Crease Features of Tensor Invariants

or,

Modeling White Matter Fiber Tracts Without Tractography

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Outline

- Background: DTI and fiber tracking
- Goal: Extracting white matter structure
- Invariants for measuring shape
- Method 1: Tensor field topology
- Method 2: Crease feature detection
- Combination: Fiber structures and boundaries
- Discussions & Ongoing Work

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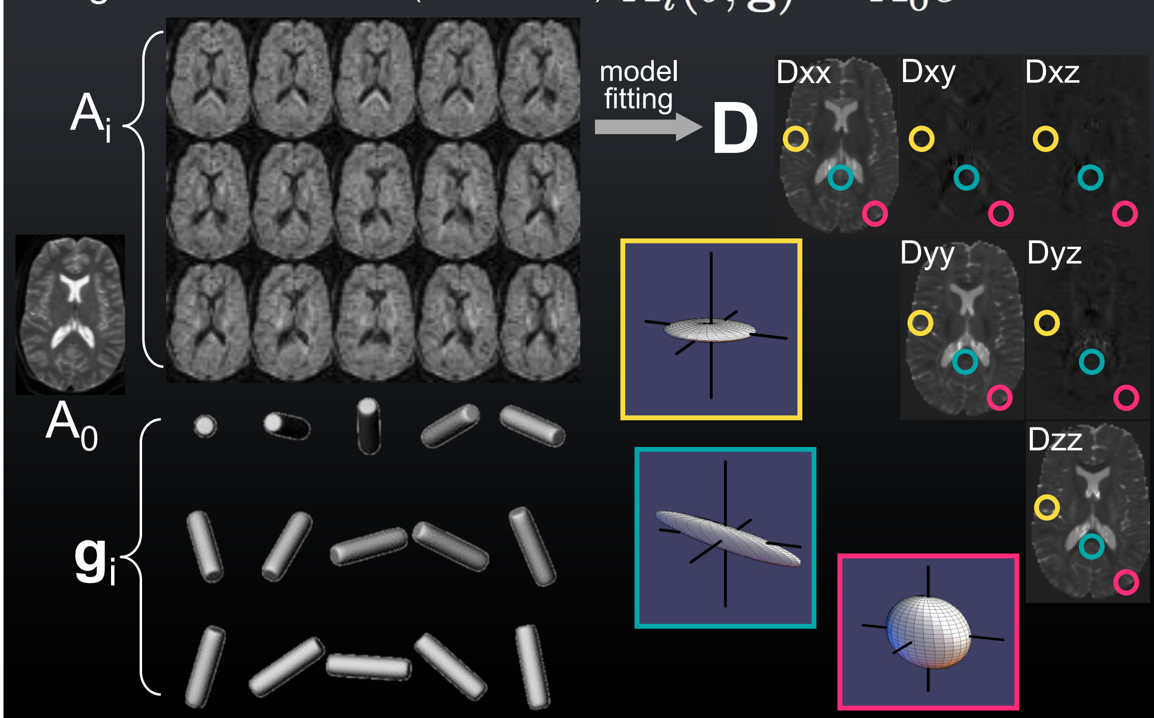
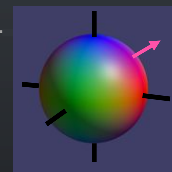
Diffusion Weighted, Diffusion Tensor MRI

Single Tensor Model (Basser 1994) $A_i(b, \mathbf{g}) = A_0 e^{-b\mathbf{g}_i^T \mathbf{D} \mathbf{g}_i}$



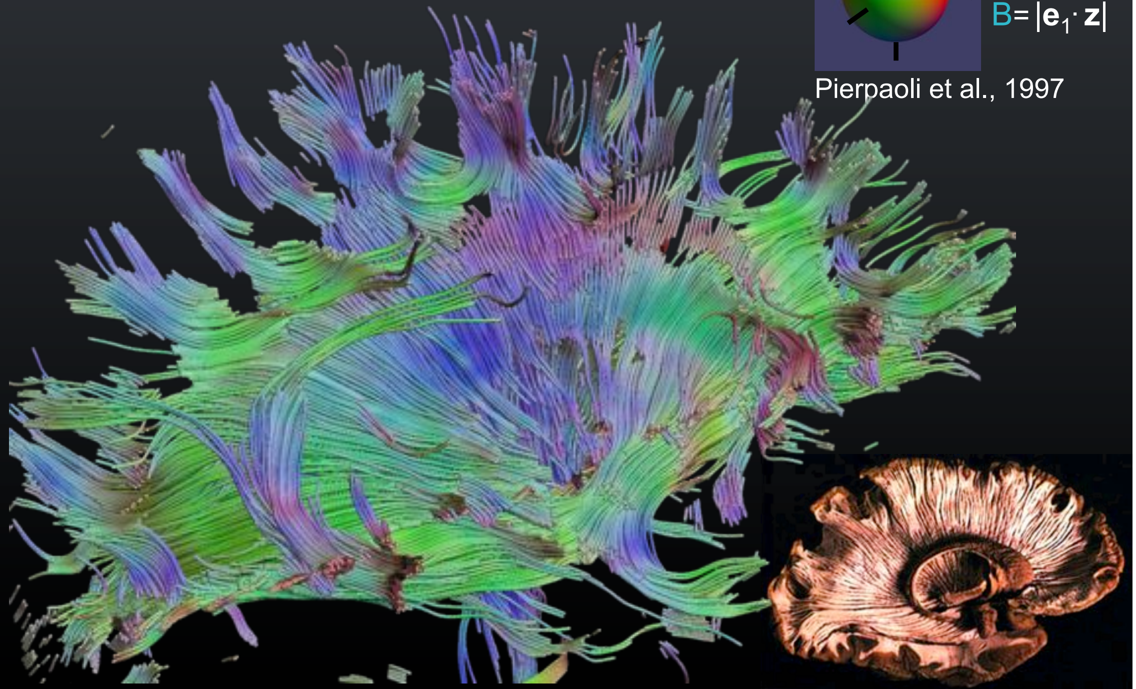
Fiber Tractography

Integrate paths along $\mathbf{e}_1$



$R = |\mathbf{e}_1 \cdot \mathbf{x}|$
$G = |\mathbf{e}_1 \cdot \mathbf{y}|$
$B = |\mathbf{e}_1 \cdot \mathbf{z}|$

Pierpaoli et al., 1997



Virtual Hospital (www.vh.org)

