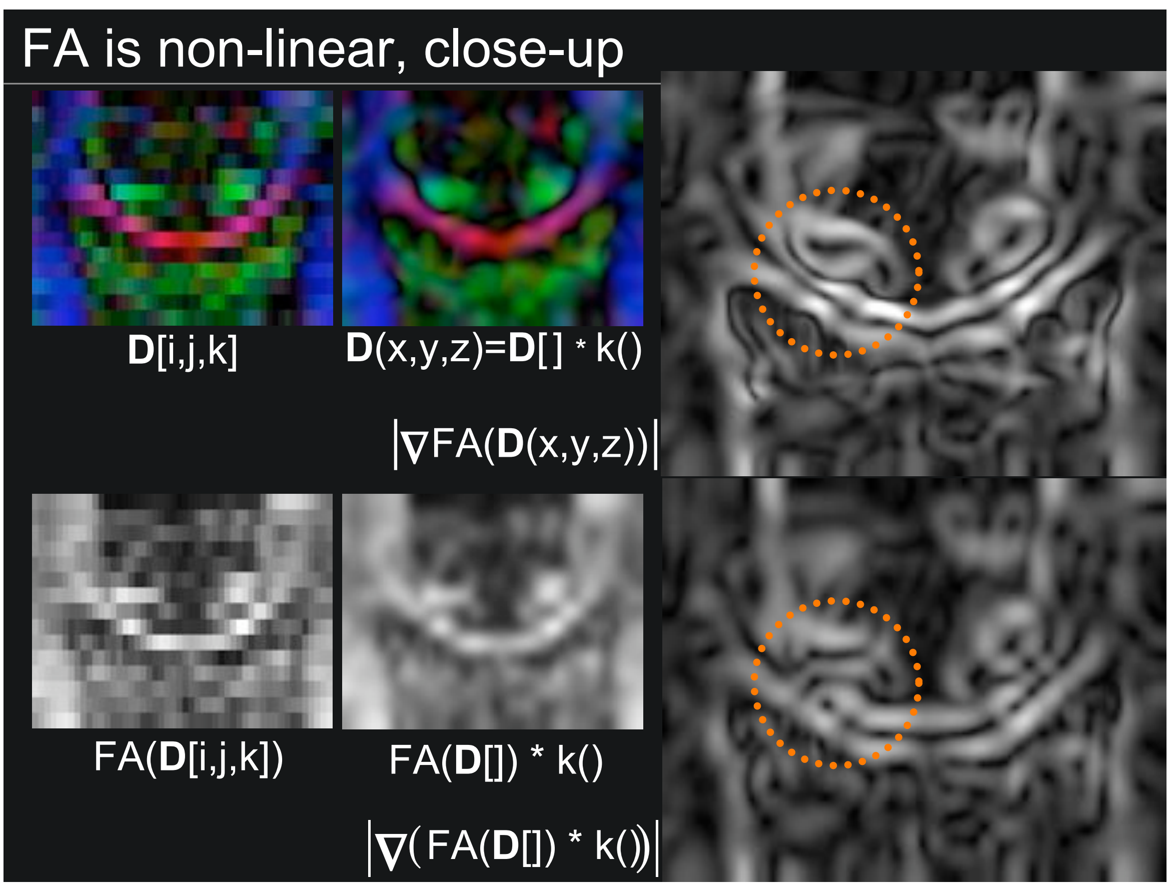
Differentiation: convolve w/ derivative of reconstruction kernel



Non-linear transform of data




Fractional Anisotropy (FA) is non-linear
f[i]
*
k(x)
=
f(t)
FA
FA
h(t) =
h[i] =
k'(x)
*
!=


FA is non-linear, close-up
D[i,j,k]
D(x,y,z)=D[] * k()
|∇FA(D(x,y,z))|
FA(D[i,j,k])
FA(D[]) * k()
|∇(FA(D[]) * k())|

FA from invariants, from coefficients

$$\mathrm{FA} \equiv \sqrt{\frac{3\, D{:}D}{2\, \mathbf{D}{:}\mathbf{D}}} \qquad D = \mathbf{D} - \langle \mathbf{D} \rangle \mathbf{I} \qquad \mathbf{D}{:}\mathbf{D} = \mathrm{tr}(\mathbf{D}\mathbf{D}^T)$$

$$J_2 = \begin{array}{c} D_{xx}D_{yy} + D_{xx}D_{zz} + D_{yy}D_{zz} \\ -D_{xy}^2 - D_{xz}^2 - D_{yz}^2 \end{array}$$

$$\mathrm{FA} = 3\sqrt{\frac{Q}{S}} \qquad Q = \frac{S - J_2}{9}$$

$$S = \mathbf{D}{:}\mathbf{D} = \begin{array}{c} D_{xx}^2 + D_{yy}^2 + D_{zz}^2 \\ +2D_{xy}^2 + 2D_{xz}^2 + 2D_{yz}^2 \end{array}$$

