

Measurement & Biology basics

Tensor Shape & Orientation

Data Inspection with Glyphs

Fiber Tracking and Analysis

Current Issues and Work

Measurement & Biology basics

Tensor Shape & Orientation

Data Inspection with Glyphs

Fiber Tracking and Analysis

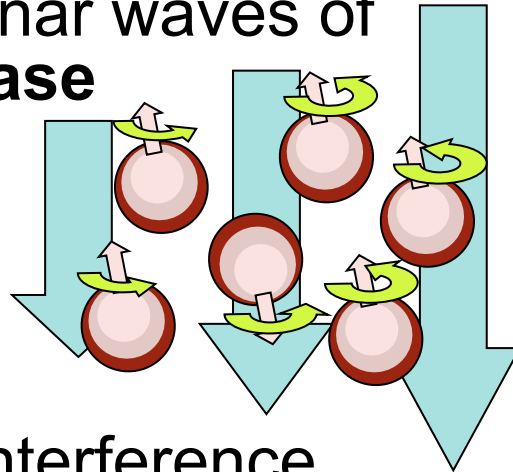
Current Issues and Work

# Diffusion-weighted MRI (DW-MRI)

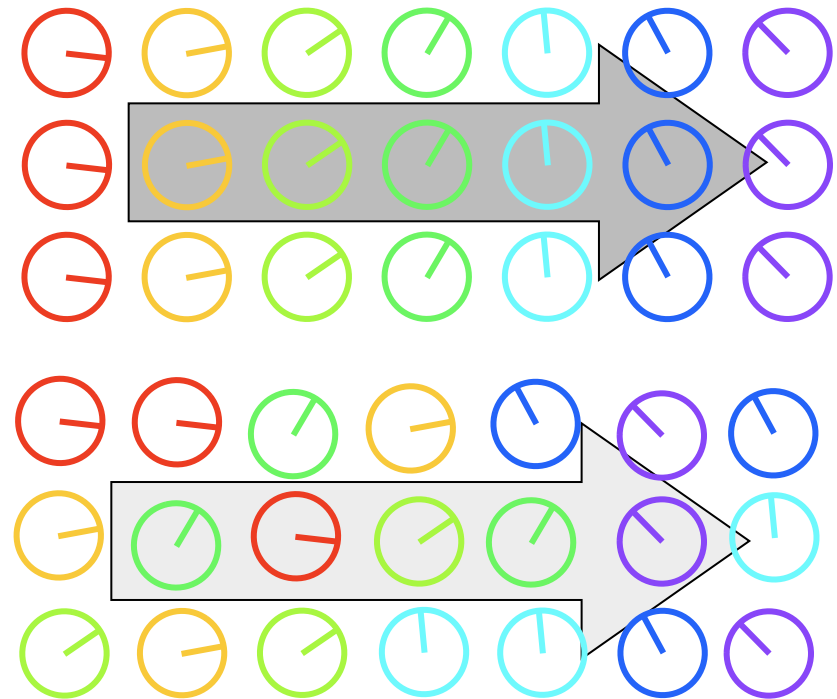
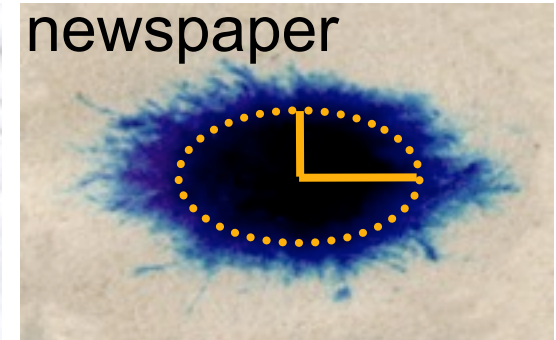
Brownian motion of one material through another

Anisotropy: diffusion rate depends on direction

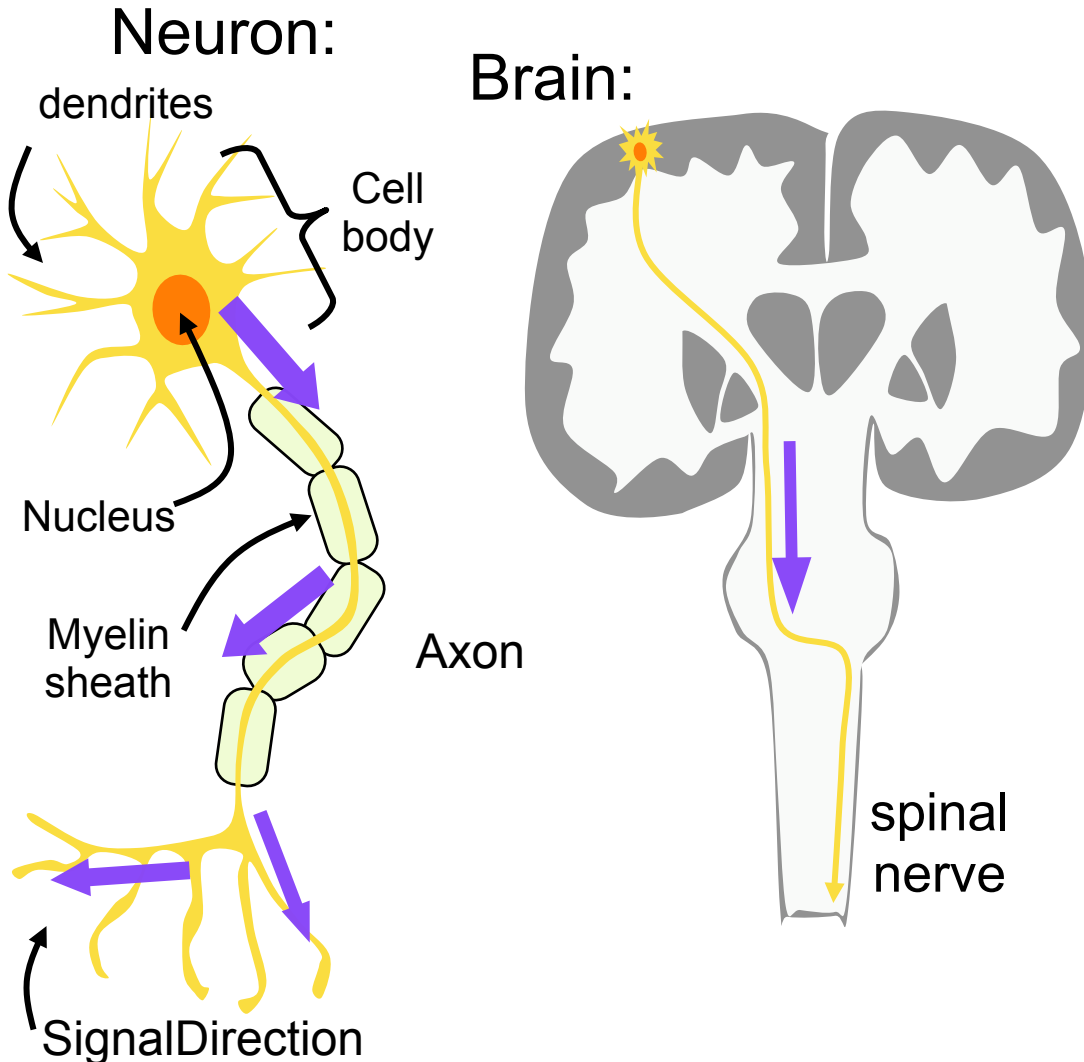
Magnetic gradients create spatial planar waves of proton **phase**



Destructive interference measures diffusion along gradient direction only



# Underlying Biology (Simplified!)



Gray matter (cortex + nuclei): cell bodies

White matter: axons

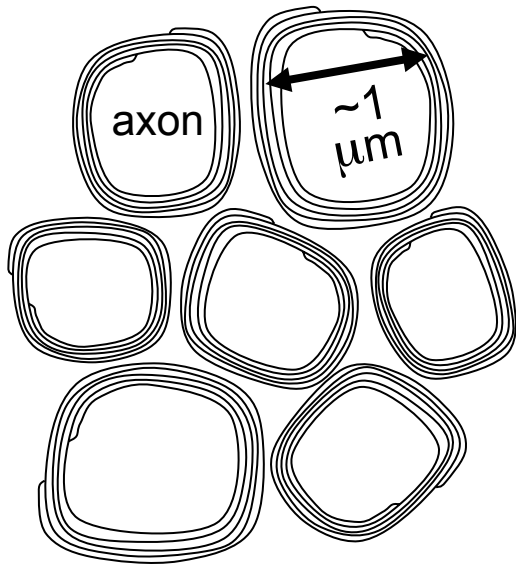
Myelin sheath speeds signal conduction

Axon + sheath = nerve fibers

Major white matter pathways aggregate many fibers into bundles

# Scales in anisotropy of DW-MRI

Fiber bundle  
Cross-section:



Microstructure of bundles directionally  
constrains water diffusion along fiber  
direction (LeBihan et al. 1985)

Intra- vs. extra-cellular diffusion?