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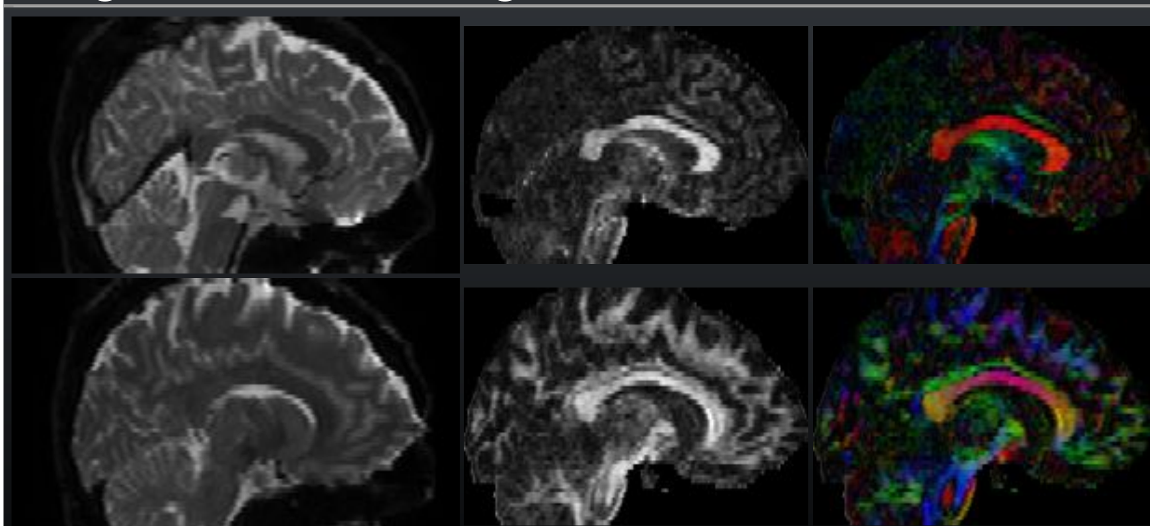
Why “tracts without tractography”

Standard pipeline:

DTI → 10,000s of fibers → ROI analysis/ clustering → anatomical regions

- Want robust way of getting at major fiber structure
  - Less parameter tuning; closer to data
- Automatic landmarks for non-rigid registration
  - Fiber tract skeleton: “Sulci for white matter”
  - Enable group differences on tensor attributes
- Surgical planning of tumor resection
  - Measure tract deformation, asymmetry

Registration challenge



T2 FA RGB($\mathbf{e}_1$)

- DTI shows major fiber orientation
- Don’t want to blur adjacent & orthogonal tracts

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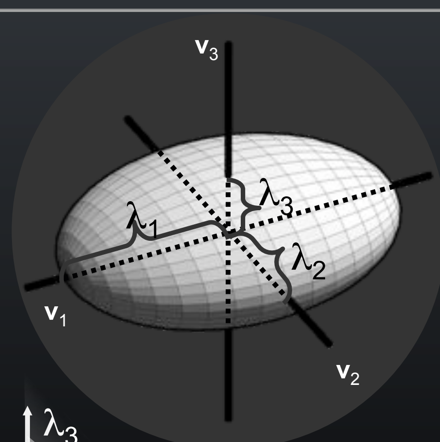
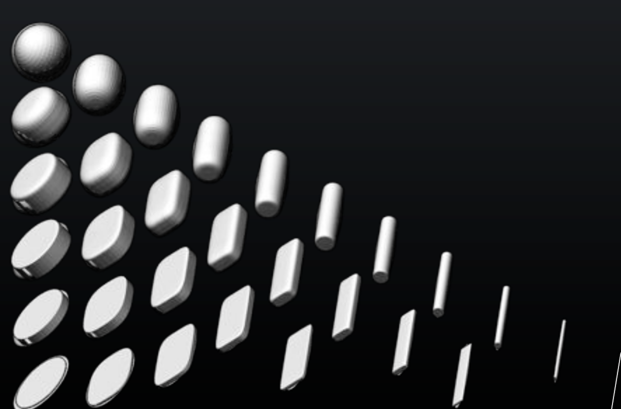
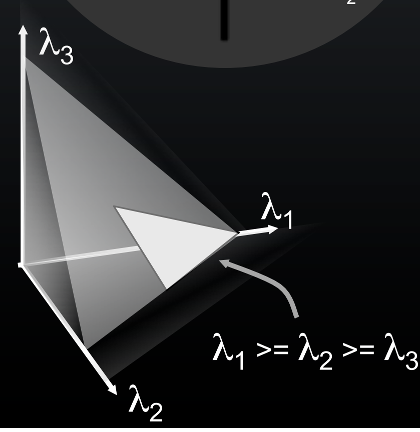
Space of Tensor Shape

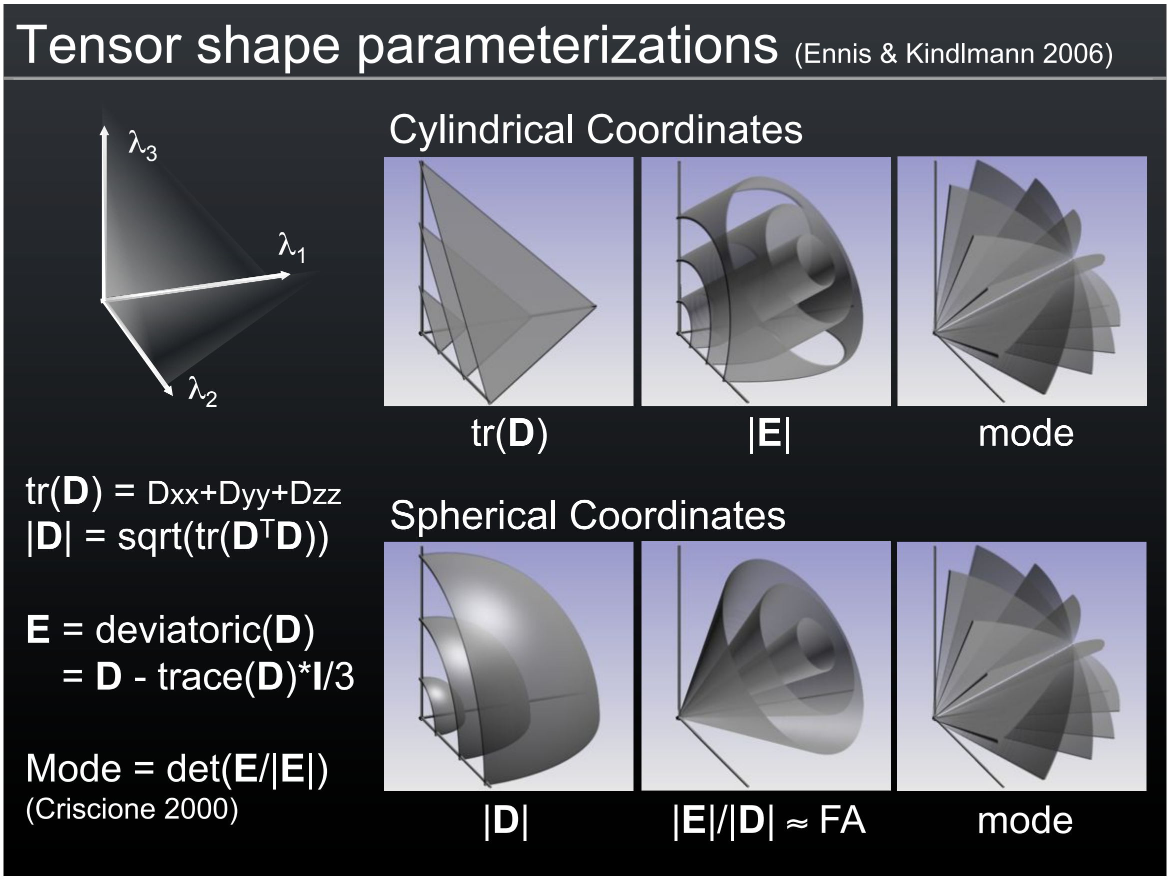
$$\mathbf{D} = \mathbf{R \Lambda R^{-1}}$$

