Anisotropy Creases and Extremal Surfaces in Diffusion Tensor Images

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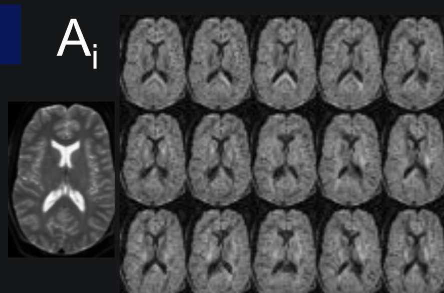
Diffusion weighted imaging, tensor imaging

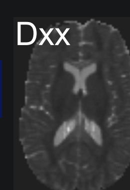
DWI A_i A_0



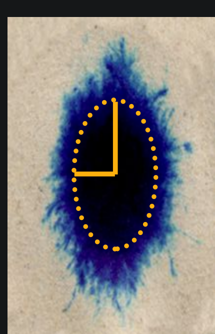
$$A_i(b, \mathbf{g}) = A_0 e^{-b\mathbf{g}_i^T \mathbf{D}\mathbf{g}_i}$$

(Basser 1994)

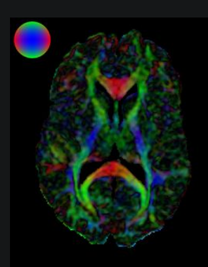
DTI



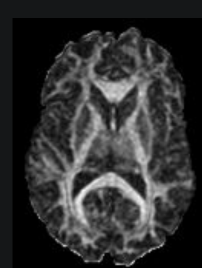
Kleenex



newspaper

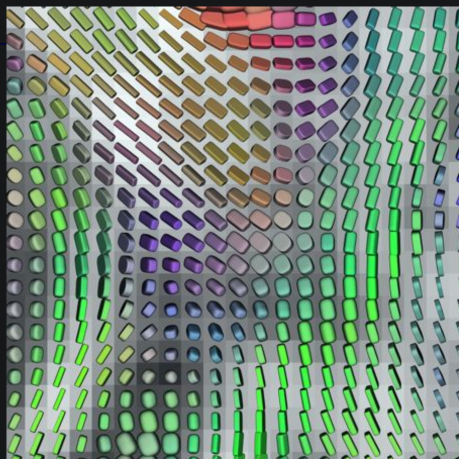


RGB($\mathbf{e}_1$)

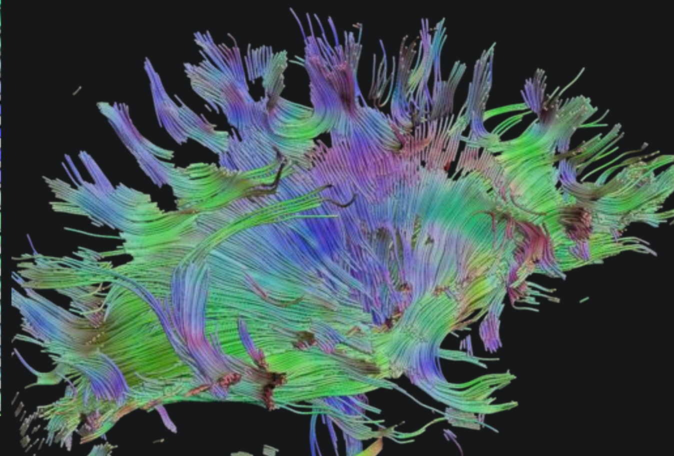


fractional anisotropy (FA)

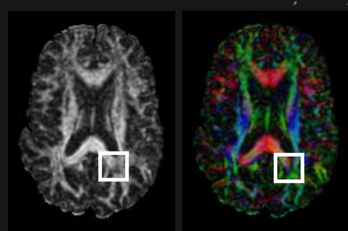
Standard visualization methods



Glyphs



Tractography,
Clustering

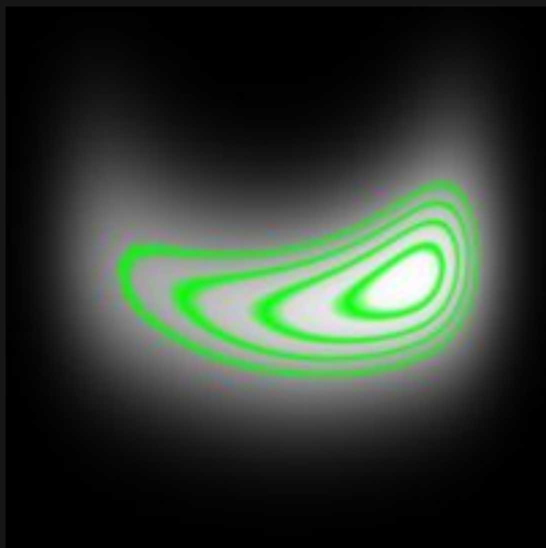


FA   RGB($\mathbf{e}_1$)

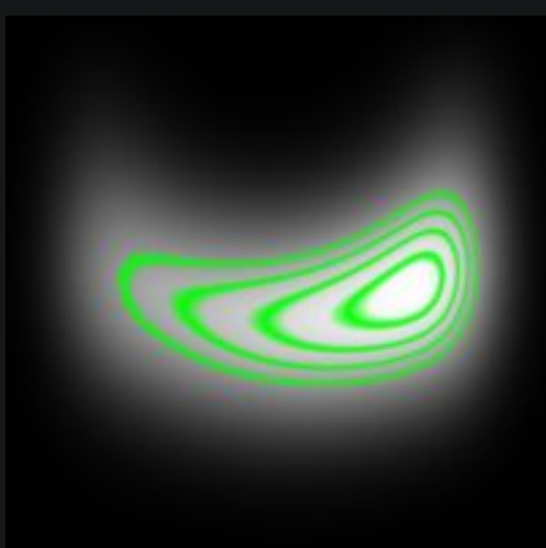
Is tractography/connectivity the only way to get useful geometric models out of DTI?

Why not something more global/structural?
An isosurface?

Isosurfaces show global structure …

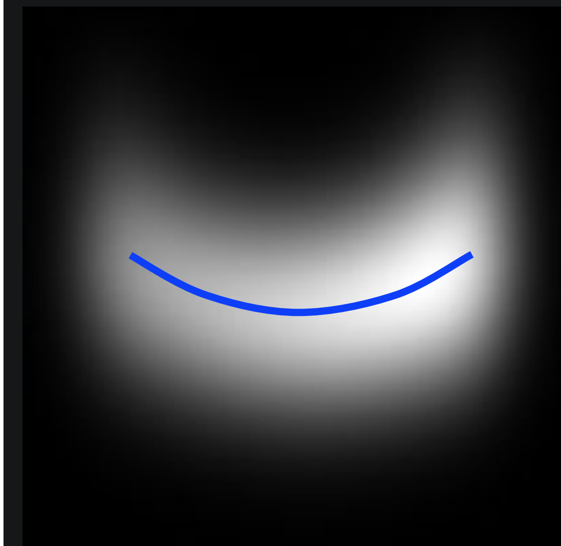


Isosurfaces show global structure …



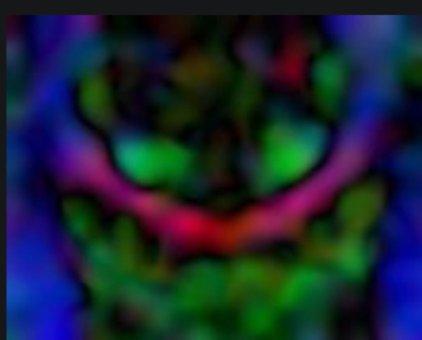
…. but don’t always show salient structure

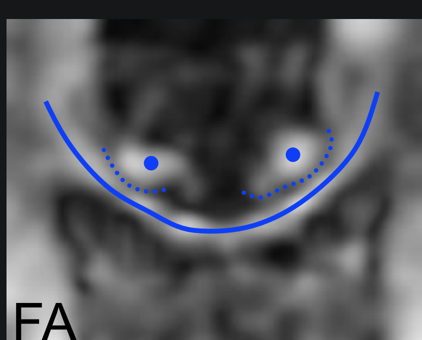
Creases!