Diffusion lengths with the time-scale of MR  
measurement ( $TE \sim 100\text{ms}$ ) on order of  
 $10\mu\text{m}$

Apparent diffusion coefficient: ADC

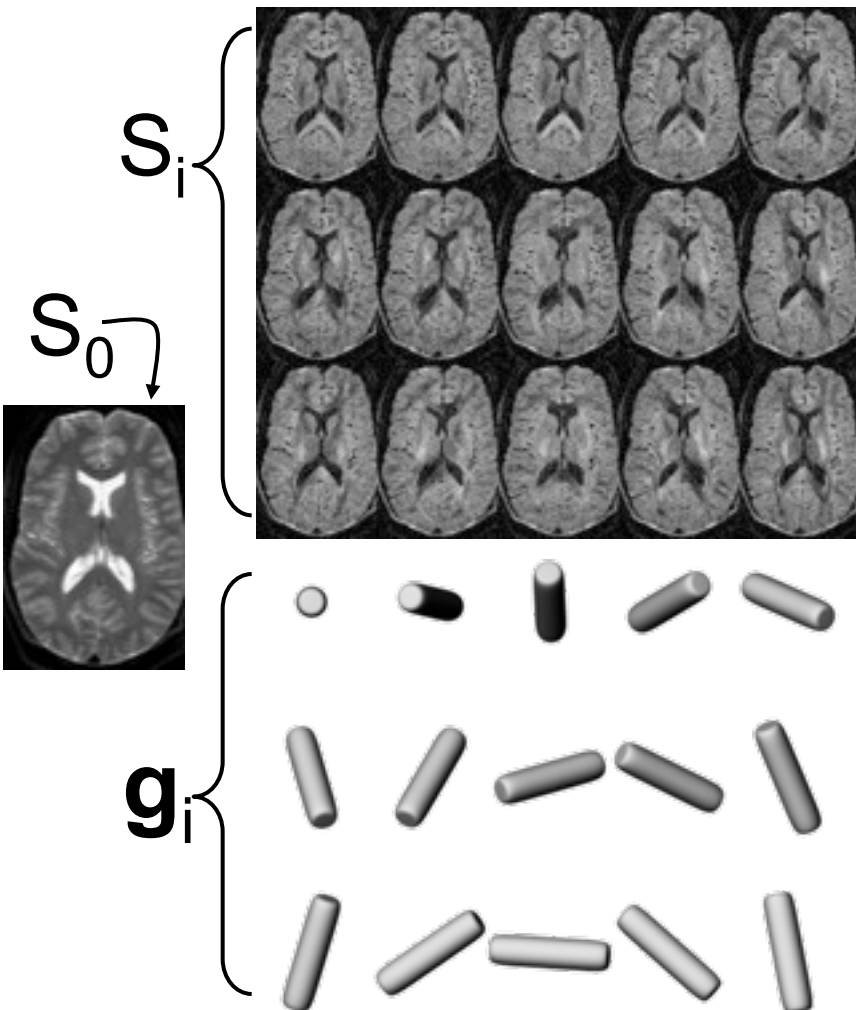
Voxels on the order of 1mm

⇒ **Two to three orders of magnitude away  
from measuring axons**

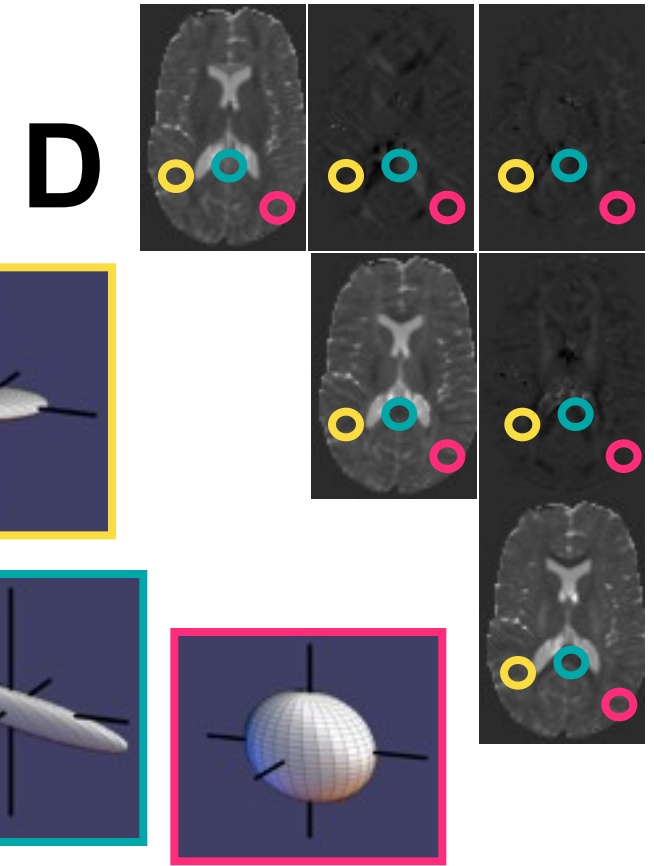
# Multiple DWI → Tensor Estimate

Single Tensor Model (Basser et al. 1994)

$$S_i(b, \mathbf{g}_i) = S_0 e^{-b \mathbf{g}_i^T \mathbf{D} \mathbf{g}_i}$$



Linear regression





Images from Virtual Hospital ([www.vh.org](http://www.vh.org))

- Neuroanatomy (in vivo)
- Neurosurgery: tumor/tract relationship
- Ischemic stroke: detection
- Degenerative diseases: ALS, Multiple Sclerosis
- Psychiatric disorders: Schizophrenia, Autism