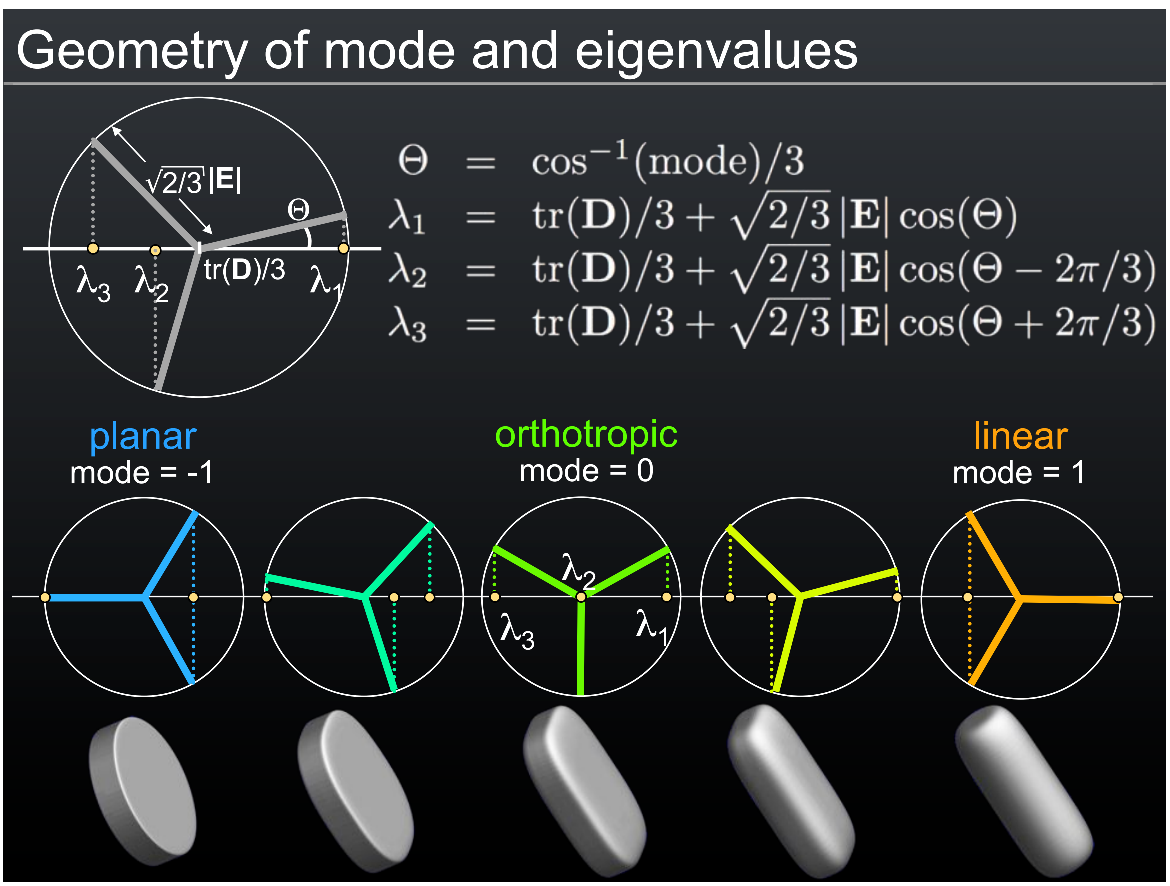
$$= \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{bmatrix}$$







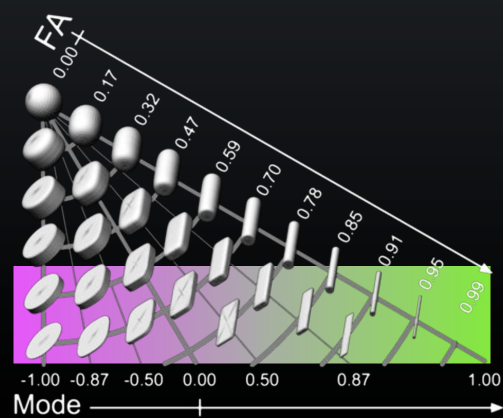

Tensor shape parameterizations (Ennis & Kindlmann 2006)
λ_3
λ_1
λ_2
Cylindrical Coordinates
tr(**D**)
|**E**|
mode
Spherical Coordinates
|**D**|
|**E**|/|**D**| ≈ FA
mode
tr(**D**) = Dxx+Dyy+Dzz
|**D**| = sqrt(tr($\mathbf{D}^T\mathbf{D}$))
E = deviatoric(**D**)
= **D** - trace(**D**)*$\mathbf{I}$/3
Mode = det(**E**/|**E**|)
(Criscione 2000)