Creases (ridges and valleys) do capture salient structure

Creases for DTI?



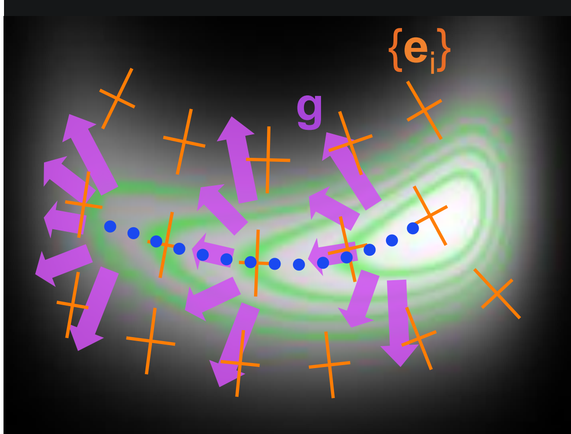


- Goal: model large-scale white matter structures
  - Robust + Repeatable
  - Few or Zero Parameters
  - “Sulci for white matter”
- Basic Idea: Creases of FA
  - Ridges: “cores”
  - Valleys: interfaces
  - Shape, not connectivity
  - “Tract-Based Spatial Statistics” Smith et al. NeuroImage ‘06

Rest of talk

- Mathematical definition of creases
- Measurement by convolution
- Analytical differentiation of FA
- Slice inspection (2-D results)
- Modified Marching Cubes for surface
- 3-D results
- Generalization: Extremal Surfaces

Crease feature definition (Eberly 1994)



Constrained extremum

Gradient **g**

Hessian eigensystem $\mathbf{e}_i$, λ_i

Crease: **g** orthogonal to one or more $\mathbf{e}_i$

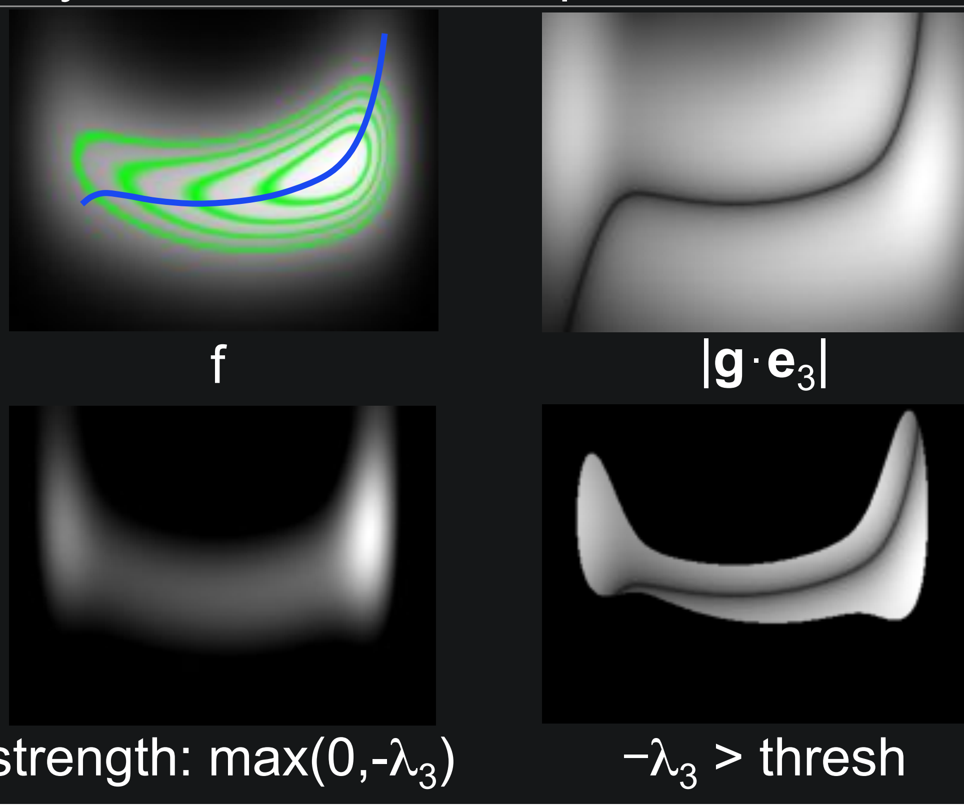
Eigenvalue gives strength

Ridge surface: $\mathbf{g} \cdot \mathbf{e}_3 = 0$; $\lambda_3 <$ thresh

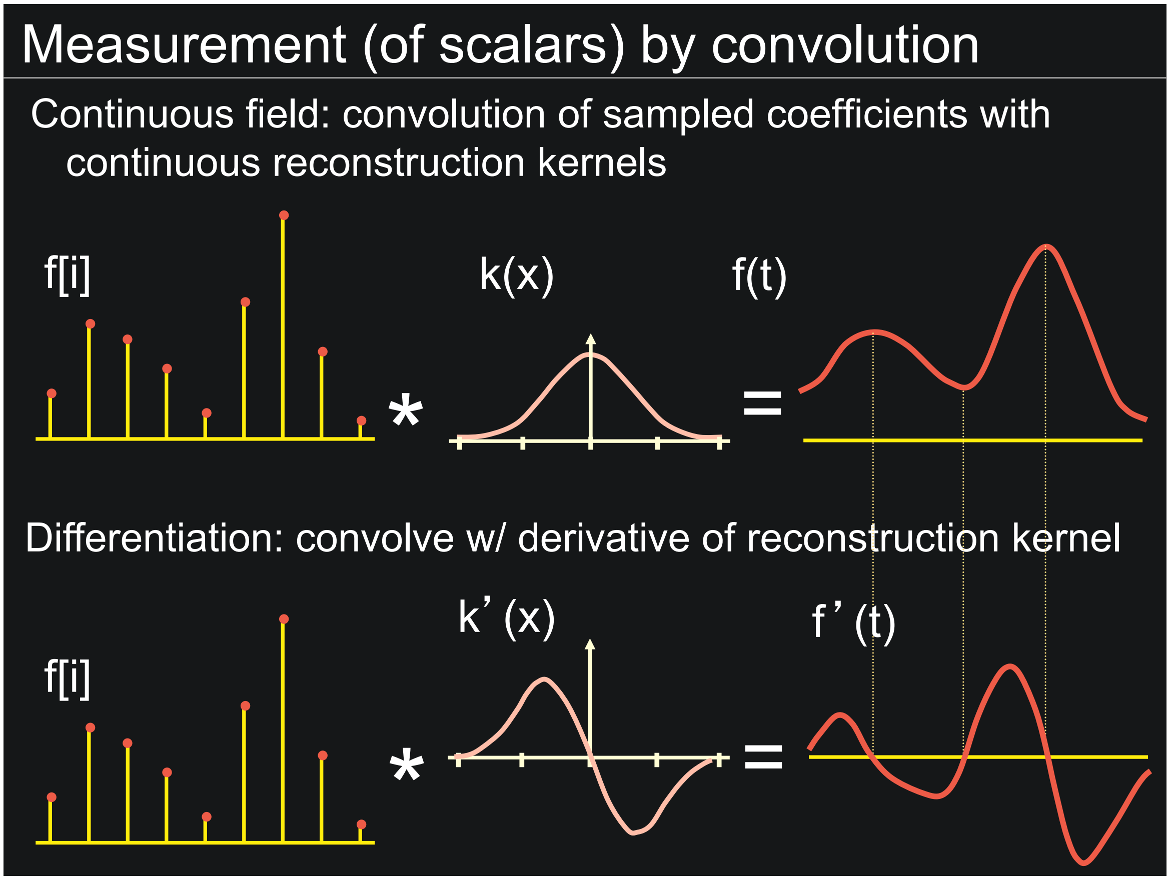
Ridge line: $\mathbf{g} \cdot \mathbf{e}_3 = \mathbf{g} \cdot \mathbf{e}_2 = 0$; $\lambda_3, \lambda_2 <$ thresh