Measurement & Biology basics

Tensor Shape & Orientation

Data Inspection with Glyphs

Fiber Tracking and Analysis

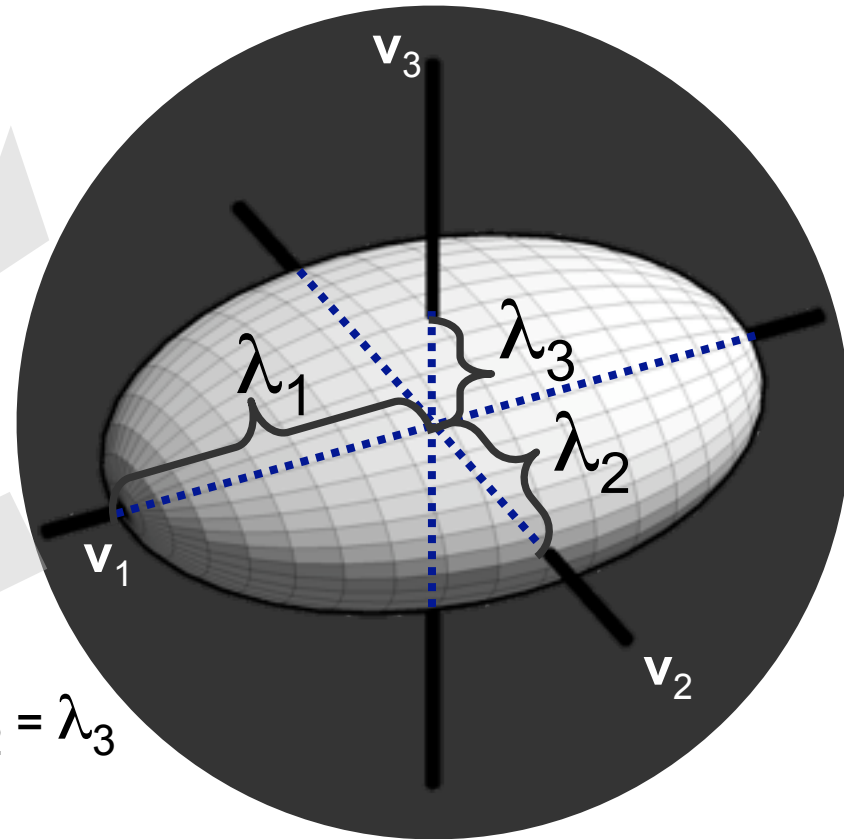
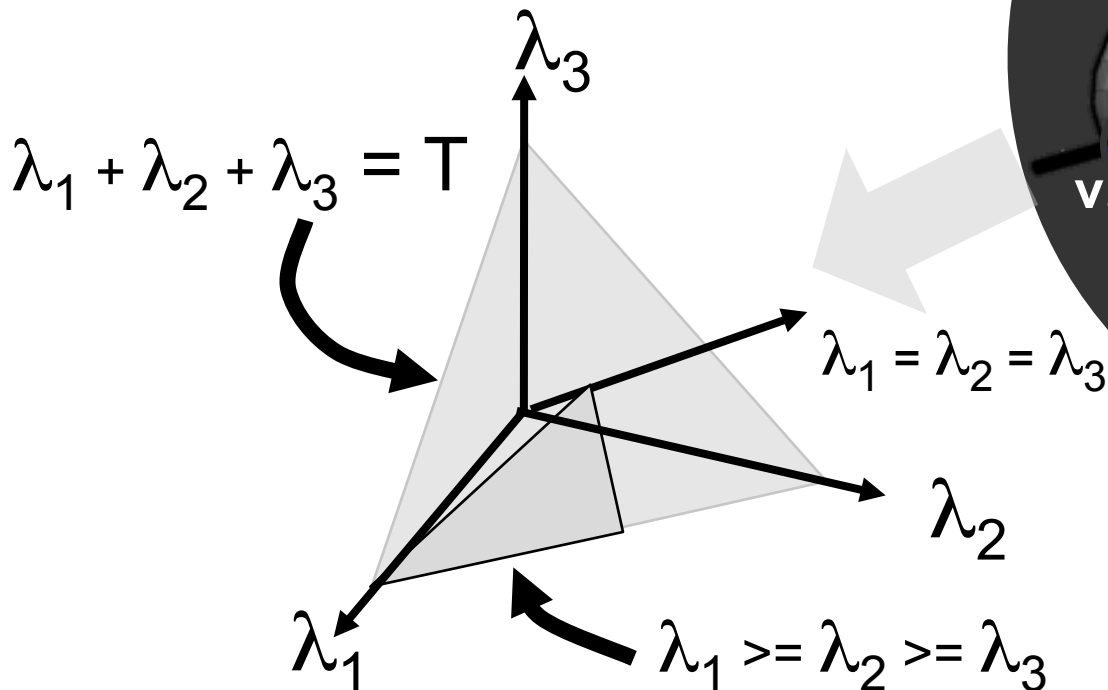
Current Issues and Work



# Eigenvalues == Shape

$$\mathbf{D} = \mathbf{R}\mathbf{\Lambda}\mathbf{R}^{-1}$$

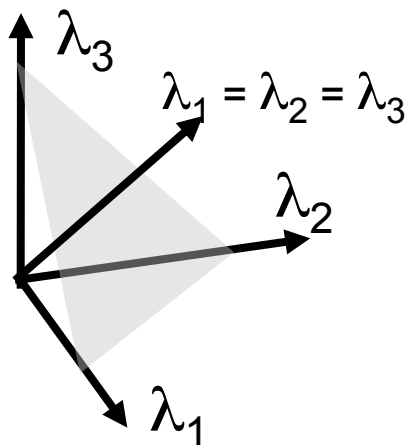
$$= \begin{bmatrix} | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \text{---} \mathbf{v}_1 \\ \text{---} \mathbf{v}_2 \\ \text{---} \mathbf{v}_3 \end{bmatrix}$$



Tensor shape always has 3 degrees of freedom

# Tensor invariants as orthogonal shape parameterizations

Cylindrical or spherical coordinates  
(Ennis & Kindlmann 2005)

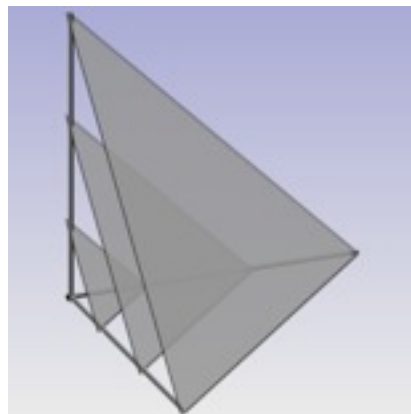


$$\text{tr}(\mathbf{D}) = D_{xx} + D_{yy} + D_{zz}$$

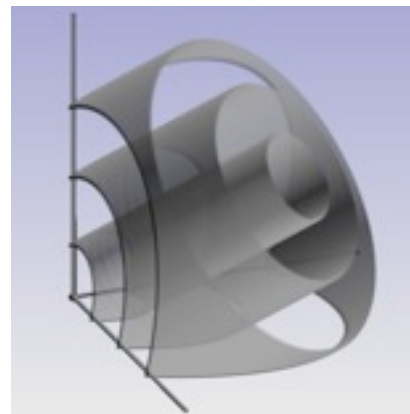
$$|\mathbf{D}| = \sqrt{\text{tr}(\mathbf{D}^T \mathbf{D})}$$

$$\mathbf{E} = \text{deviatoric}(\mathbf{D})$$

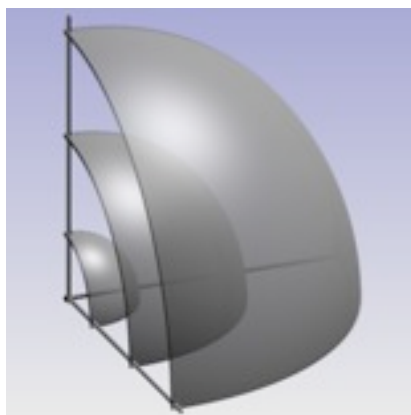
$$= \mathbf{D} - \text{trace}(\mathbf{D}) * \mathbf{I} / 3$$



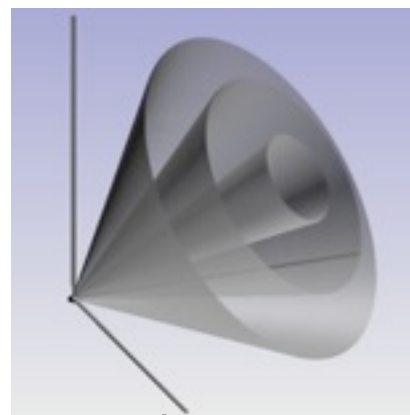
$\text{tr}(\mathbf{D})$



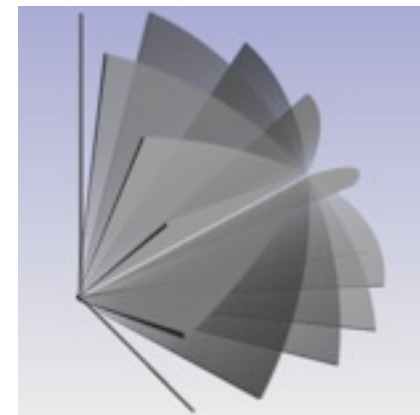
$|\mathbf{E}|$



$|\mathbf{D}|$



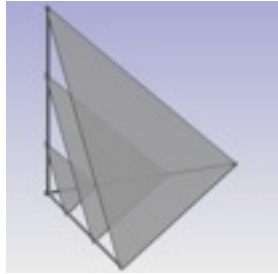
$|\mathbf{E}|/|\mathbf{D}| \sim \text{FA}$   
FA = Fractional Anisotropy



$\text{mode}(\mathbf{E})$   
 $= \det(\mathbf{E}/|\mathbf{E}|)$   
 (Criscione '00)  
 Mode measures  
 Linear vs. planar  
 anisotropy

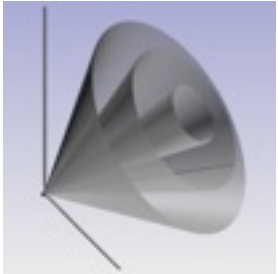
# Biological Meaning of Tensor Shape

## Size: **bulk mean diffusivity** (“ADC”)



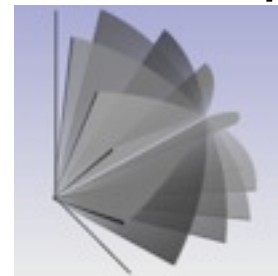
- ADC strictly speaking diffusivity along **one** direction
- Note: same across gray+white matter, high in CSF
- Indicator of acute ischemic stroke