$$\nabla J_2 = \begin{array}{c} (D_{yy} + D_{zz})\nabla D_{xx} + (D_{xx} + D_{zz})\nabla D_{yy} + (D_{xx} + D_{yy})\nabla D_{zz} \\ -2D_{xy}\nabla D_{xy} - 2D_{xz}\nabla D_{xz} - 2D_{yz}\nabla D_{yz} \end{array}$$

$$\nabla Q = \frac{\nabla S - \nabla J_2}{9}$$

$$\nabla \mathrm{FA} = \frac{3}{2}\left(\sqrt{\frac{1}{SQ}}\nabla Q - \sqrt{\frac{Q}{S^3}}\nabla S\right)$$

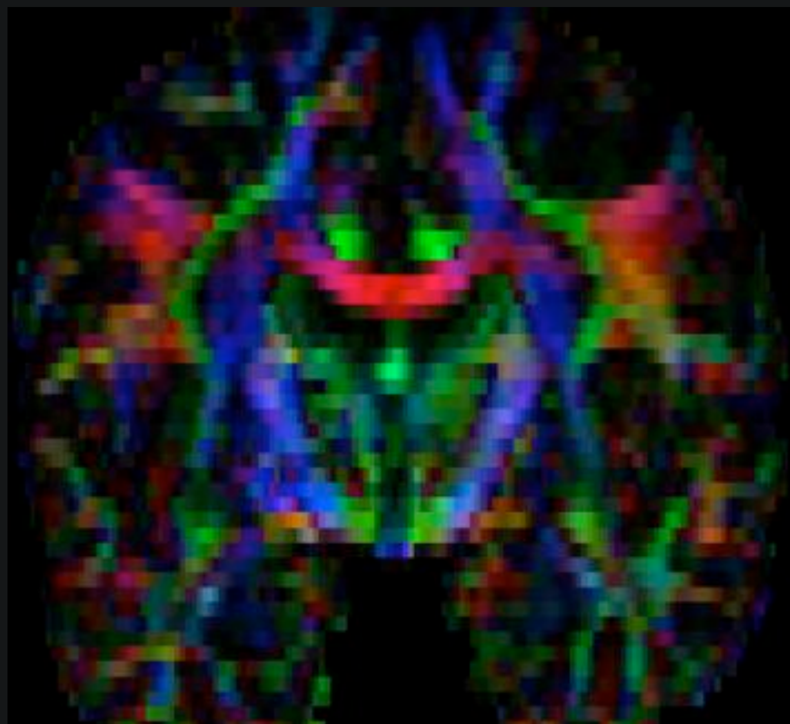
$$\nabla S = \begin{array}{c} 2D_{xx}\nabla D_{xx} + 2D_{yy}\nabla D_{yy} + 2D_{zz}\nabla D_{zz} \\ +4D_{xy}\nabla D_{xy} + 4D_{xz}\nabla D_{xz} + 4D_{yz}\nabla D_{yz} \end{array}$$

Hessian(FA) more complicated, but similarly derived

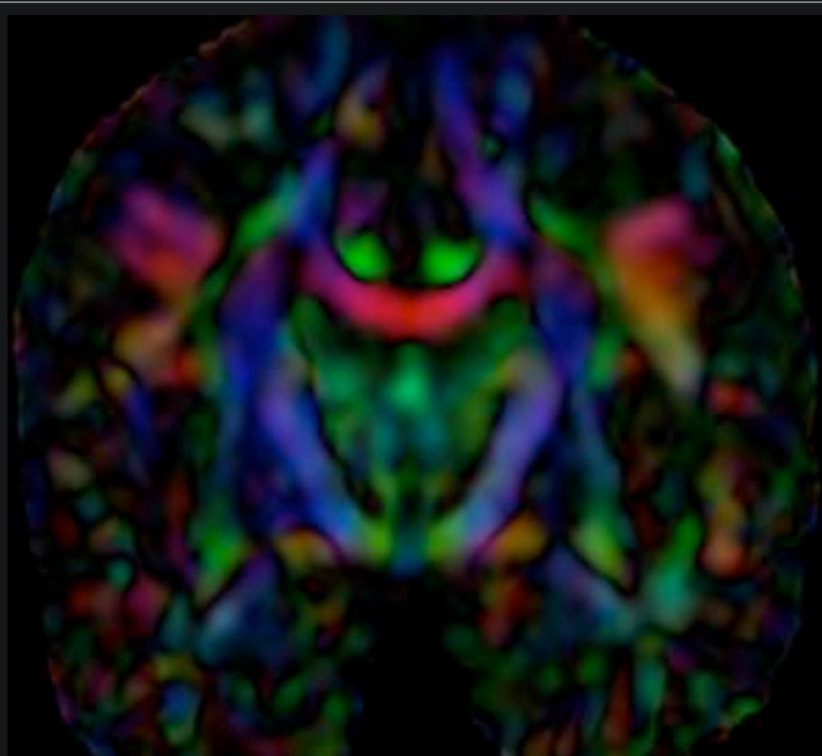
Cute math.