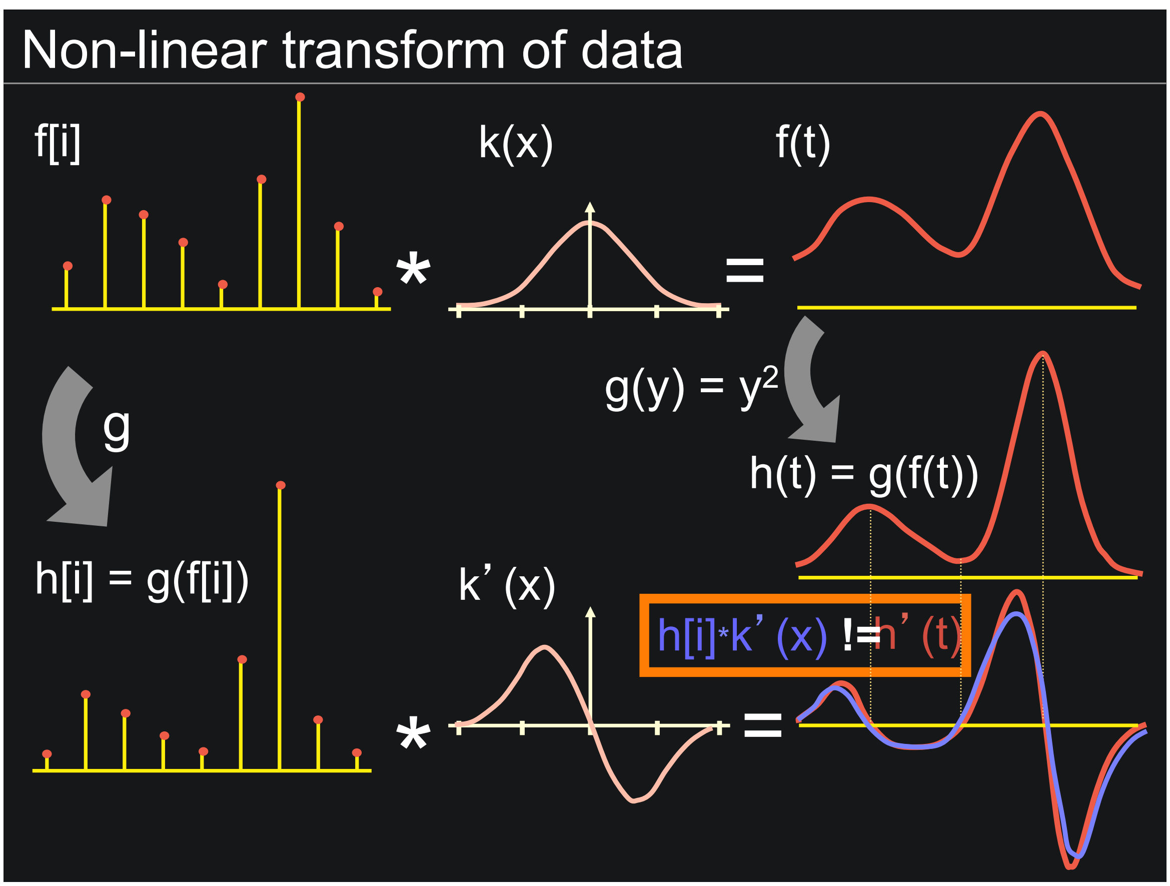
Valley surface: $\mathbf{g} \cdot \mathbf{e}_1 = 0$; $\lambda_1 >$ thresh