## Anisotropy (e.g. FA): directional microstructure



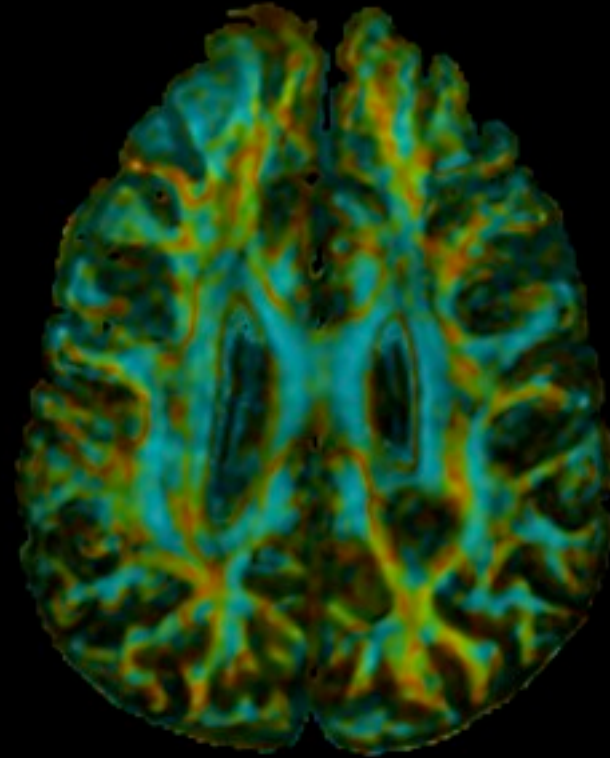
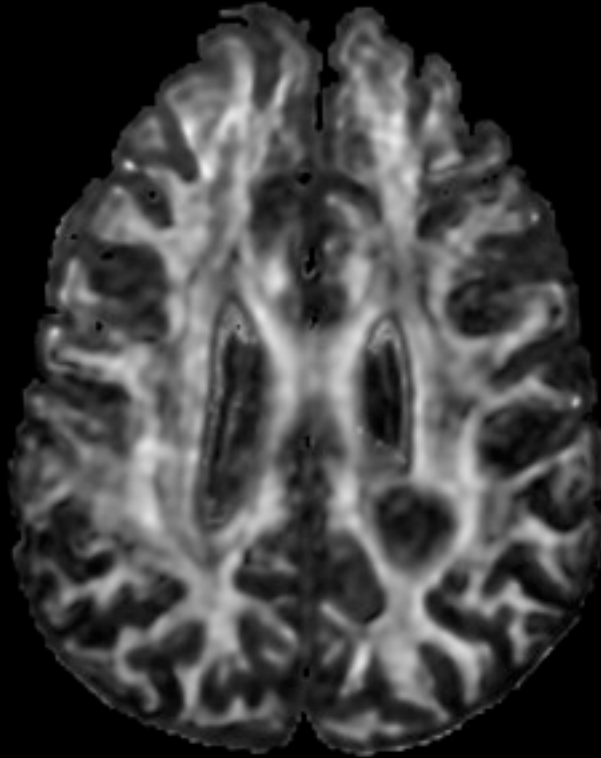
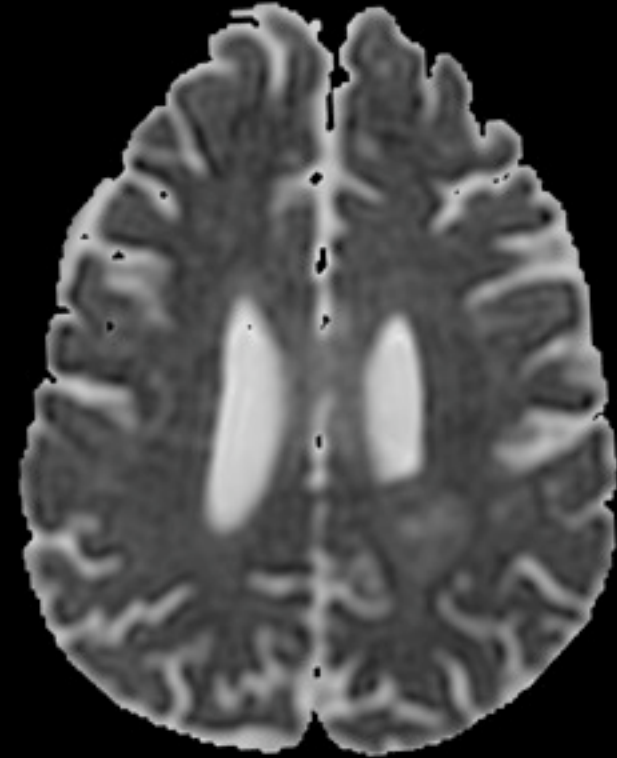
- High in white matter, low in gray matter and CSF
- Increases with myelination, decreases in some diseases (Multiple Sclerosis)

## Mode: linear versus planar



- Partial voluming of adjacent orthogonal structures
- Fine-scale mixing of diverse fiber directions
- Tensor fitting error increases with planarity (Tuch 2002)

# Tensor shape on one slice

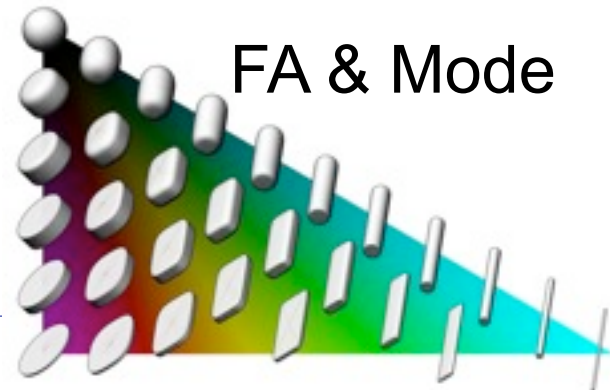


Trace

Fractional **A**nisotropy

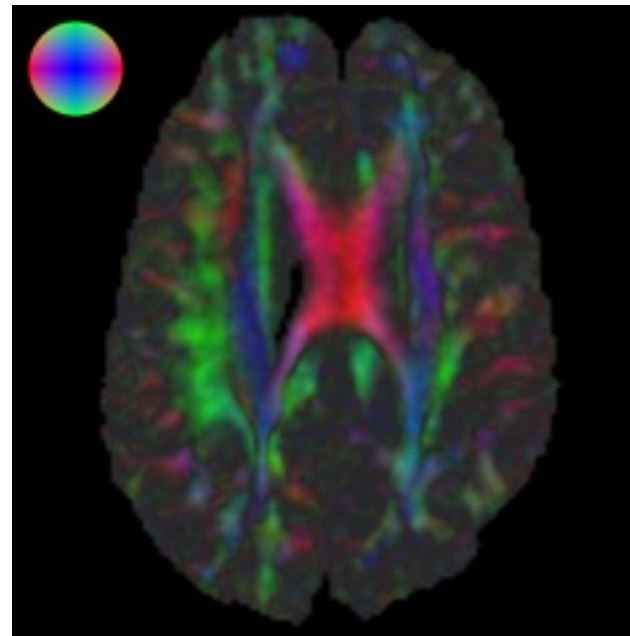
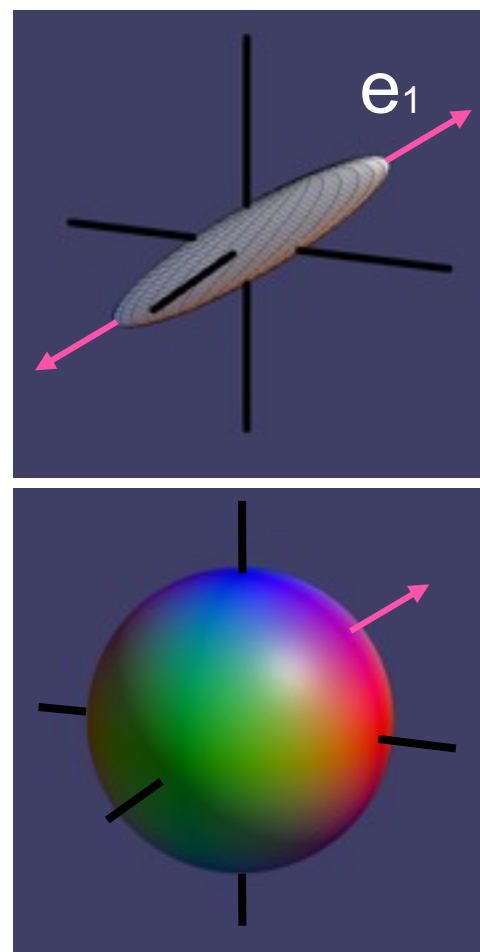
FA & Mode