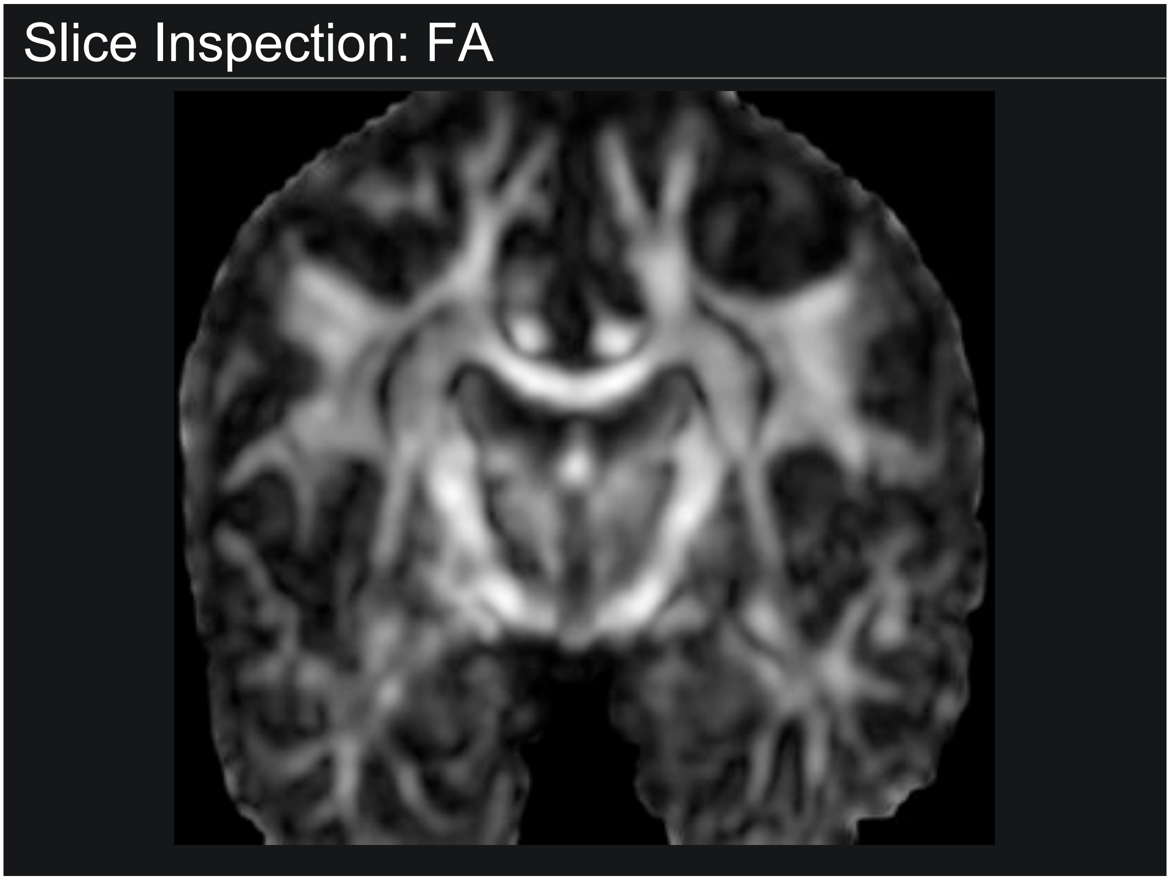
Does it work?