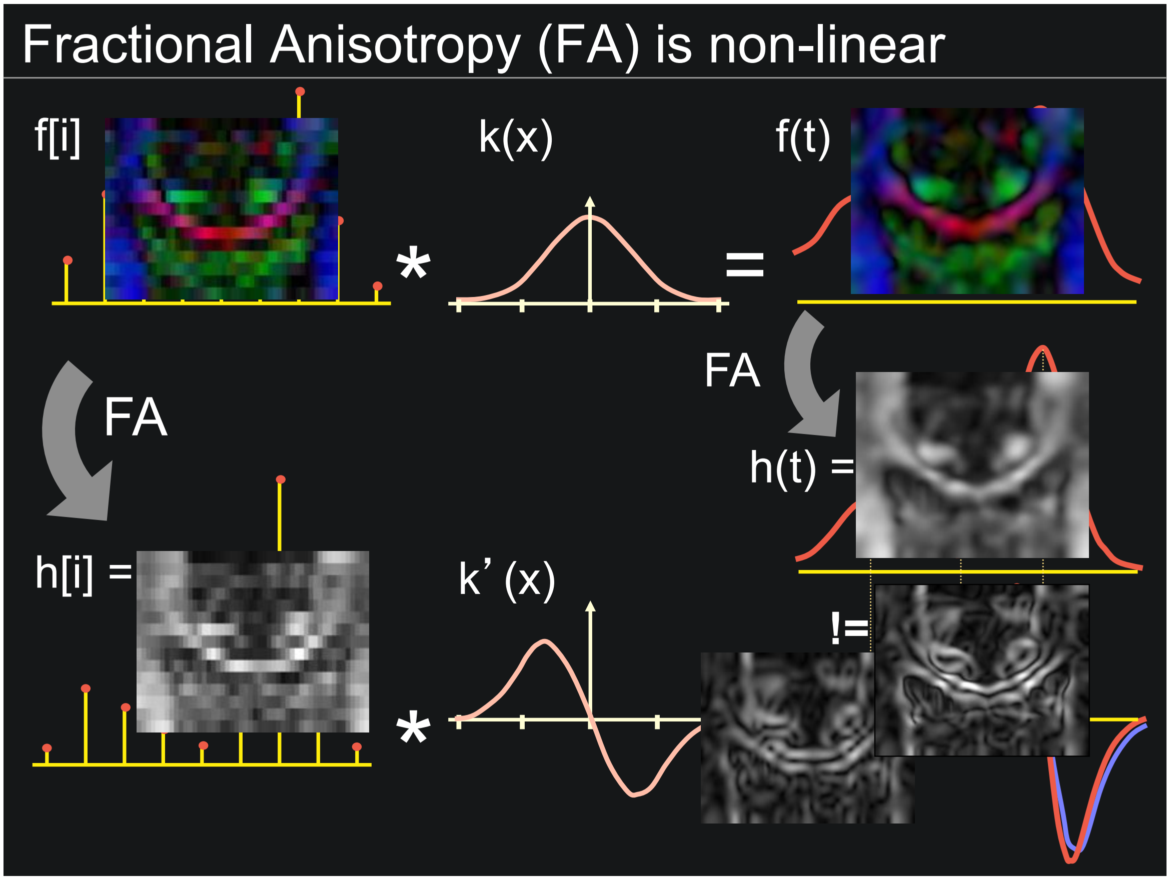
2-D Synthetic Scalar Example

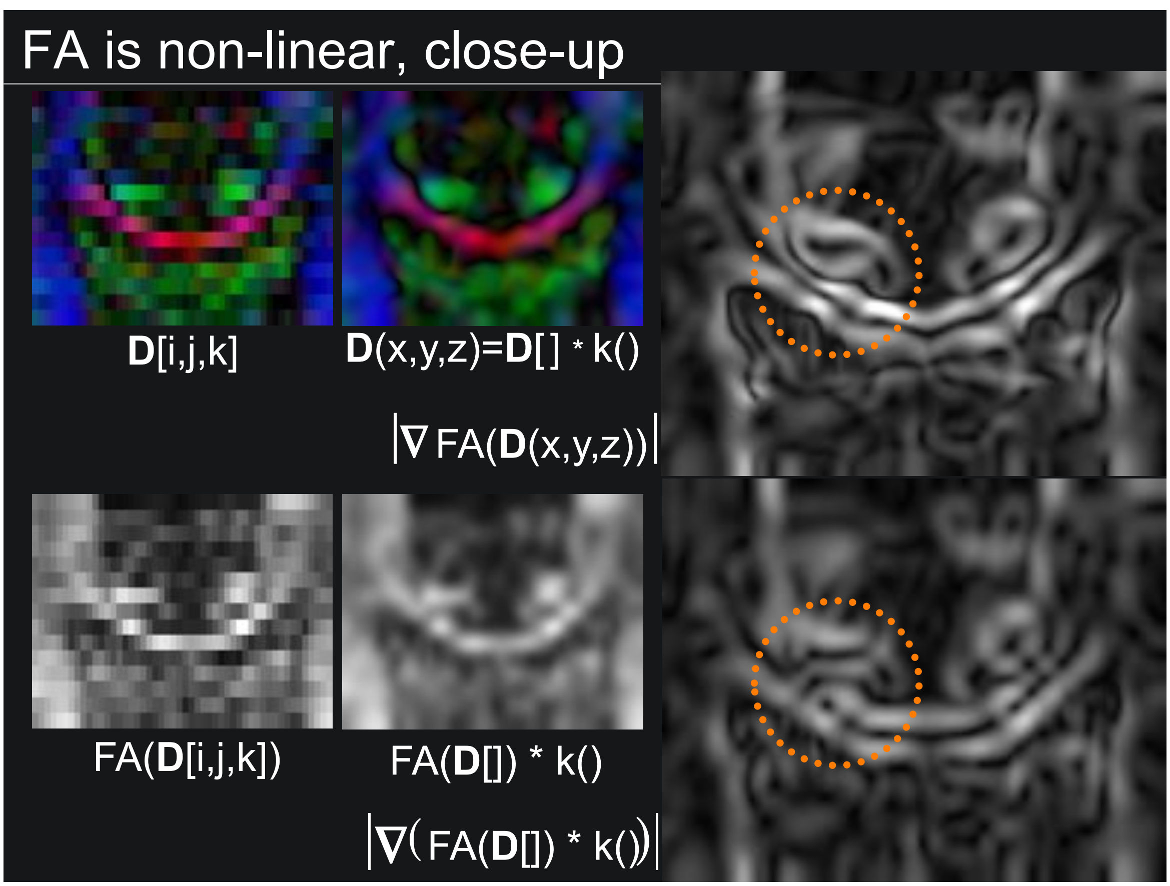


How do you measure derivatives in the tensor field?


Measurement (of scalars) by convolution
Continuous field: convolution of sampled coefficients with continuous reconstruction kernels
f[i]
k(x)
f(t)
*
=
Differentiation: convolve w/ derivative of reconstruction kernel
f[i]
k' (x)
f' (t)
*
=

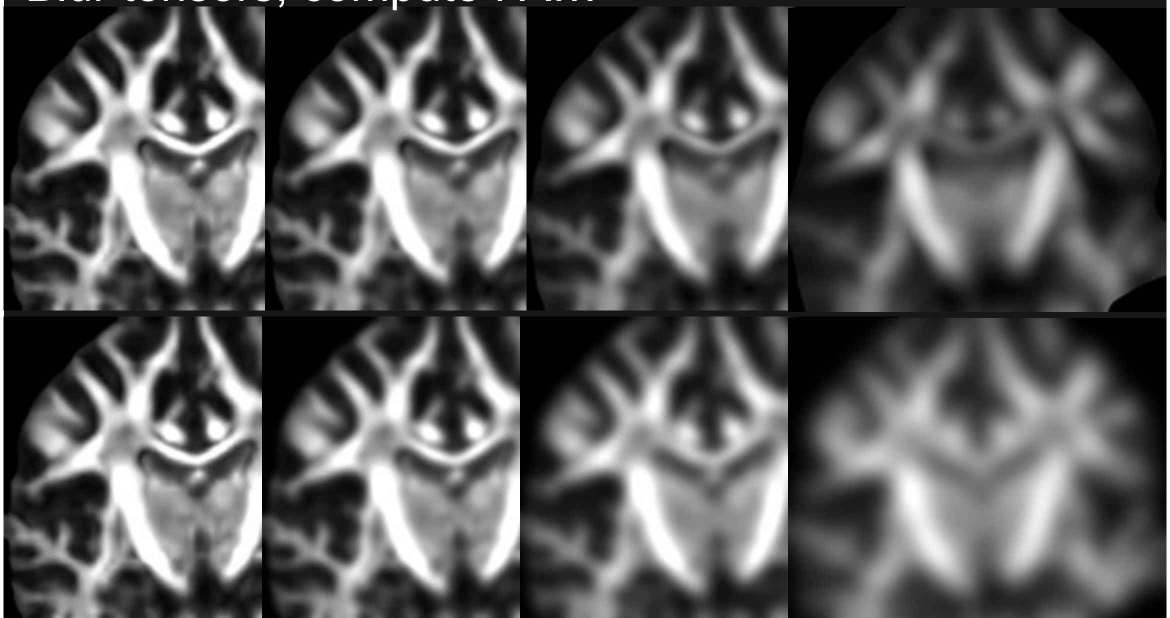

Non-linear transform of data
f[i]
k(x)
f(t)
*
=
g
g(y) = y²
h(t) = g(f(t))
h[i] = g(f[i])
k' (x)
h[i]*k' (x) != h' (t)
*
=


Fractional Anisotropy (FA) is non-linear
f[i]
k(x)
f(t)
*
=
FA
FA
h(t) =
h[i] =
k' (x)
!=
*


FA is non-linear, close-up
D[i,j,k]
D(x,y,z)=D[] * k()
|∇ FA(D(x,y,z))|
FA(D[i,j,k])
FA(D[]) * k()
|∇(FA(D[]) * k())|

Scale Space of FA is non-linear

Blur tensors, compute FA…



Compute FA, blur FA…

FA from invariants, from coefficients

$$\mathrm{FA} \equiv \sqrt{\frac{3\, D{:}D}{2\, \mathbf{D}{:}\mathbf{D}}} \qquad D = \mathbf{D} - \langle \mathbf{D} \rangle \mathbf{I} \qquad \mathbf{D}{:}\mathbf{D} = \mathrm{tr}(\mathbf{D}\mathbf{D}^T)$$

$$\mathrm{FA} = 3\sqrt{\frac{Q}{S}} \qquad Q = \frac{S - J_2}{9}$$

$$J_2 = \begin{array}{c} D_{xx}D_{yy} + D_{xx}D_{zz} + D_{yy}D_{zz} \\ -D_{xy}^2 - D_{xz}^2 - D_{yz}^2 \end{array}$$

$$S = \mathbf{D}{:}\mathbf{D} = \begin{array}{c} D_{xx}^2 + D_{yy}^2 + D_{zz}^2 \\ +2D_{xy}^2 + 2D_{xz}^2 + 2D_{yz}^2 \end{array}$$

$$\nabla J_2 = \begin{array}{c} (D_{yy} + D_{zz})\nabla D_{xx} + (D_{xx} + D_{zz})\nabla D_{yy} + (D_{xx} + D_{yy})\nabla D_{zz} \\ -2D_{xy}\nabla D_{xy} - 2D_{xz}\nabla D_{xz} - 2D_{yz}\nabla D_{yz} \end{array}$$