Geometry of mode and eigenvalues
$\sqrt{2/3}|\mathbf{E}|$
Θ
λ_3 λ_2 tr(**D**)/3 λ_1
$\Theta = \cos^{-1}(\text{mode})/3$
$\lambda_1 = \text{tr}(\mathbf{D})/3 + \sqrt{2/3}\,|\mathbf{E}|\cos(\Theta)$
$\lambda_2 = \text{tr}(\mathbf{D})/3 + \sqrt{2/3}\,|\mathbf{E}|\cos(\Theta - 2\pi/3)$
$\lambda_3 = \text{tr}(\mathbf{D})/3 + \sqrt{2/3}\,|\mathbf{E}|\cos(\Theta + 2\pi/3)$
planar
mode = -1
orthotropic
mode = 0
linear
mode = 1
λ_2
λ_3
λ_1

Software Demo

Human brain dataset (2x2x3 mm, 3T, 5 B0 + 30 DWI)

Purpose of visualizations: develop intuition for relationship between the spatial patterns of invariants and the underlying anatomy



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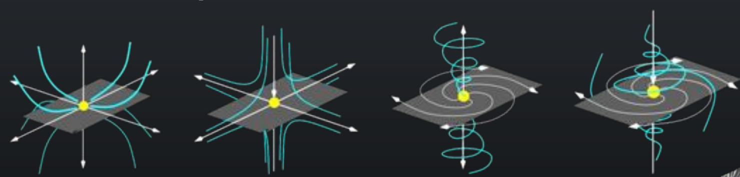
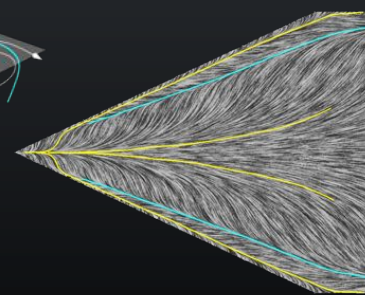
Method 1: Tensor Field Topology

- Vector Field Topology (Helman & Hesselink 1989)
  - Critical points: **v** = 0





  - Paths connecting critical points
  - Separatrices decompose field
- Applied to tensor fields (Delmarcelle & Hesselink 1994)
  - Degenerate points: 2 or 3 eigenvalues equal
  - Decompose field into eigenvector flows

Points where 2 eigenvalues equal

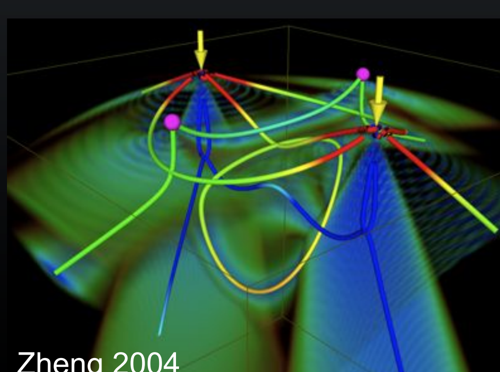
- Generically are lines (co-dimension 2, counting argument)
- Zheng et al. 2004, 2005
- Explicit root finding on

Tensor Discriminant:

$D_3 = (\lambda_1-\lambda_2)^2 (\lambda_1-\lambda_3)^2 (\lambda_2-\lambda_3)^2$

$$
\begin{aligned}
D_3(T) = \; & f_x(T)^2 + f_{y1}(T)^2 + f_{y2}(T)^2 + f_{y3}(T)^2 + \\
& 15 f_{z1}(T)^2 + 15 f_{z2}(T)^2 + 15 f_{z3}(T)^2
\end{aligned}
$$

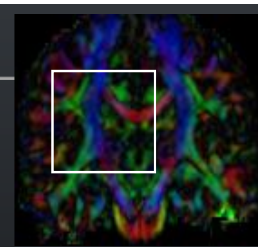
$$
\begin{aligned}
f_x(T) = \; & T_{00}(T_{11}^2 - T_{22}^2) + T_{00}(T_{01}^2 - T_{02}^2) + T_{11}(T_{22}^2 - T_{00}^2) + \\
& T_{11}(T_{12}^2 - T_{01}^2) + T_{22}(T_{00}^2 - T_{11}^2) + T_{22}(T_{02}^2 - T_{12}^2) \\
f_{y1}(T) = \; & T_{12}(2(T_{12}^2 - T_{00}^2) - (T_{02}^2 + T_{01}^2) + 2(T_{11}T_{00} + T_{22}T_{00} \\
& -T_{11}T_{22})) + T_{01}T_{02}(2T_{00} - T_{22} - T_{11}) \\
f_{y2}(T) = \; & T_{02}(2(T_{02}^2 - T_{11}^2) - (T_{01}^2 + T_{12}^2) + 2(T_{22}T_{11} + T_{00}T_{11} \\
& -T_{22}T_{00})) + T_{12}T_{01}(2T_{11} - T_{00} - T_{22}) \\
f_{y3}(T) = \; & T_{01}(2(T_{01}^2 - T_{22}^2) - (T_{12}^2 + T_{02}^2) + 2(T_{00}T_{22} + T_{11}T_{22} \\
& -T_{00}T_{11})) + T_{02}T_{12}(2T_{22} - T_{11} - T_{00}) \\
f_{z1}(T) = \; & T_{12}(T_{02}^2 - T_{01}^2) + T_{01}T_{02}(T_{11} - T_{22}) \\
f_{z2}(T) = \; & T_{02}(T_{01}^2 - T_{12}^2) + T_{12}T_{01}(T_{22} - T_{00}) \\
f_{z3}(T) = \; & T_{01}(T_{12}^2 - T_{02}^2) + T_{02}T_{12}(T_{00} - T_{11})
\end{aligned}
$$