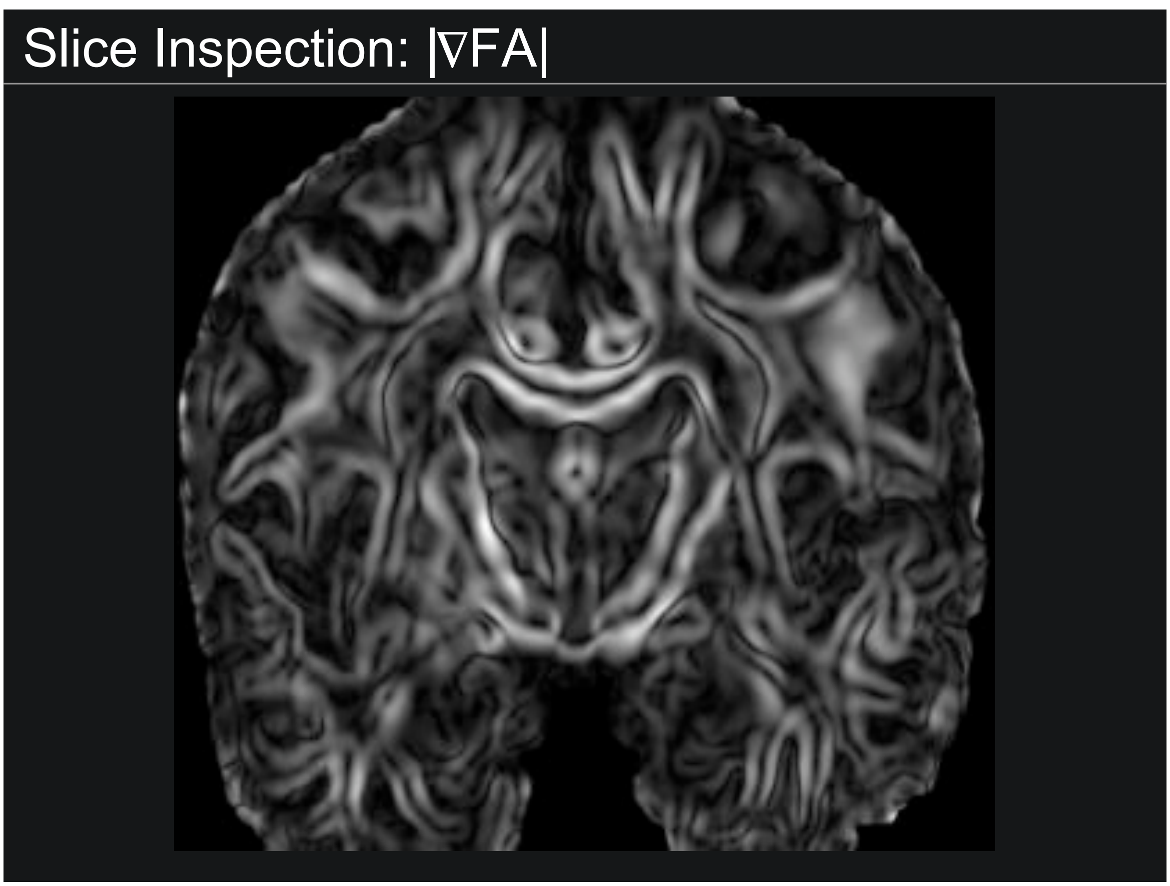
Slice Inspection: RGB($\mathbf{e}_1$) (original data)



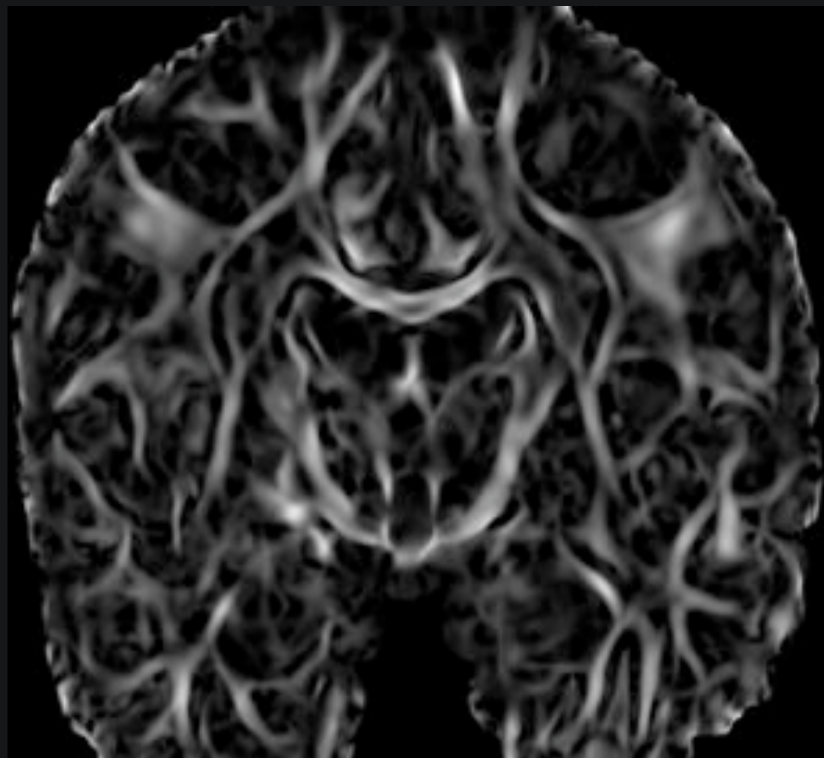
Slice Inspection: RGB($\mathbf{e}_1$)