“Anisotropy” is a bivariate quantity

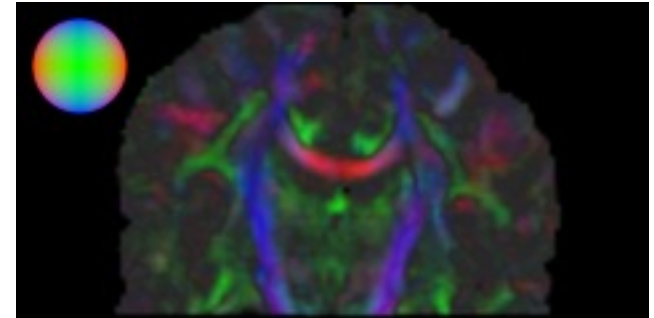


# Tensor Orientation: 1st Eigenvector

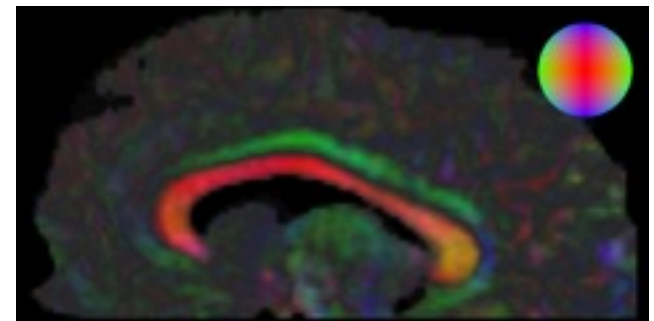
Principal eigenvector gives axon bundle direction



Axial



Coronal



Sagittal

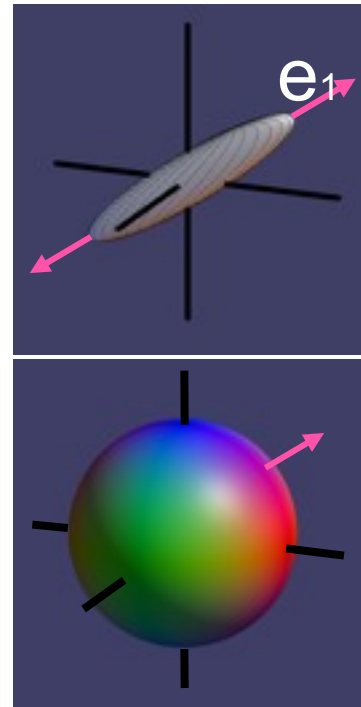
$$R = | \mathbf{e}_1 \cdot \mathbf{x} |$$

$$G = | \mathbf{e}_1 \cdot \mathbf{y} |$$

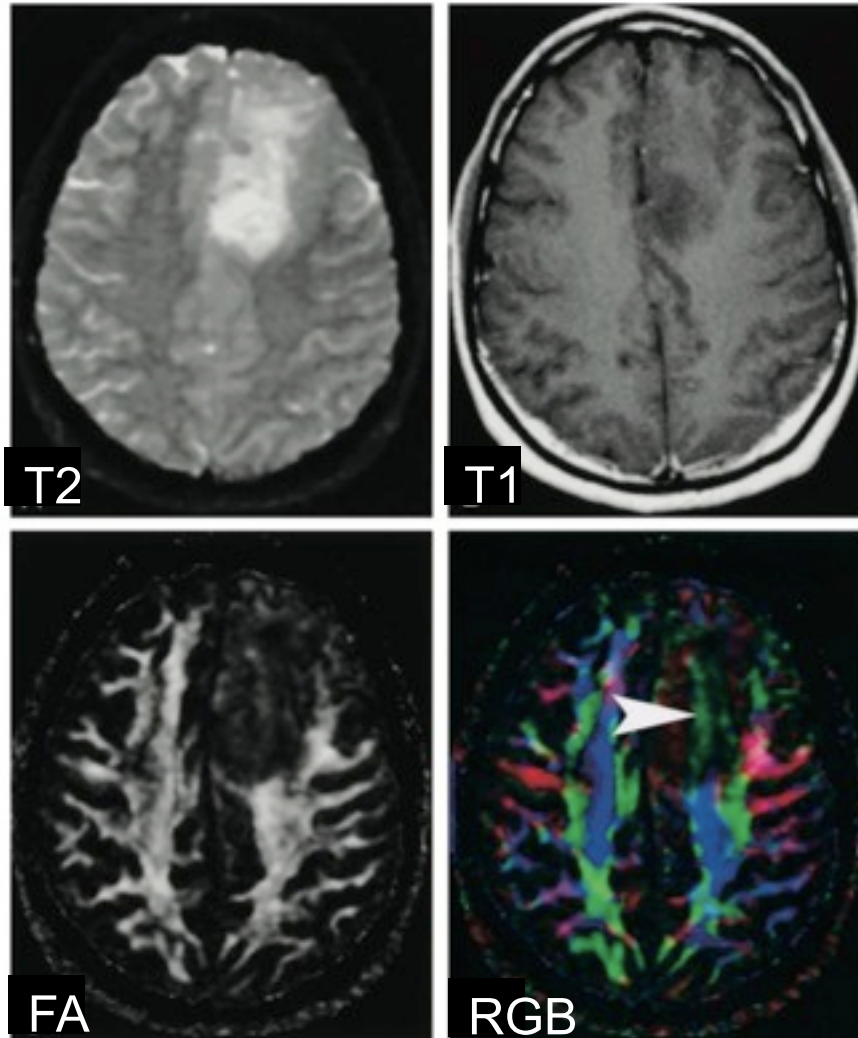
$$B = | \mathbf{e}_1 \cdot \mathbf{z} |$$

(Pajevic & Pierpaoli, 1999)

# RGB maps for tumors



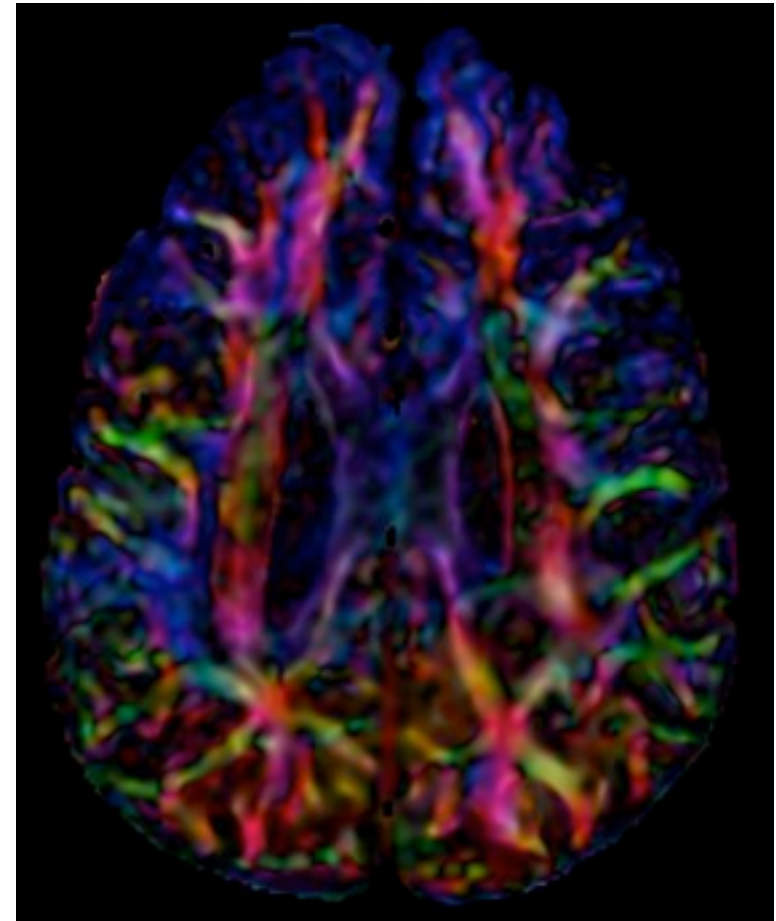
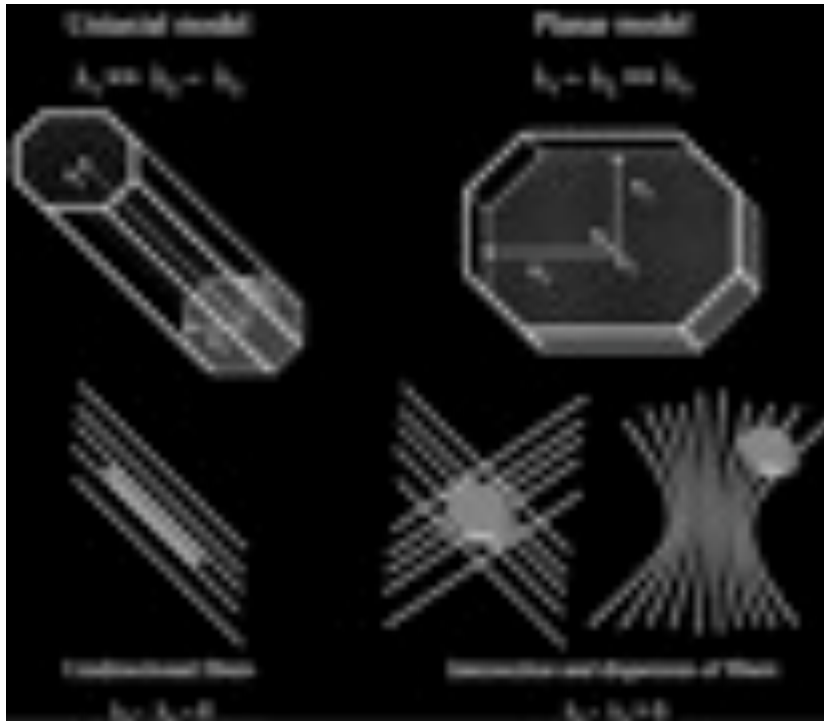
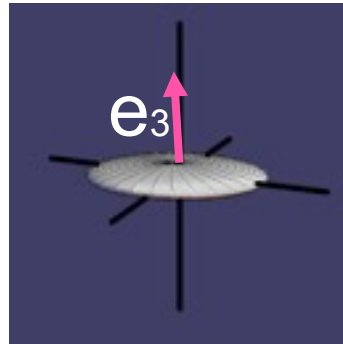
Jellison et al.  
2004