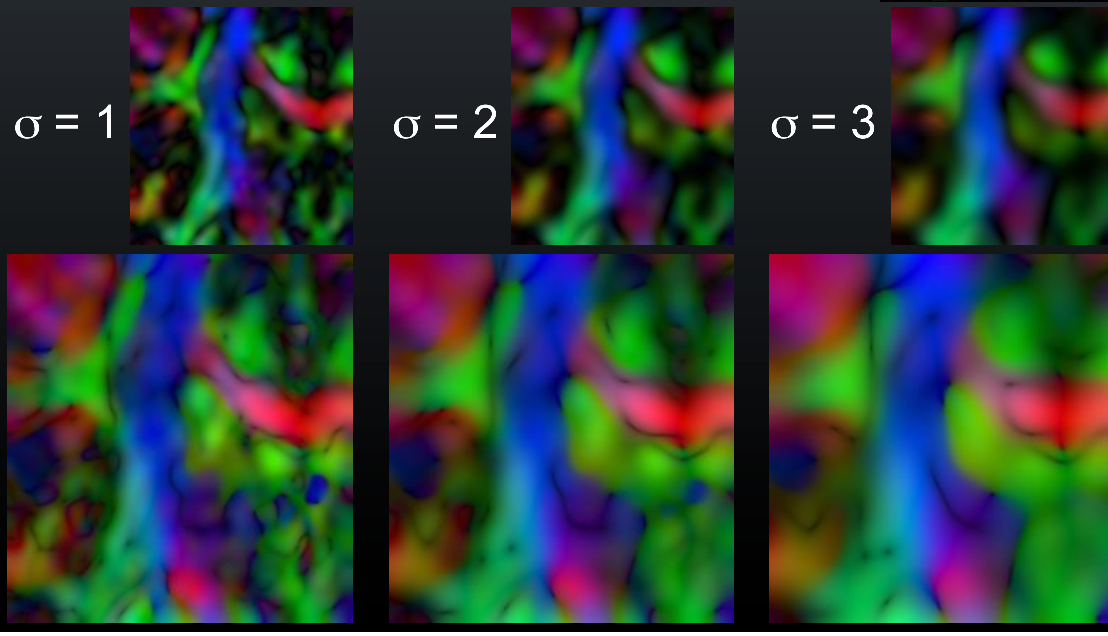
Numerically find simultaneous roots of seven cubic polynomials

Synthetic Elastic Stress Tensor

Degenerate Tensors in Real Data?



- Densely sampled discriminant in sagittal slice, at different scales (σ)



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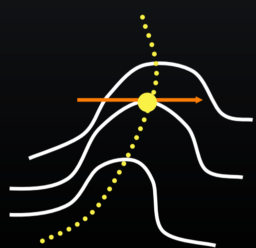
Method 2: Ridge/Valley Detection

- Medial Axis Transform (Blum 1973)
  - For binary Images
- Ridges and Valleys
  - For gray-scale images
  - Saint-Venant 1852, Haralick 1983, Pizer et al. 1994, Eberly et al. 1994, Morse 1994, Koenderick & van Doorn 1994, Lindeberg 1996, and others …



Morse 1994

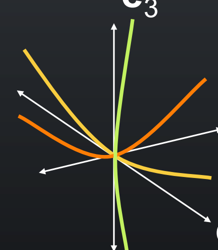
Crease feature definition (Eberly 1994)

- Taylor expansion → Hessian

$$f(\mathbf{x}_0 + \mathbf{d}) \approx f(\mathbf{x}_0) + \mathbf{d}\cdot\mathbf{g}(\mathbf{x}_0) + \mathbf{d}^{\mathrm{T}}\mathbf{H}(\mathbf{x}_0)\mathbf{d}/2$$

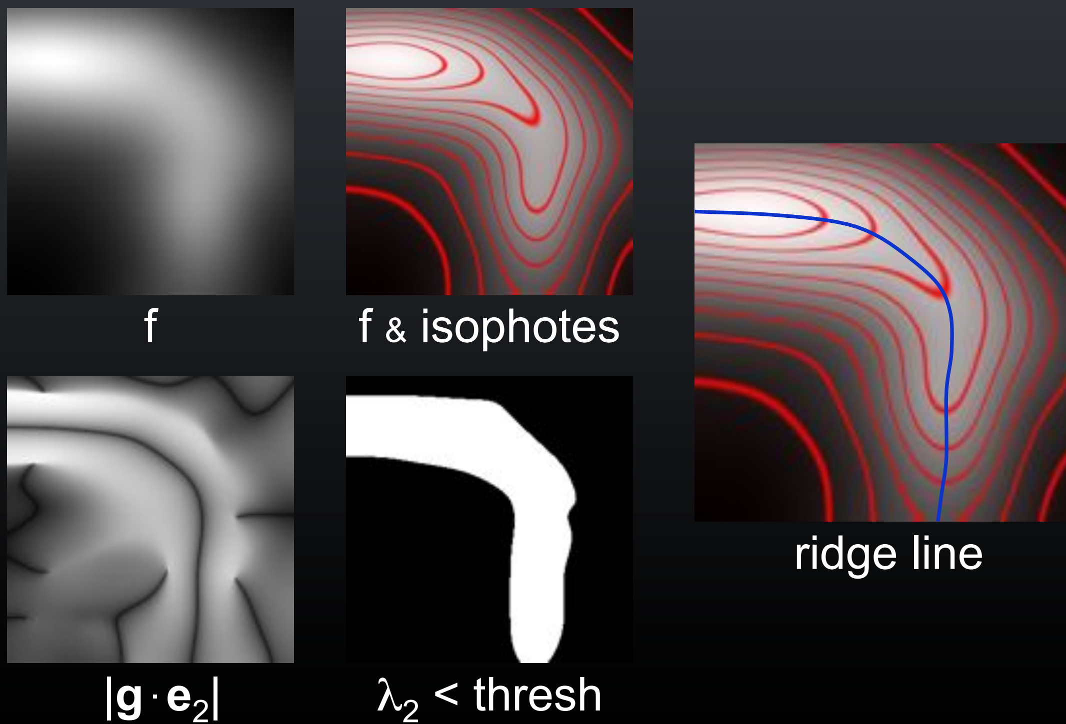
$$\mathbf{g} = \nabla f = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \\ \partial f/\partial z \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} \partial^2 f/\partial x^2 & \partial^2 f/\partial x \partial y & \partial^2 f/\partial x \partial z \\ \partial^2 f/\partial x \partial y & \partial^2 f/\partial y^2 & \partial^2 f/\partial y \partial z \\ \partial^2 f/\partial x \partial z & \partial^2 f/\partial y \partial z & \partial^2 f/\partial z^2 \end{bmatrix}$$



- Eigenvectors(**H**): 2nd-order structure orientation
- Extrema when **g** orthogonal to constraint surface
- **Crease**: Gradient orthogonal to one or two $\mathbf{e}_i$
  - ridge surface: $\mathbf{g}\cdot\mathbf{e}_3 = 0$; $\lambda_3 <$ thresh
  - valley line: $\mathbf{g}\cdot\mathbf{e}_1 = 0$; $\mathbf{g}\cdot\mathbf{e}_2 = 0$; $\lambda_1, \lambda_2 <$ thresh