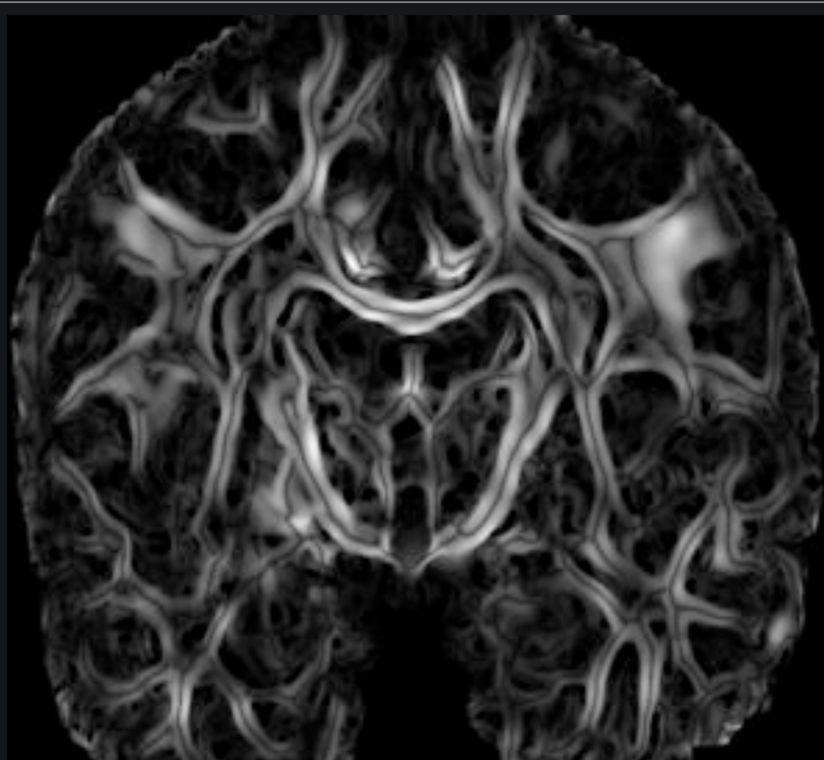

Slice Inspection: FA


Slice Inspection: |∇FA|

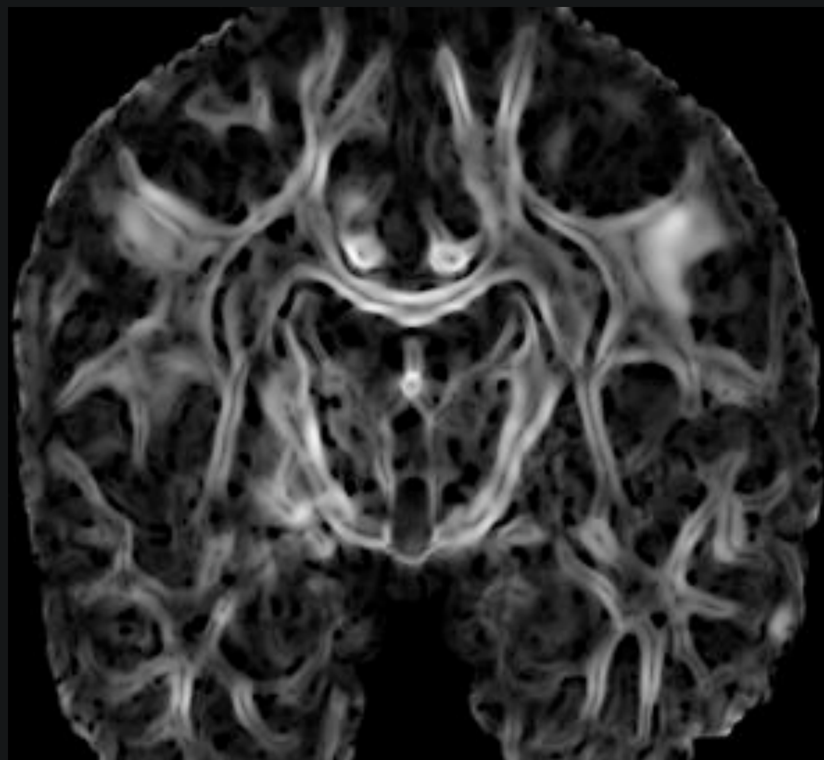
Slice Inspection: ridge strength: $\max(0,-\lambda_3)$



Slice Inspection: $|\mathbf{g}\cdot\mathbf{e}_3|$ (modulated by strength)



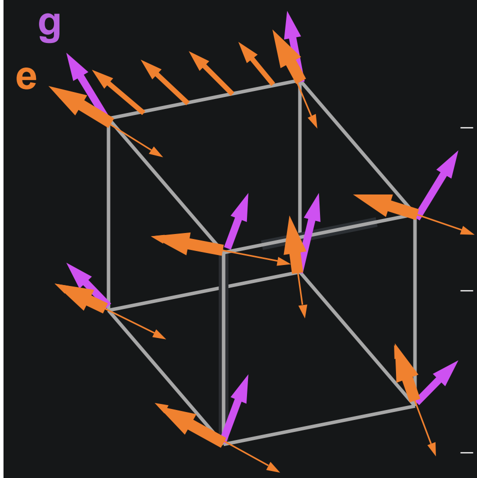
Slice Inspection: sqrt($(\mathbf{g}\cdot\mathbf{e}_3)^2 + (\mathbf{g}\cdot\mathbf{e}_2)^2$)



Cute pictures.