$$\nabla Q = \frac{\nabla S - \nabla J_2}{9}$$

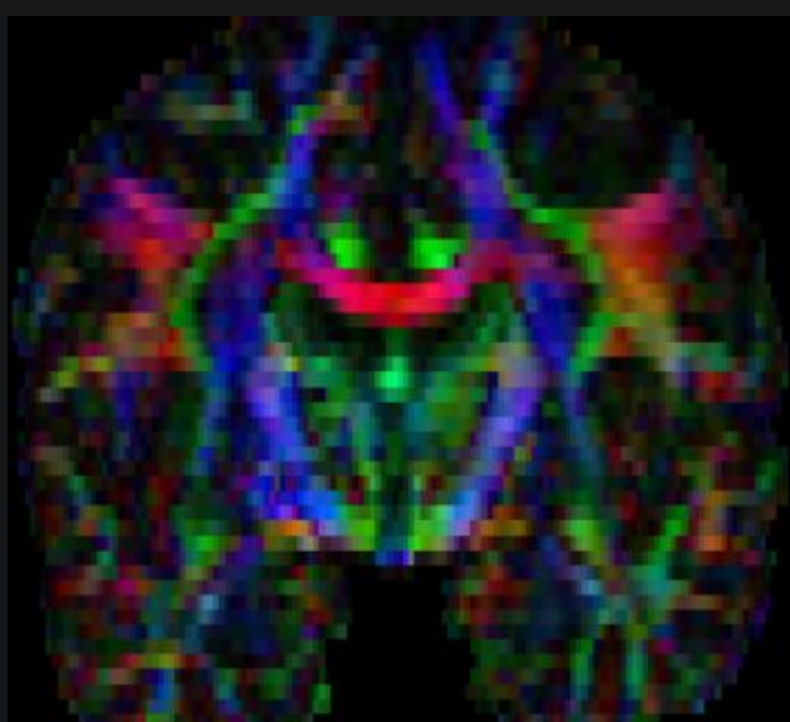
$$\nabla \mathrm{FA} = \frac{3}{2}\left(\sqrt{\frac{1}{SQ}}\nabla Q - \sqrt{\frac{Q}{S^3}}\nabla S\right)$$

$$\nabla S = \begin{array}{c} 2D_{xx}\nabla D_{xx} + 2D_{yy}\nabla D_{yy} + 2D_{zz}\nabla D_{zz} \\ +4D_{xy}\nabla D_{xy} + 4D_{xz}\nabla D_{xz} + 4D_{yz}\nabla D_{yz} \end{array}$$

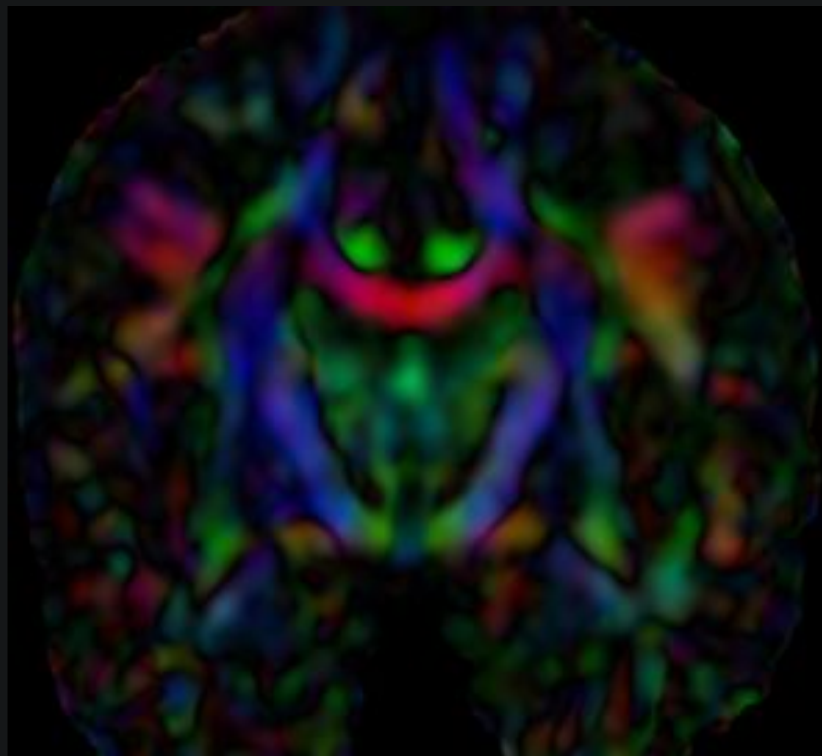
Hessian(FA) more complicated, but similarly derived

So, does it work?

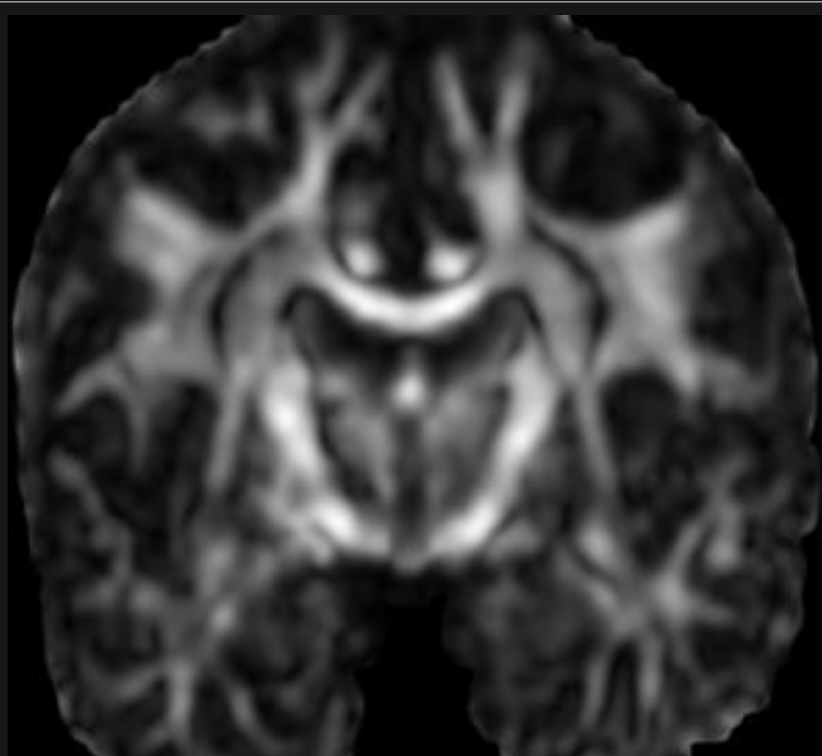
Slice Inspection: RGB($\mathbf{e}_1$) (original data)

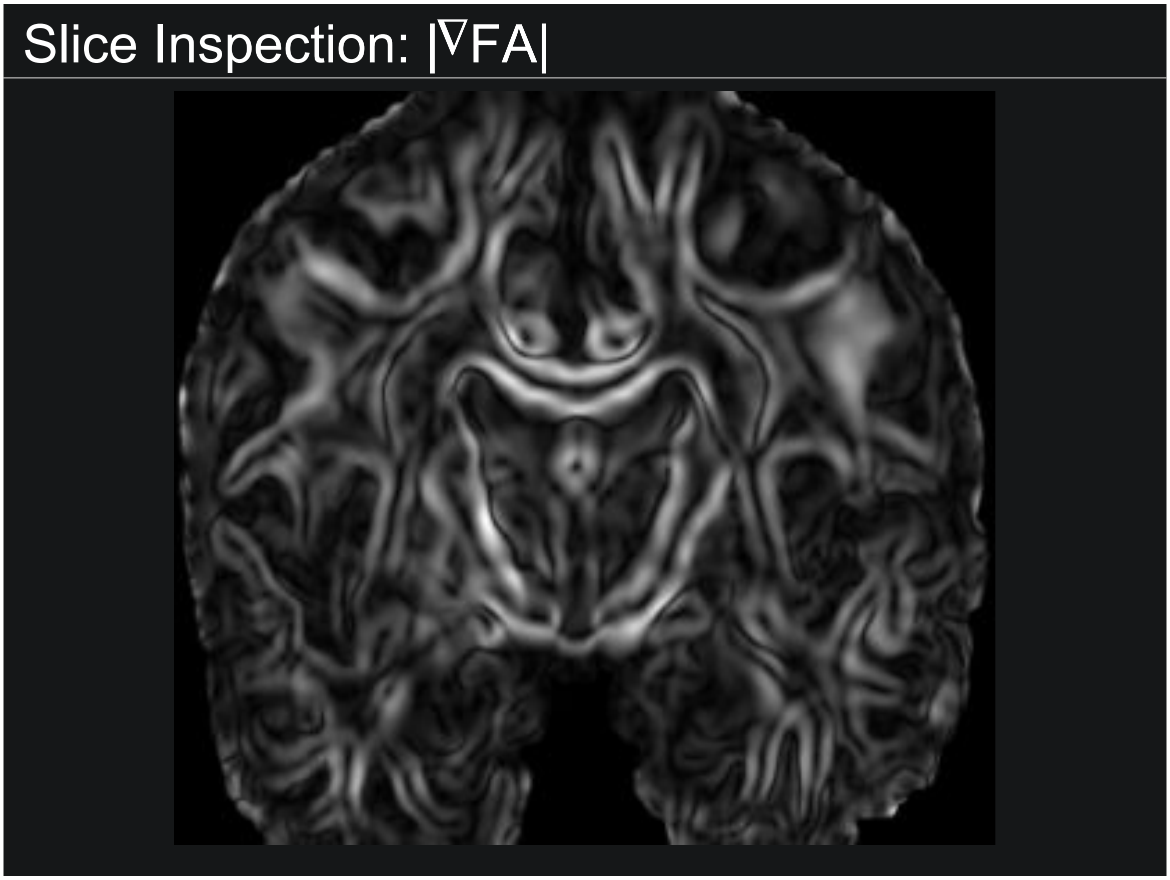


Slice Inspection: RGB($\mathbf{e}_1$)