2D Ridge Line Example



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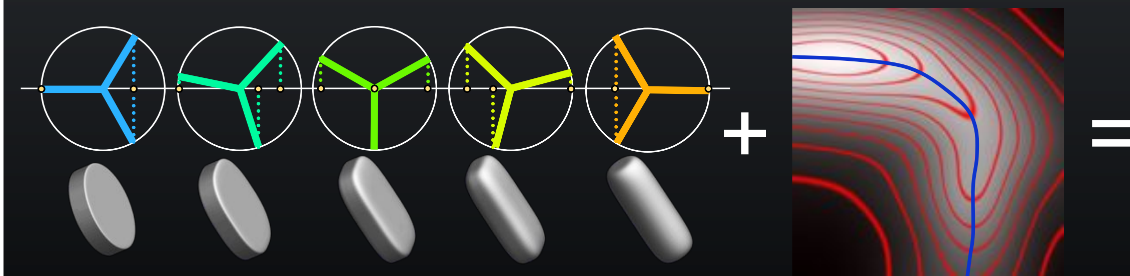
Basic Idea

Discriminant $(\lambda_1-\lambda_2)^2 (\lambda_1-\lambda_3)^2 (\lambda_2-\lambda_3)^2 = 0$

$\Rightarrow$ Two eigenvalues equal

$\Rightarrow$ Mode at **global** extrema (-1 or 1)



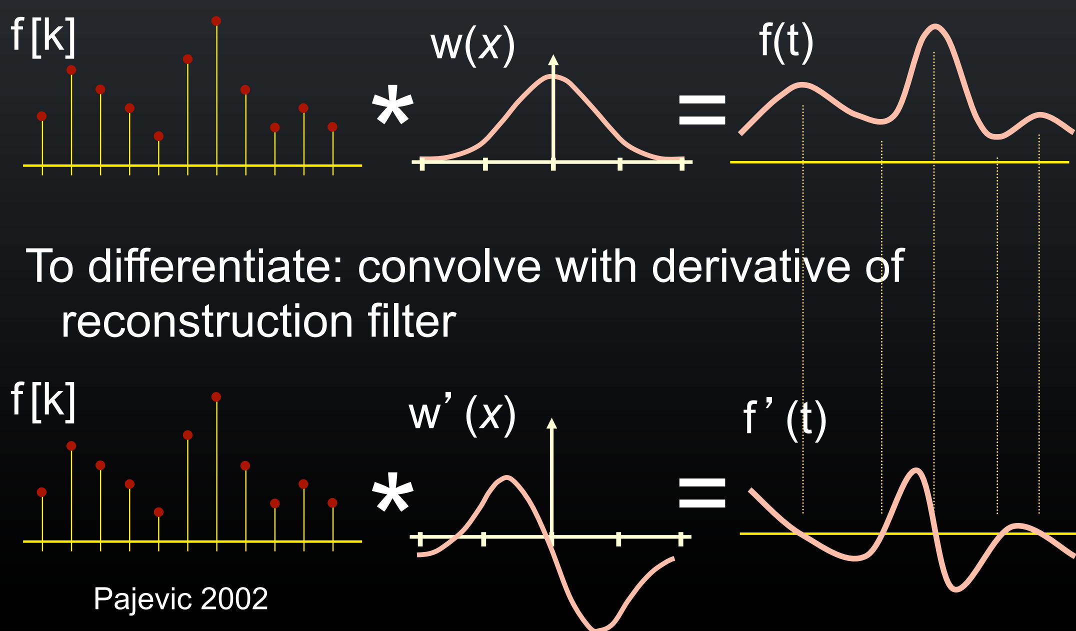
- Generalize tensor field topology
  - Crease features in tensor mode
  - Crease features in FA

Crease features of tensor invariants

- Create smooth continuous tensor field
- 1st, 2nd derivatives of field (rank 3,4 tensors)
- Gradient, Hessian of invariant
- Initial results

Measurement by convolution

Continuous tensor field: convolution of sampled coefficients with continuous reconstruction filters



Pajevic 2002

Computing invariant derivatives

$$\mathrm{FA} = 3\sqrt{\frac{Q}{S}} \qquad Q = \frac{S - J_2}{9}$$

$$J_2 = \begin{matrix} D_{xx}D_{yy} + D_{xx}D_{zz} + D_{yy}D_{zz} \\ -D_{xy}^2 - D_{xz}^2 - D_{yz}^2 \end{matrix}$$

$$S = \mathbf{D}{:}\mathbf{D} = \begin{matrix} D_{xx}^2 + D_{yy}^2 + D_{zz}^2 \\ +2D_{xy}^2 + 2D_{xz}^2 + 2D_{yz}^2 \end{matrix}$$

$$\nabla J_2 = \begin{matrix} (D_{yy} + D_{zz})\nabla D_{xx} + (D_{xx} + D_{zz})\nabla D_{yy} + (D_{xx} + D_{yy})\nabla D_{zz} \\ -2D_{xy}\nabla D_{xy} - 2D_{xz}\nabla D_{xz} - 2D_{yz}\nabla D_{yz} \end{matrix}$$

$$\nabla Q = \frac{\nabla S - \nabla J_2}{9}$$

$$\nabla \mathrm{FA} = \frac{3}{2}\left(\sqrt{\frac{1}{SQ}}\nabla Q - \sqrt{\frac{Q}{S^3}}\nabla S\right)$$

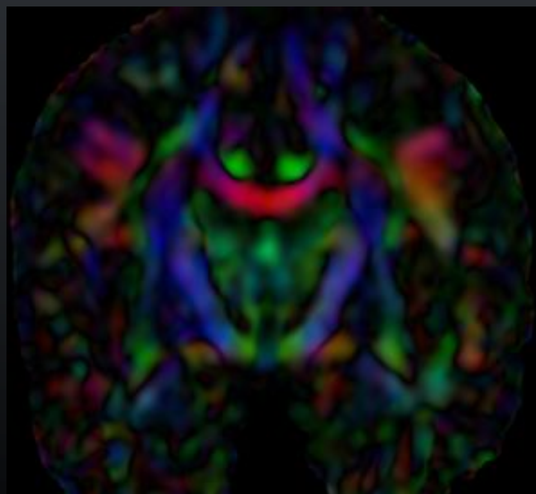
$$\nabla S = \begin{matrix} 2D_{xx}\nabla D_{xx} + 2D_{yy}\nabla D_{yy} + 2D_{zz}\nabla D_{zz} \\ +4D_{xy}\nabla D_{xy} + 4D_{xz}\nabla D_{xz} + 4D_{yz}\nabla D_{yz} \end{matrix}$$

$$\mathrm{mode} = \frac{R}{\sqrt{Q^3}} \qquad R = \frac{-5\,\mathrm{tr}(\mathbf{D})J_2 + 27\det(\mathbf{D}) + 2\,\mathrm{tr}(\mathbf{D})S}{54}$$