Where are the geometric models?

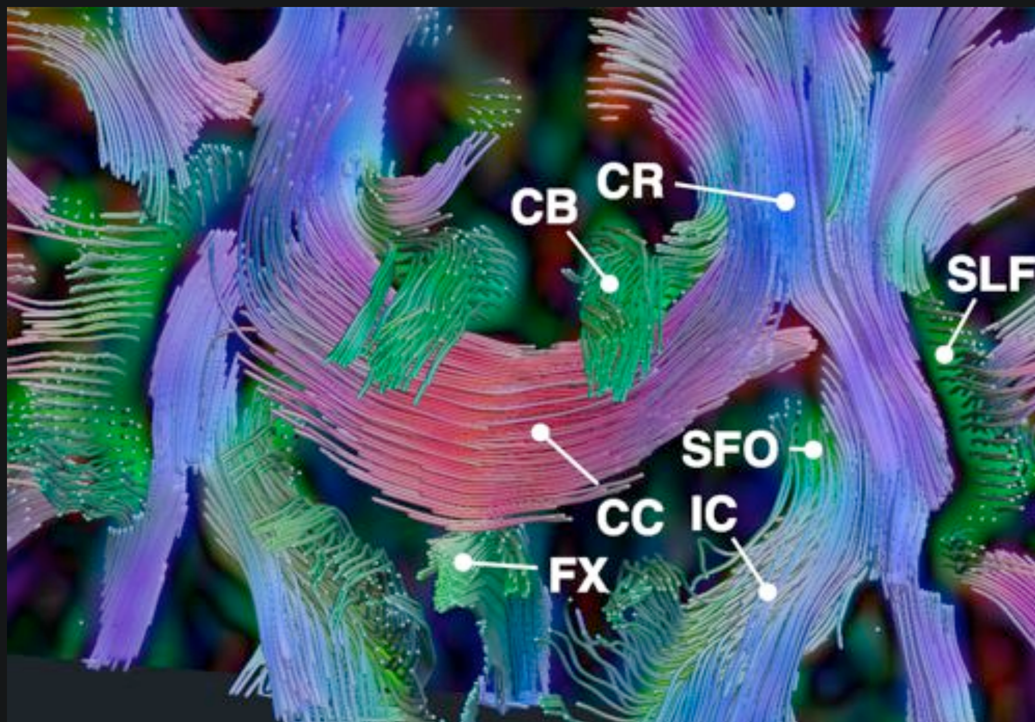
Modified Marching Cubes for Surfaces



- Crease surface is isosurface (zero-crossing) of **g**·**e**$_i$, but…
- Eigenvectors lack sign: enforce intra-voxel sign consistency
- Propagate eigenvector at one corner to all others
- **g**·**e** dot products, then MC case table

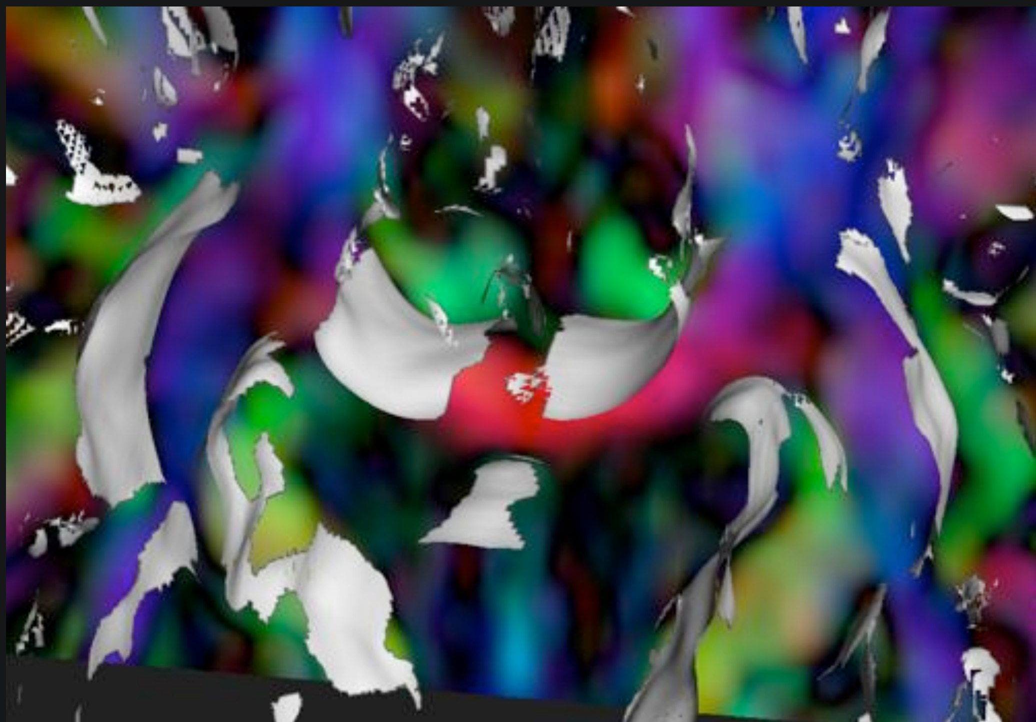
(torus demo)