- Allows view of affected region in and around tumor
- 2D views still common for brain surgery planning
- RGB map illustrates changes in white matter

# Tensor Orientation: 3rd Eigenvector

**Minor**  
eigenvector:  
direction of  
**least** diffusion



Significant and coherent planar anisotropy, e.g. around tumor

Measurement & Biology basics

Tensor Shape & Orientation

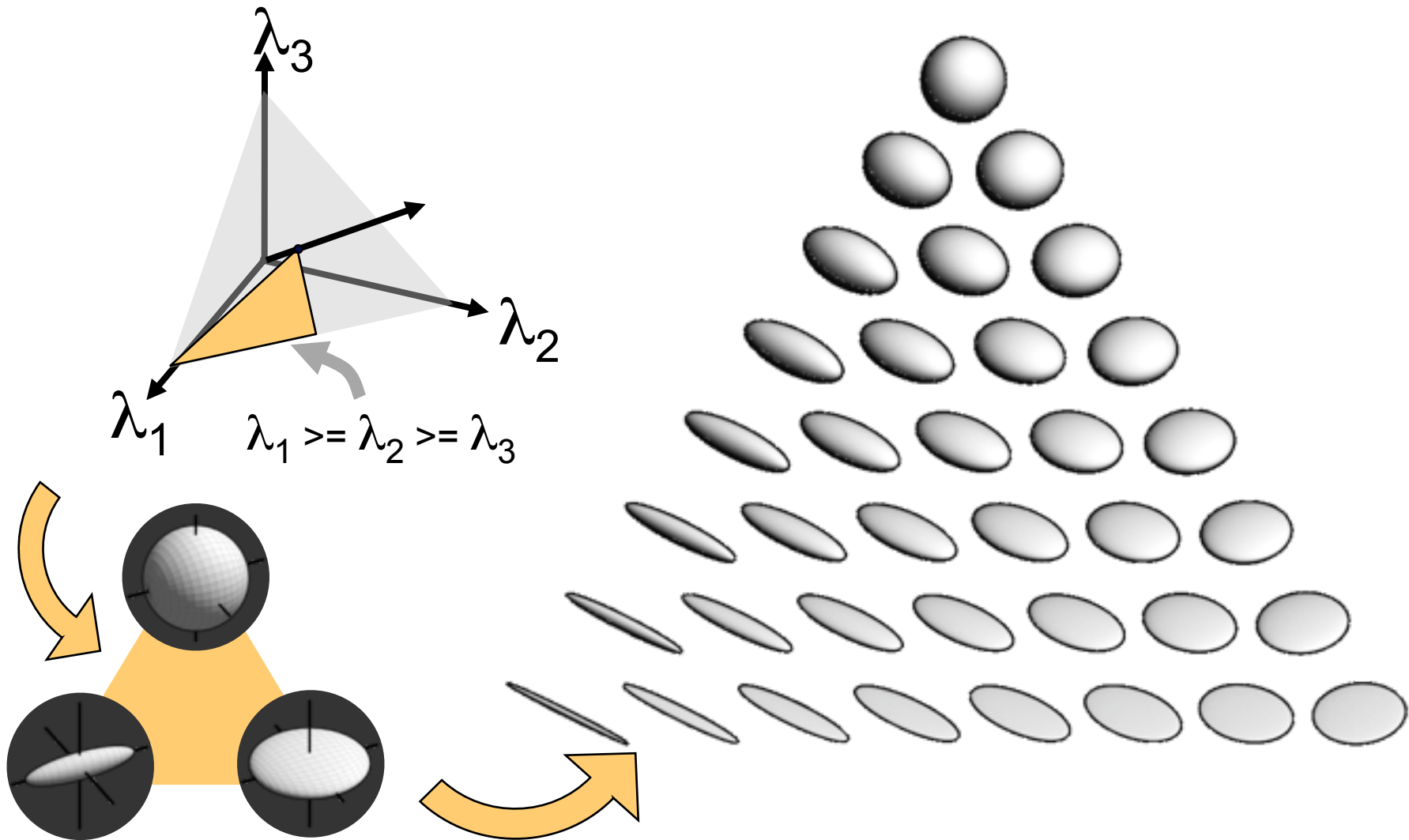
Data Inspection with Glyphs

Fiber Tracking and Analysis

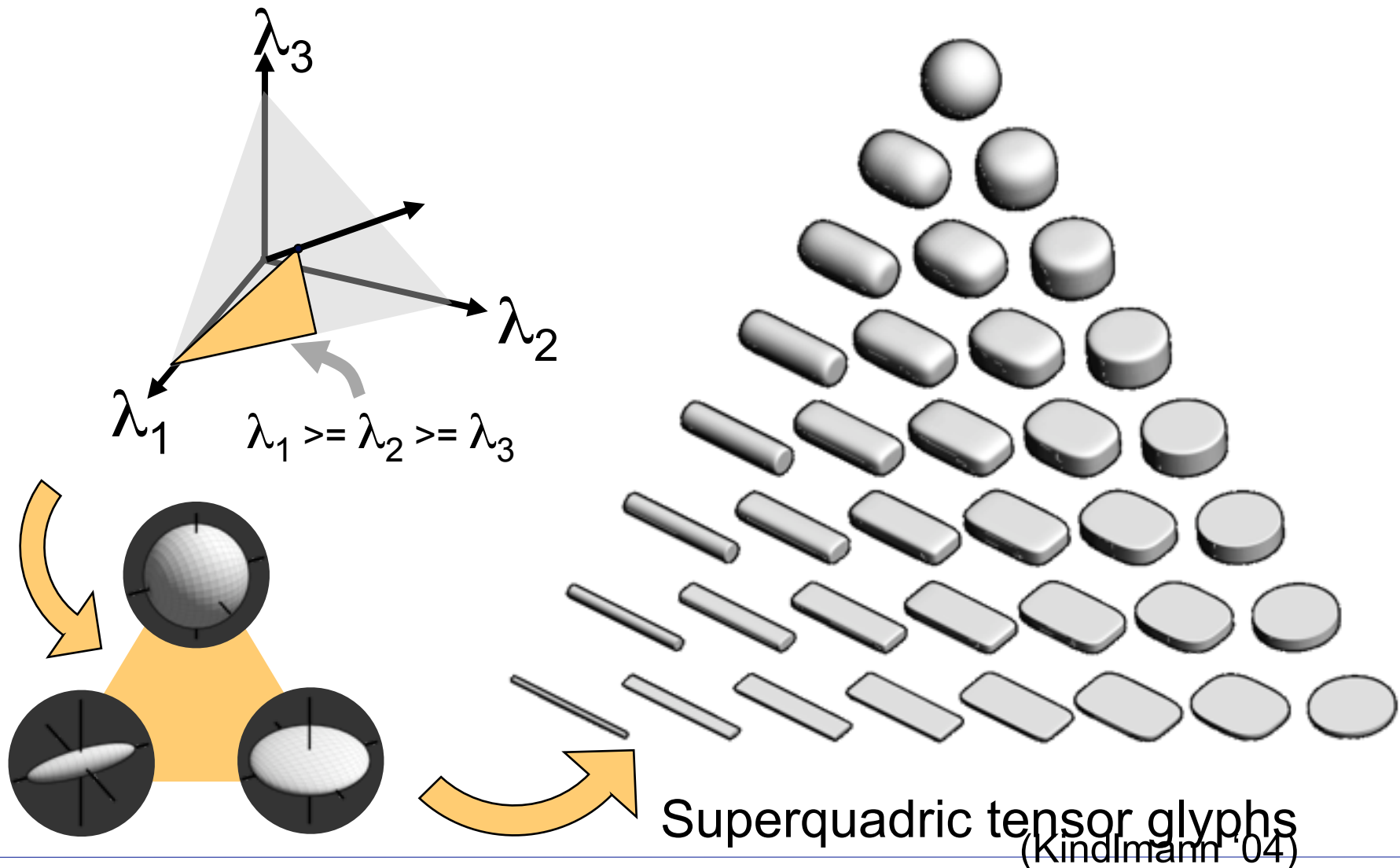
Current Issues and Work



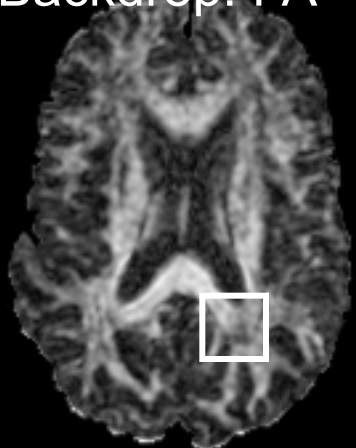
# Glyphs for Range of Tensor Shape



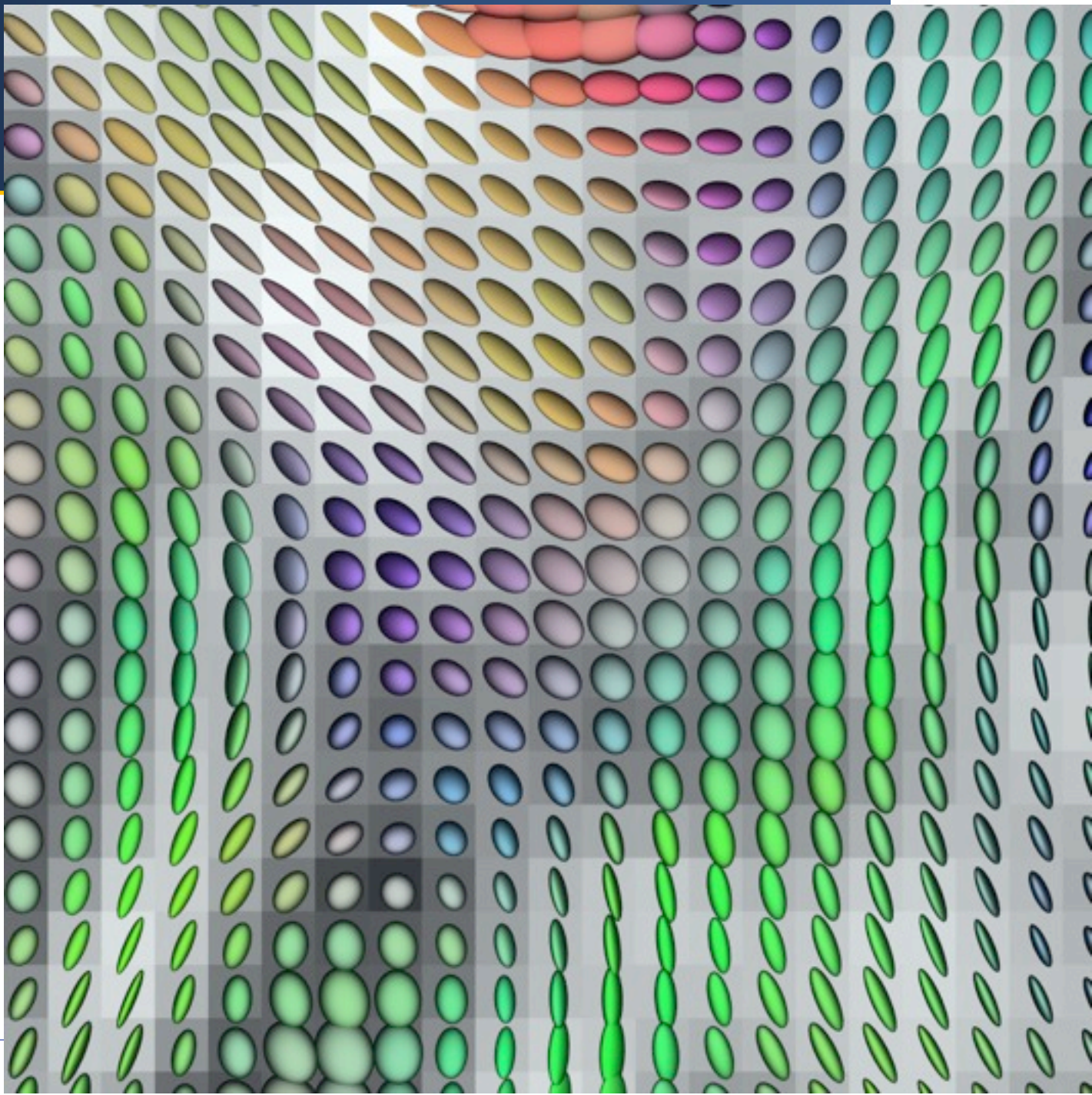
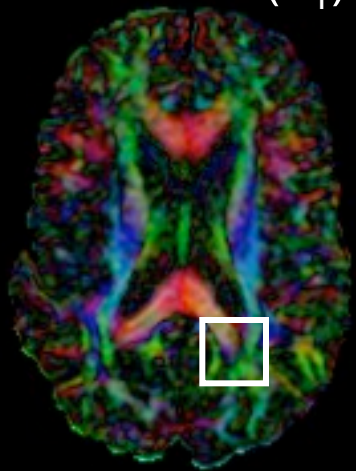
# Glyphs for Range of Tensor Shape