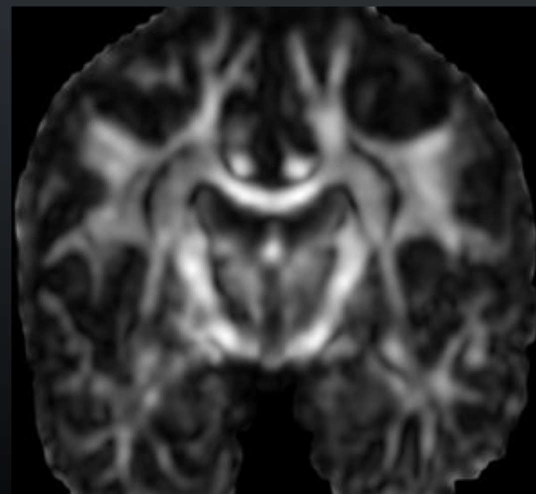
Hessian(FA), Hessian(mode) more involved

Why: Invariants and ∇ don’t commute; storage

Initial results

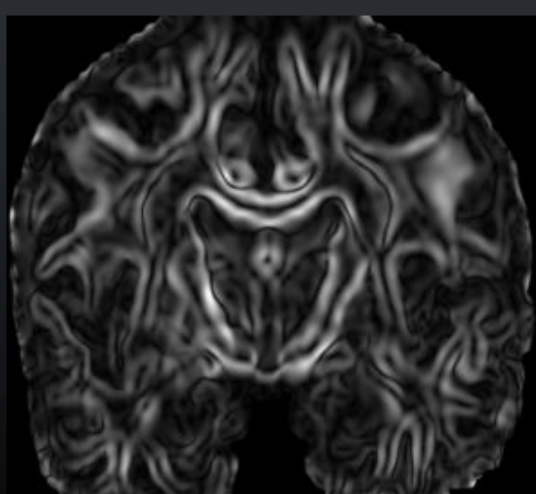


RGB($\mathbf{e}_1$)