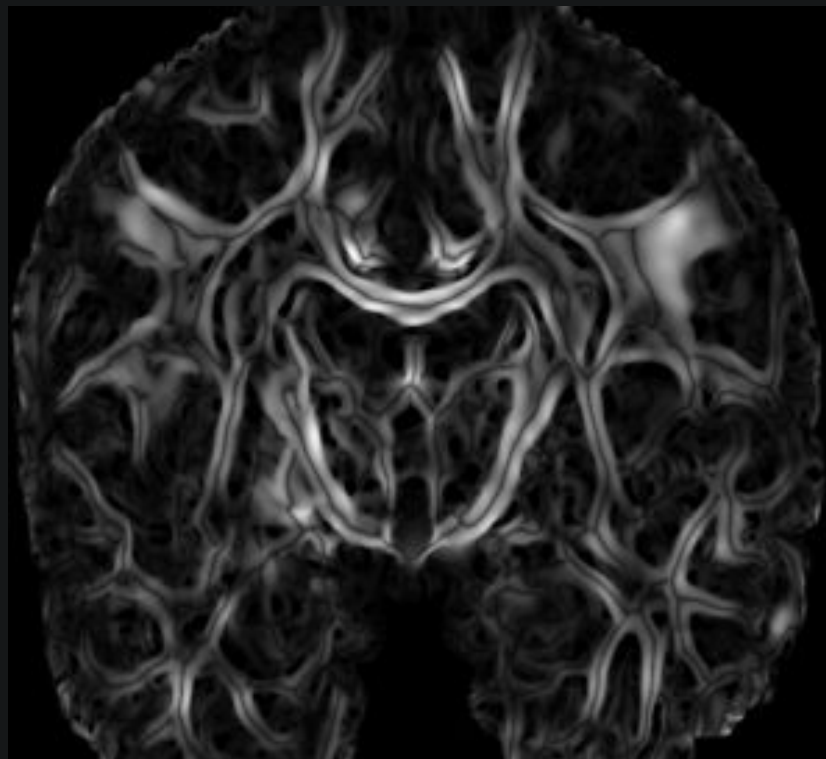
Slice Inspection: FA



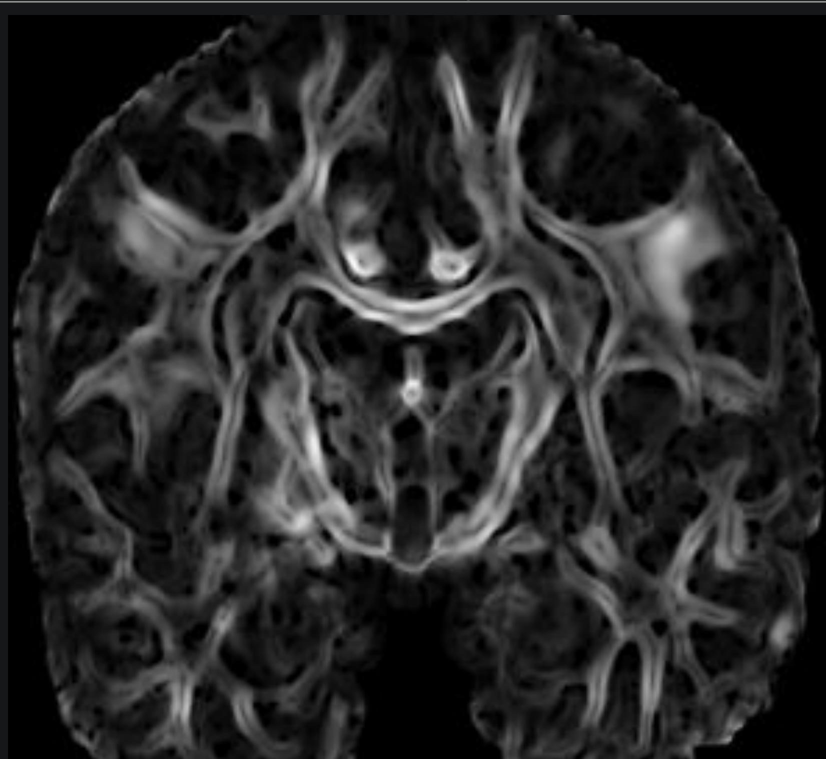

Slice Inspection: |∇FA|


Slice Inspection: ridge strength: max(0,-λ_3)

Slice Inspection: $|\mathbf{g}\cdot\mathbf{e}_3|$ (modulated by strength)

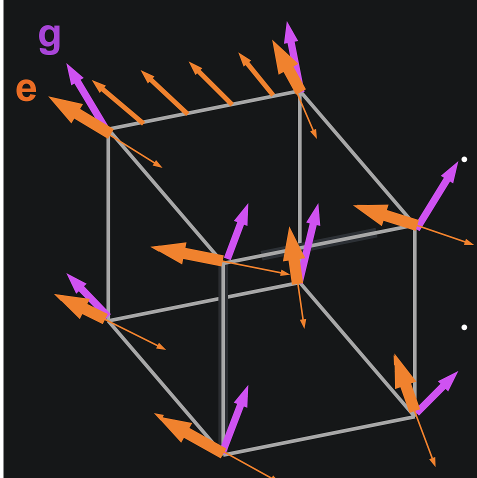


Slice Inspection: sqrt$((\mathbf{g}\cdot\mathbf{e}_3)^2 + (\mathbf{g}\cdot\mathbf{e}_2)^2)$



How are geometric models created?

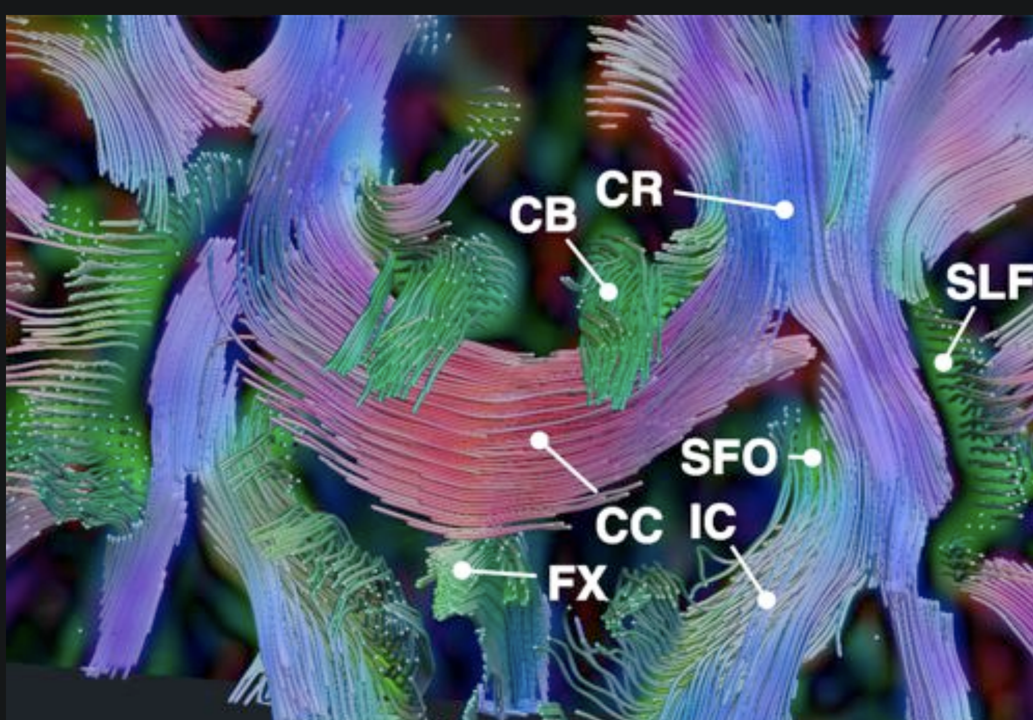
Modified Marching Cubes for Surfaces



- Crease surface is isosurface (zero-crossing) of $\mathbf{g} \cdot \mathbf{e}_i$, but…
- Eigenvectors lack sign: enforce intra-voxel sign consistency
- Propagate eigenvector at one corner to all others
- $\mathbf{g} \cdot \mathbf{e}$ dot products, then MC case table