3-D Results: coronal fibers



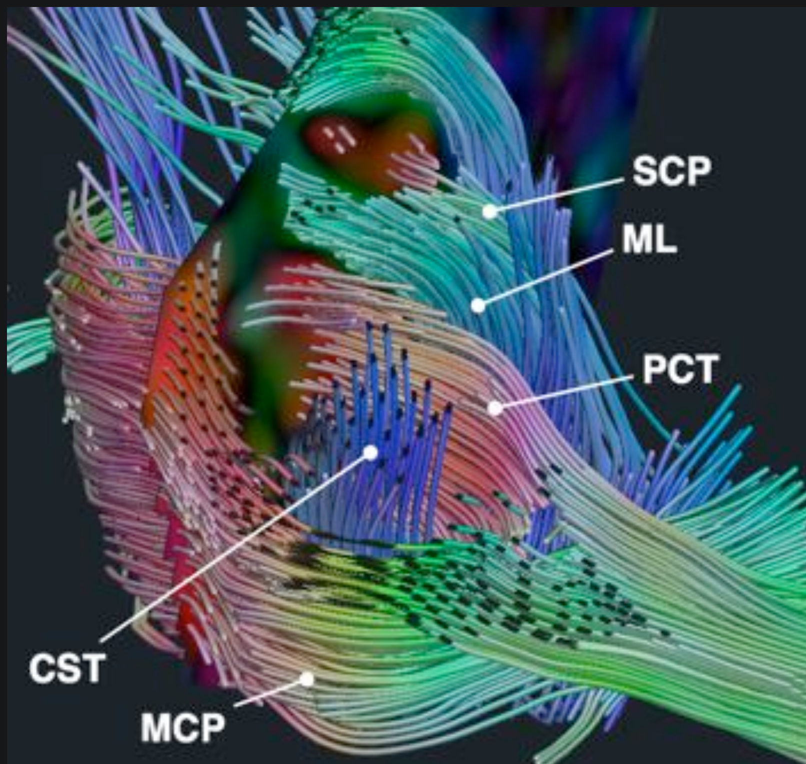
3-D Results: ridge surfaces

3-D Results: valley surfaces

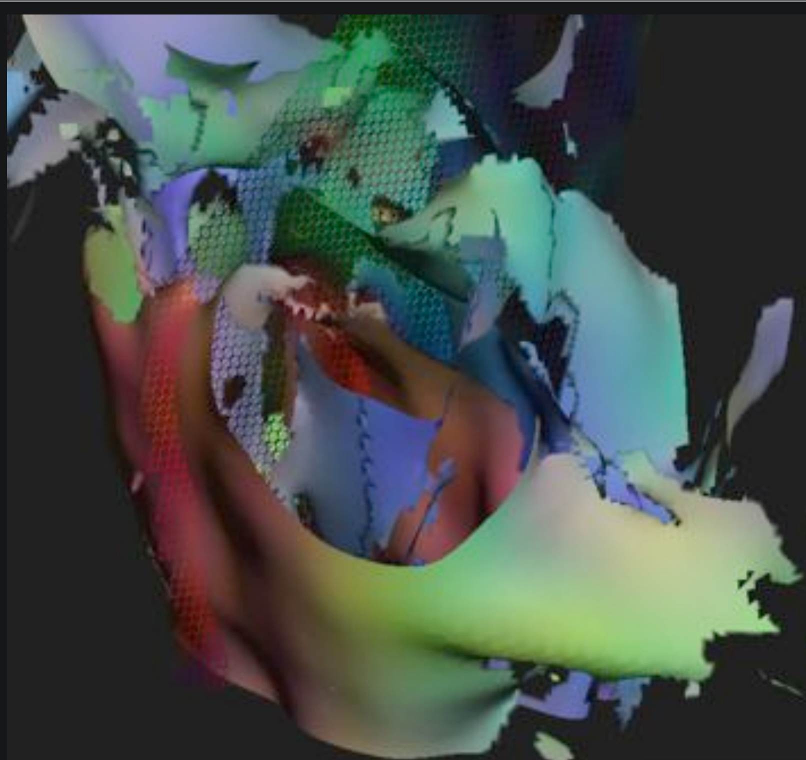


3-D Results: valley surfaces with fibers

3-D Results: brainstem fibers

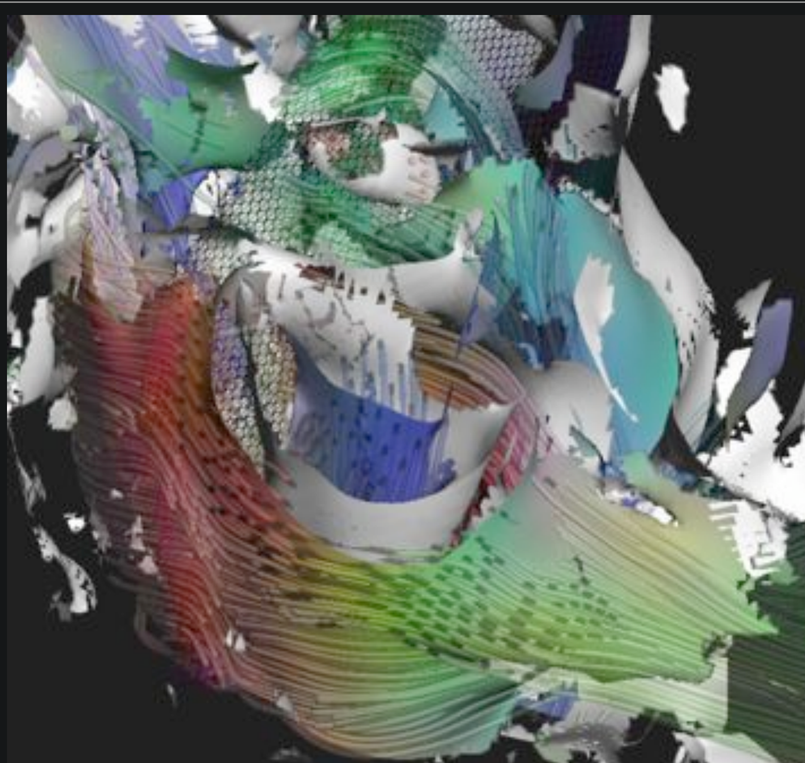


3-D Results: brainstem ridge surfaces



3-D Results: brainstem valley surfaces

3-D Results: combined results



(coronal slab brain demo)

Cute models.

What do they represent?

Extremal Surfaces (Amenta SIGGRAPH '04)

Generalization of MLS (moving least squares)

Implicit surface generation from scattered point set

Minimize distance to local planar weighted fit of points

Ingredients:

- Scalar function
- Line field

Ridge surfaces:

Line field is minor eigenvector of Hessian

Why not minor eigenvector of tensor field itself?

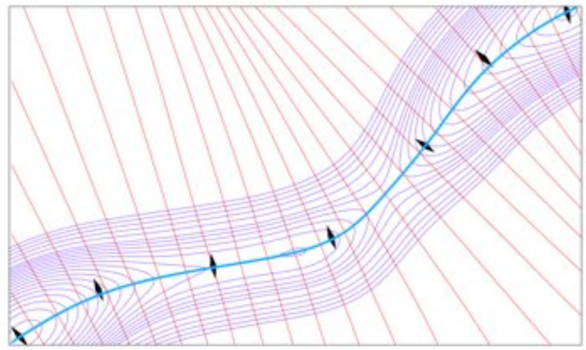


Figure 4: Streamlines (red) of a vector field $n(x)$, and iso-contours (blue) of an energy function $s(x)$. The heavy blue line is the extremal surface determined by n and s, running neatly along the "valley" in the energy landscape and passing through the minima of s. The streamlines of n and the iso-contours of s are tangent at the surface points. Here n and s were computed using the point-set surface for surfels introduced in Section 7; the input surfels are shown as black diamonds, with the long diagonal pointed in the direction of the surfel normal.

Discussion & Ongoing Work

- Novel Aspects:
  - Application of computer vision to DTI
  - Extracting geometry from differential DTI structure
- Scale space: <u>interfaces</u> are easier than "cores"
- Connected components!
- Crease line extraction; line vs. surface decision
- Evaluation on more datasets
- **So, what you do with this?**

Acknowledgements

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- URL for paper + software info: http://lmi.bwh.harvard.edu/~gk/miccai06/

thank you