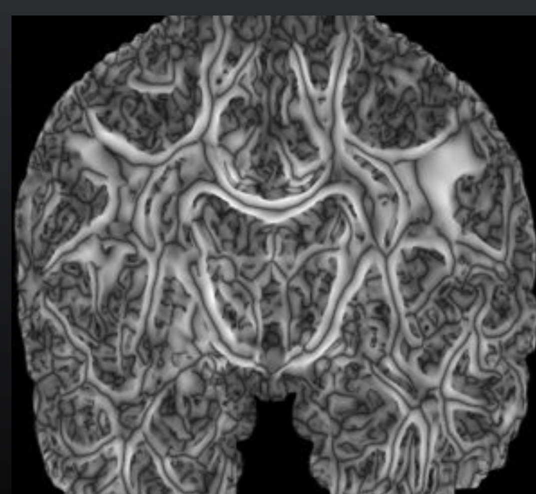


FA

Initial results



$|\nabla(\text{FA})|$



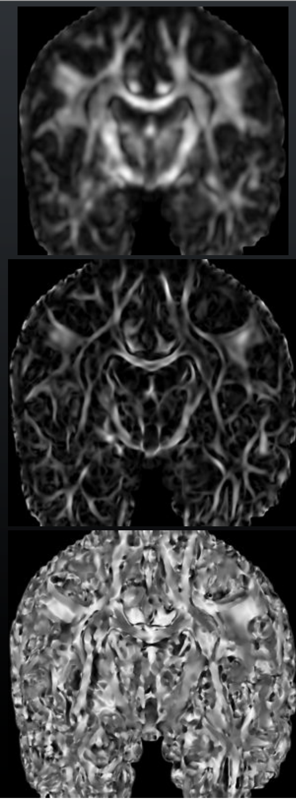
FA ridge surface

Initial results

FA

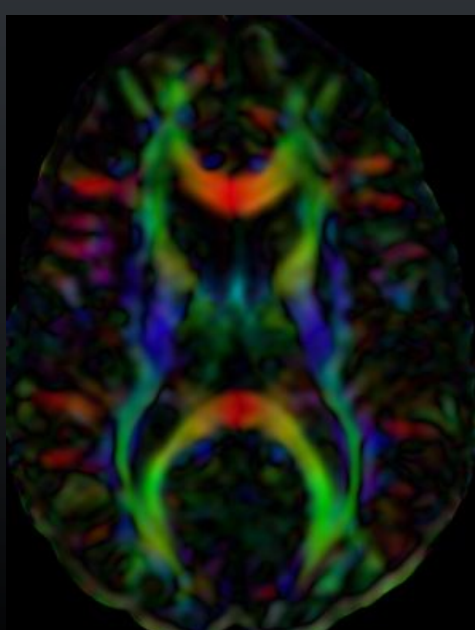
$-\lambda_3(H(FA))$

$1 - |\mathbf{e}_3(H(FA)) \cdot \mathbf{e}_1(\mathbf{D})|$

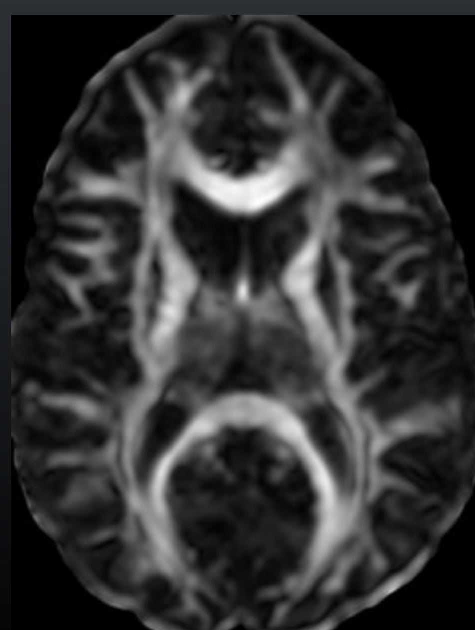


FA ridge surface, weighted

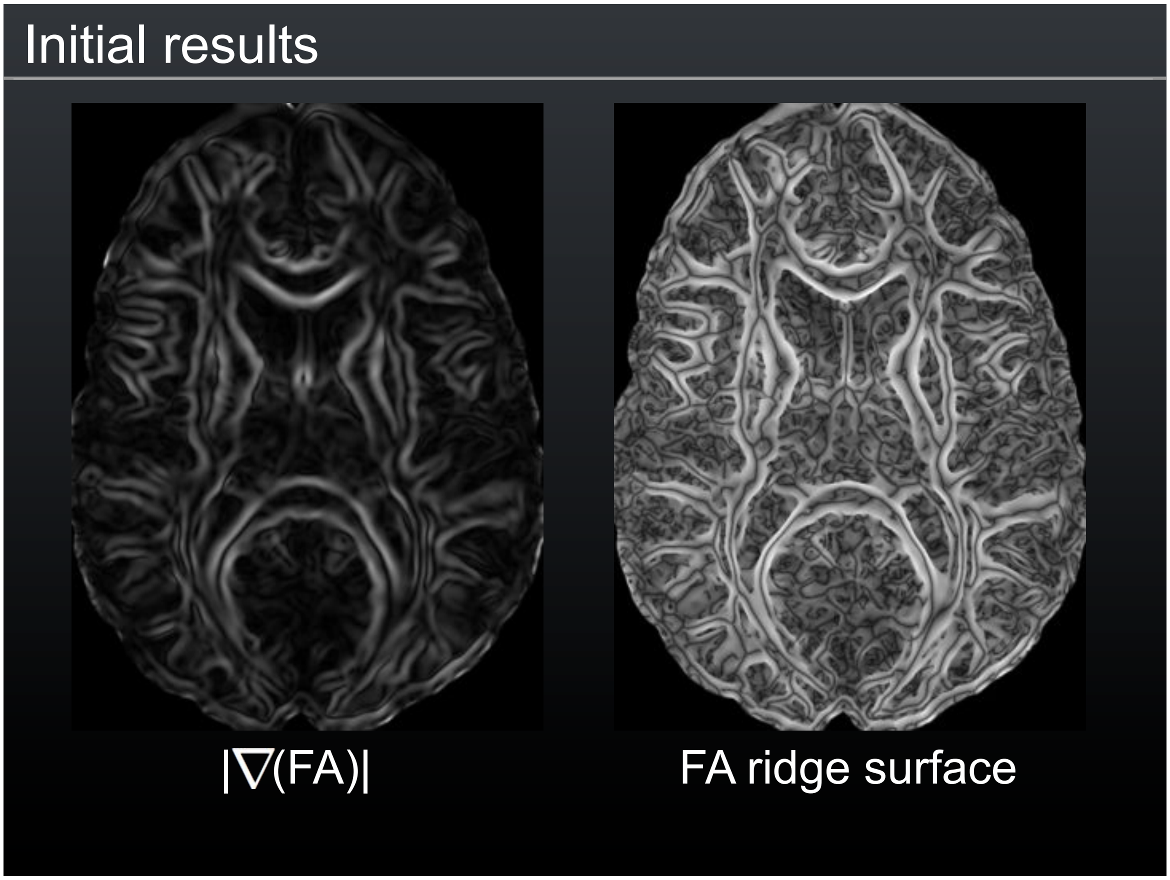
Initial results

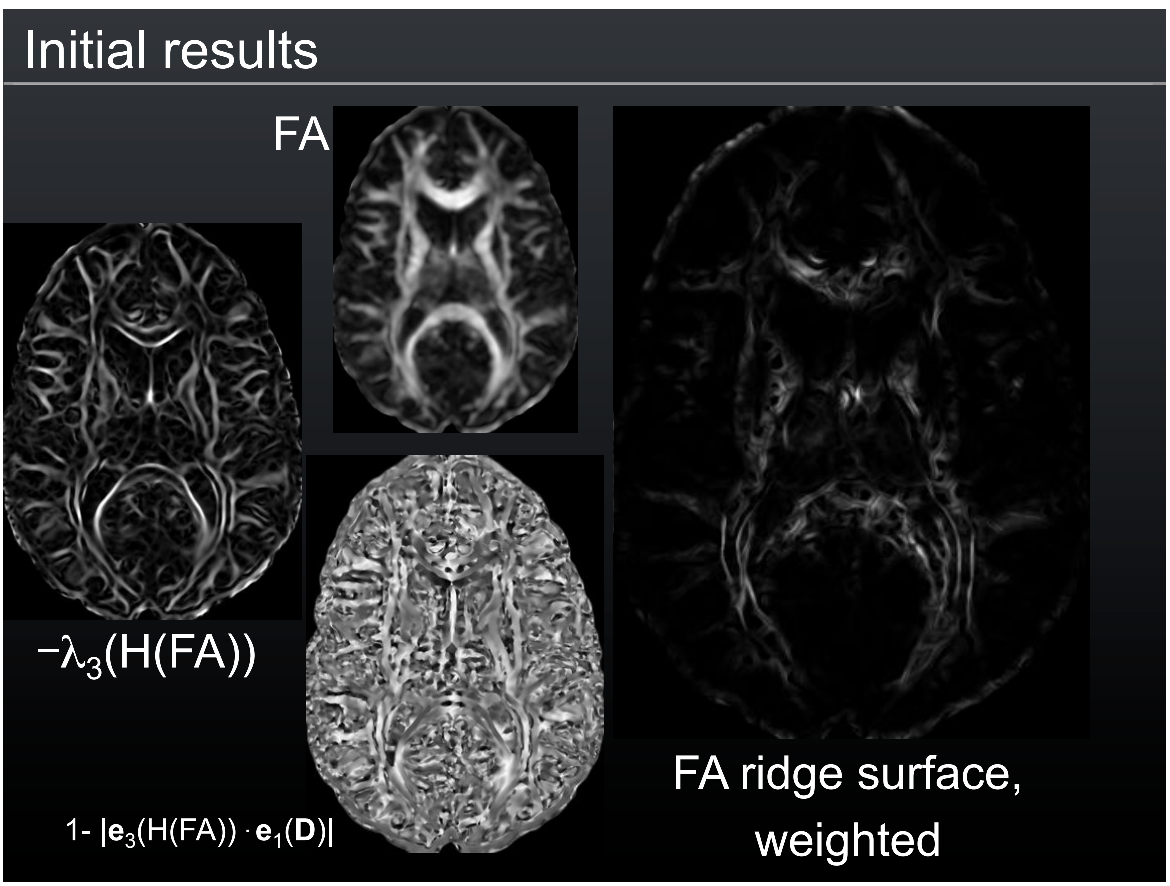


RGB($\mathbf{e}_1$)



FA


Initial results
|∇(FA)|
FA ridge surface


Initial results
FA
−λ3(H(FA))
1- |e3(H(FA)) · e1(D)|
FA ridge surface,
weighted

Discussion & Ongoing Work

- Contributions
  - Extracting geometry from differential structure
  - Combining tensor topology & crease detection
- Scale space: interfaces versus cores
- Tensor eigensystem orientation
- Crease lines
- Evaluation on more datasets

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