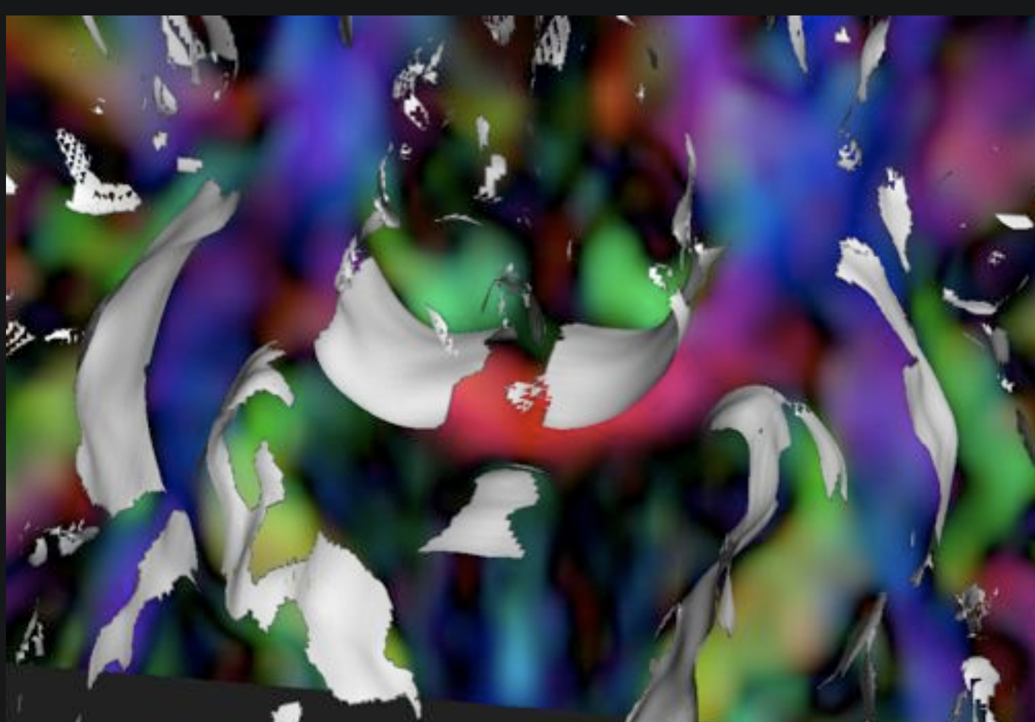
(torus demo)

3-D Results: coronal fibers

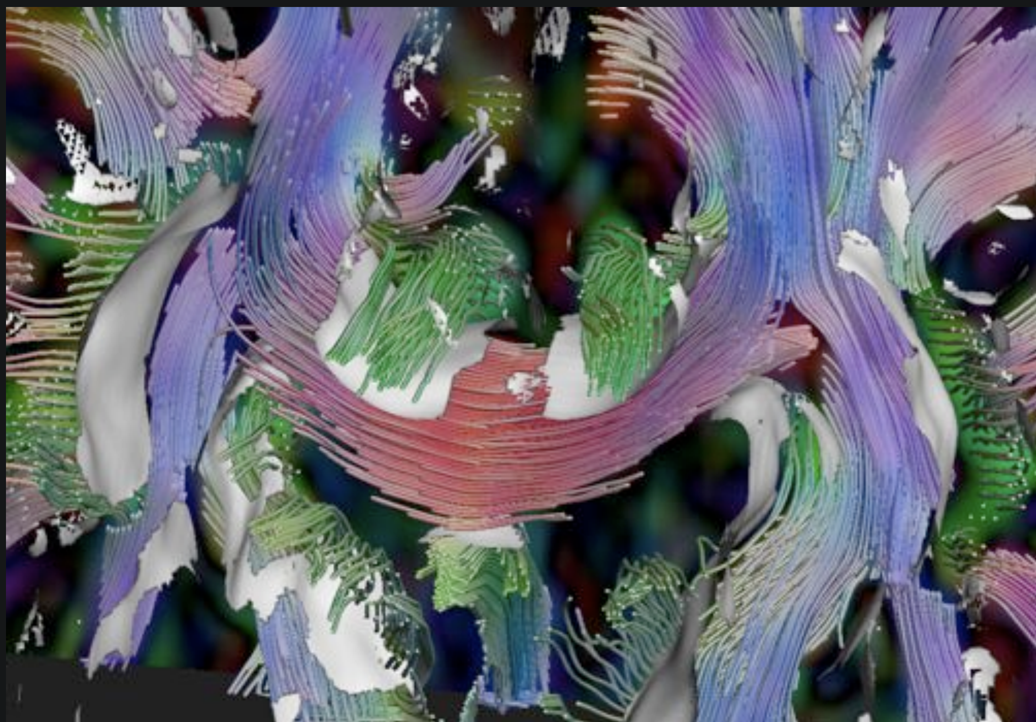


3-D Results: ridge surfaces

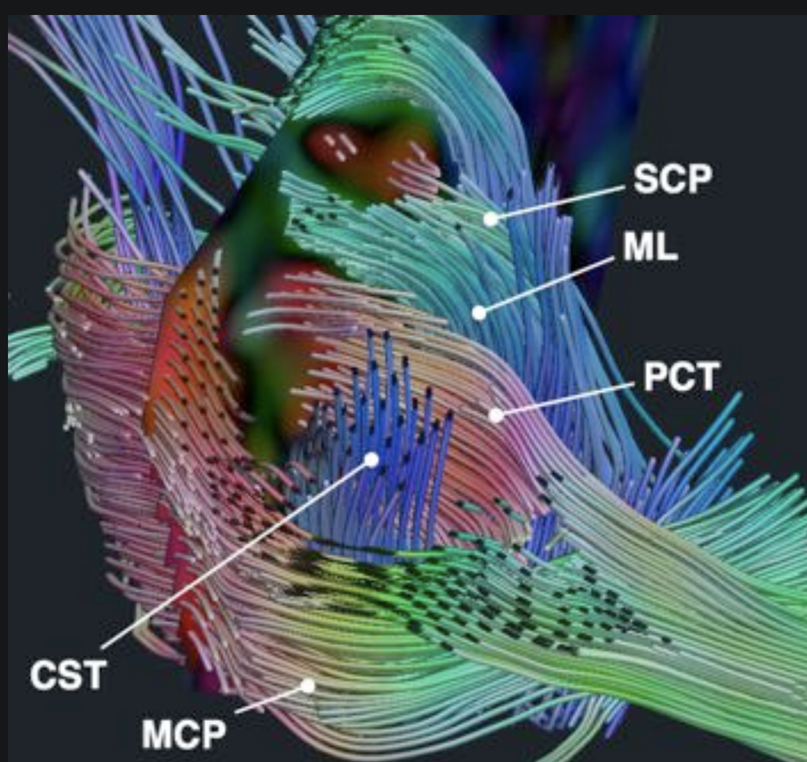
3-D Results: valley surfaces



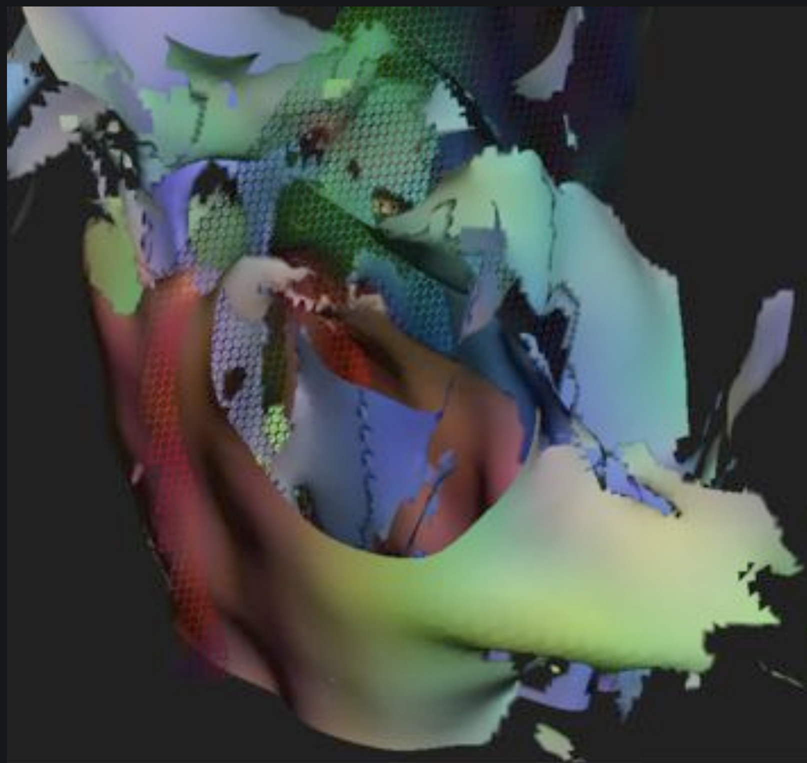
3-D Results: valley surfaces with fibers



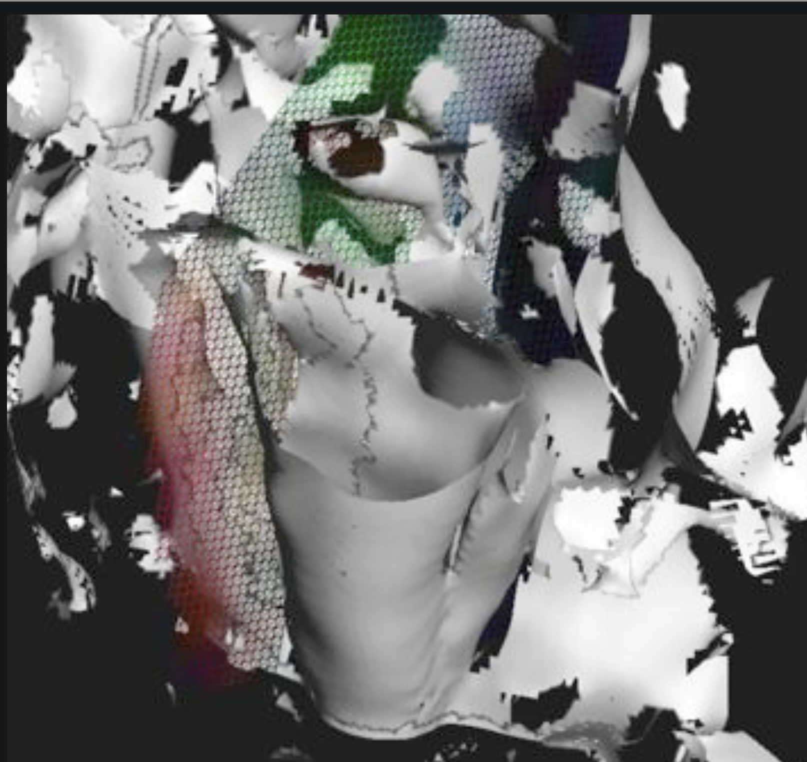
3-D Results: brainstem fibers



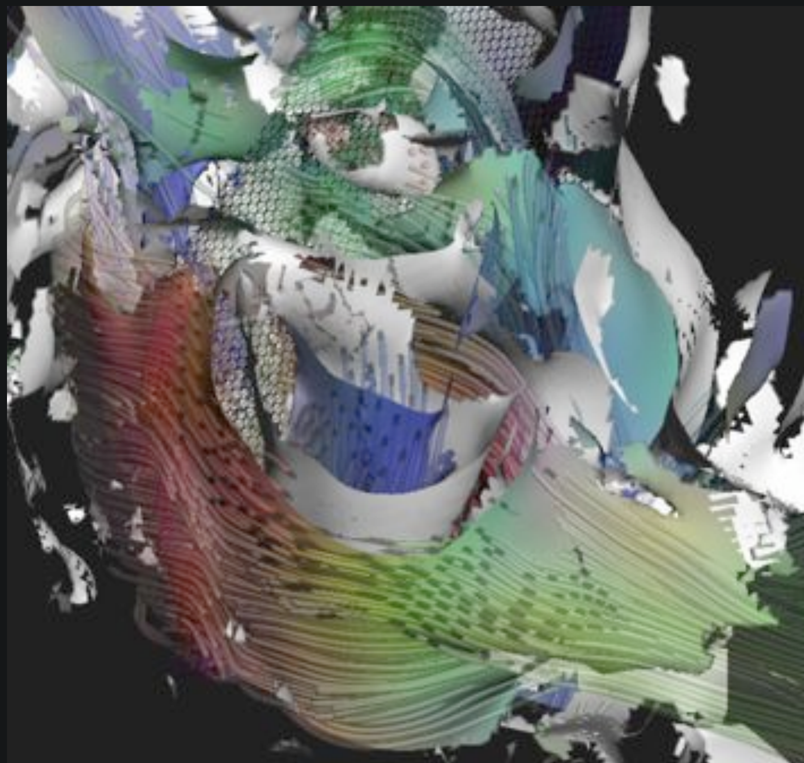
3-D Results: brainstem ridge surfaces



3-D Results: brainstem valley surfaces



3-D Results: combined results



Extremal Surfaces (Amenta SIGGRAPH '04)

Generalization of MLS (moving least squares)

Implicit surface generation from scattered point set

Minimize distance to local planar weighted fit of points

Ingredients:

- Scalar function
- Line field

Ridge surfaces:

Line field is minor eigenvector of Hessian

Why not minor eigenvector of tensor field itself?

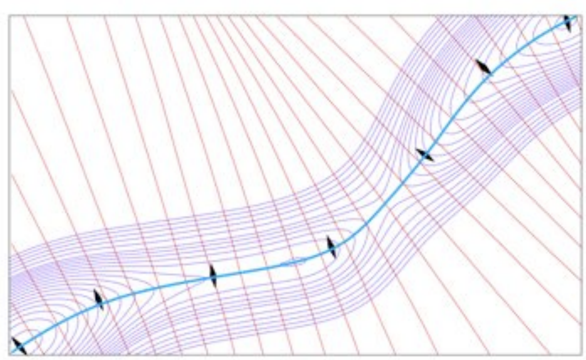


Figure 4: Streamlines (red) of a vector field $n(x)$, and iso-contours (blue) of an energy function $s(x)$. The heavy blue line is the extremal surface determined by n and s, running neatly along the "valley" in the energy landscape and passing through the minima of s. The streamlines of n and the iso-contours of s are tangent at the surface points. Here n and s were computed using the point-set surface for surfels introduced in Section 7; the input surfels are shown as black diamonds, with the long diagonal pointed in the direction of the surfel normal.

Discussion & Ongoing Work

- Novel Aspects:
  - Application of computer vision to DTI
  - Extracting geometry from differential DTI structure
- Scale space: interfaces are easier than “cores”
- Comparison to Tract-Based Spatial Statistics
- Crease line extraction; line vs. surface decision
- Evaluation on more datasets
- Clinical applications?

Acknowledgements

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thank you