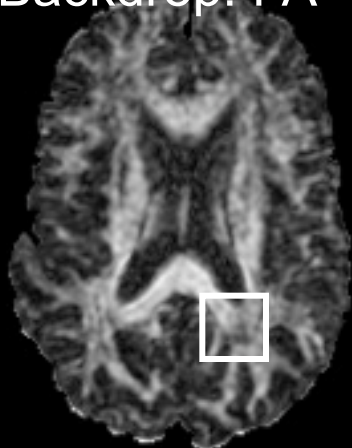
Backdrop: FA



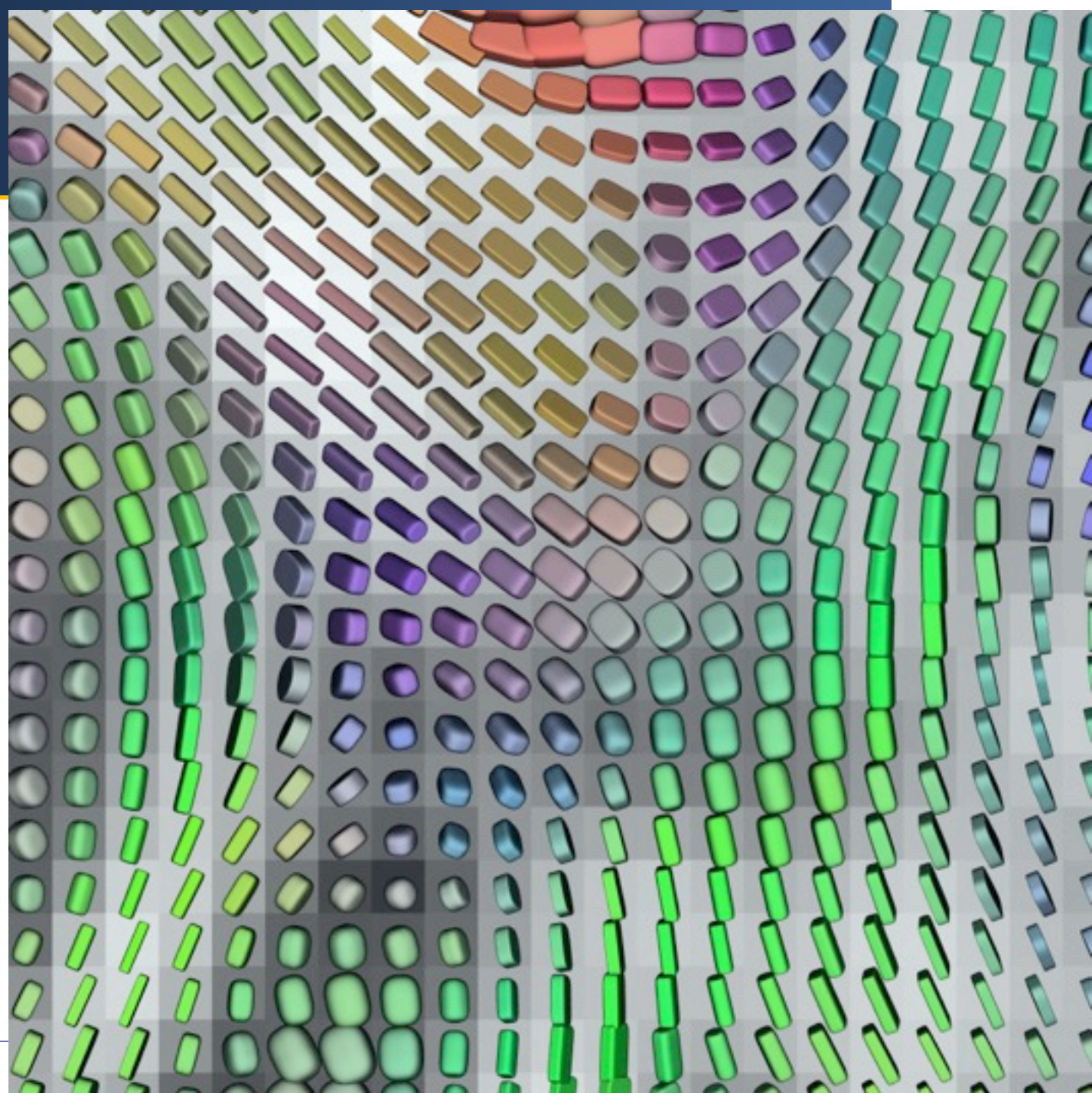
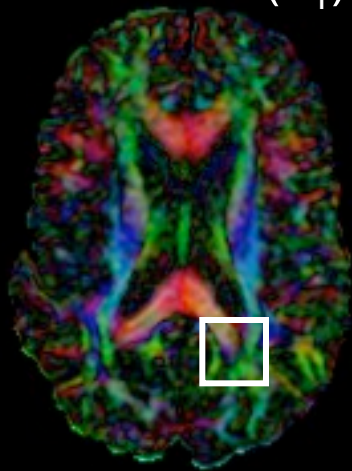
Color: RGB( $e_1$ )



Backdrop: FA



Color: RGB( $e_1$ )



# Glyph Placement

Particle systems (from graphics and anisotropic mesh generation) to “pack” glyphs (Kindlmann & Westin Vis’ 06)

Ongoing work with neurosurgeon

Friday, 10:45 -11:45  
Vector/Tensor Visualization II



Measurement & Biology basics

Tensor Shape & Orientation

Data Inspection with Glyphs

Fiber Tracking and Analysis

Current Issues and Work

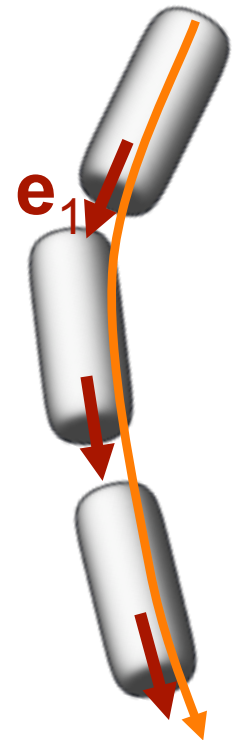
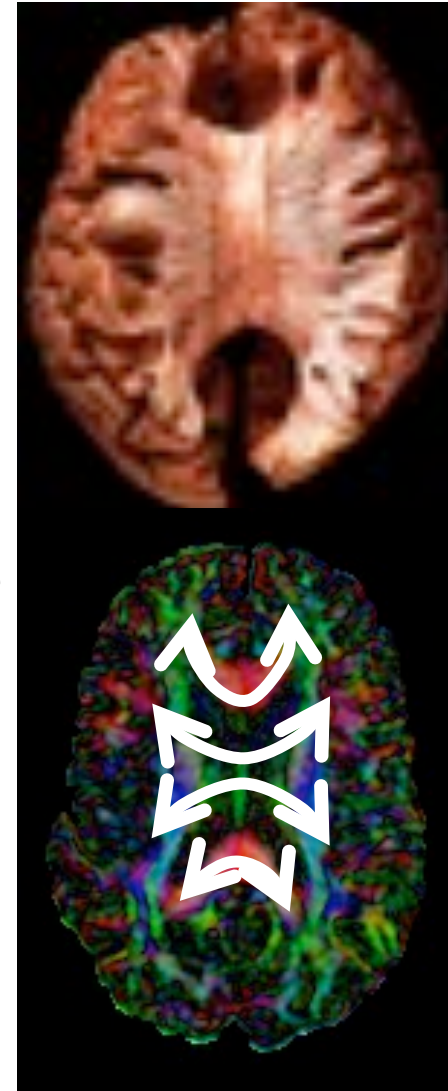
# Fiber tractography (Basser 1999)

Path integration along principal eigenvector

Idea/Fantasy: follow paths of individual axons!

- Reality: 2-3 orders of magnitude too coarse

Essentially same as simplified hyperstreamlines (Delmarcelle 1993)



(demo anisotropy  
isosurfaces + glyphs  
+ fiber tracking)



# Fiber Tracking Issues



- Tensor Field Interpolation/Filtering
- Integration quality, step size
  - Eigensolve at every sample non-trivial
- Seedpoint selection determines path
- Termination criteria
- Parameter space → Reproducibility, Validation

Measurement & Biology basics

Tensor Shape & Orientation

Data Inspection with Glyphs

Fiber Tracking and Analysis

Current Issues and Work

Single Tensor Model limited: fiber crossings

→ High Angular Resolution Diffusion Imaging

Spatial Resolution still somewhat coarse

→ Parallel imaging, better post-processing

Tractography validation not common

→ Histological comparison

Fusion with other modalities: fMRI, MEG

→ Wide open visualization research problem!

# Acknowledgements



**Carl-Fredrik Westin, PhD**, Laboratory of Mathematics in Imaging,  
Brigham & Women's Hospital, Harvard Medical School

**Alexandra Golby, MD**, Stephen Whalen, Department of  
Neurosurgery, Brigham & Women's Hospital, Harvard Medical School

This work supported by NIH grants NIBIB T32-  
EB002177, P41-RR13218 (NAC), and U41-  
RR019703-01A2, as well as the Brain Science  
Foundation.

# References



- Le Bihan D, Breton E. Imagerie de diffusion in-vivo par resonance magnetique nucleaire. Cr. Acad. Sci. (Paris) 1985; 301: 1109–1112.
- PJ Basser, J Mattiello, and D Le Bihan. MR diffusion tensor spectroscopy and imaging. Biophysics Journal, 66(1):259–267, 1994
- DB Ennis and G Kindlmann. “Orthogonal tensor invariants and the analysis of diffusion tensor magnetic resonance images.” Magnetic Resonance in Medicine 55(1):136–146 (2006)
- DS Tuch, TG Reese, MR Wiegell, N Makris, JW Belliveau, and VJ Wedeen. High angular resolution diffusion imaging reveals intravoxel white matter fiber heterogeneity. Magnetic Resonance in Medicine, 48:577–582, 2002.
- S Pajevic and C Pierpaoli. Color schemes to represent the orientation of anisotropic tissues from diffusion tensor data: Application to white matter fiber tract mapping in the human brain. Magnetic Resonance in Medicine, 42(3):526–540, 1999.
- Jellison BJ, Field AS, Medow J, Lazar M, Salamat MS, Alexander AL. Diffusion tensor imaging of cerebral white matter: a pictorial review of physics, fiber tract anatomy, and tumor imaging patterns. AJNR Am J Neuroradiol. 2004 Mar;25(3):356-69.
- MR Wiegell, HBW Larsson, and VJ Wedeen. Fiber crossing in human brain depicted with diffusion tensor MR imaging. Radiology, 217(3):897–903, Dec 2000.
- G Kindlmann. Superquadric tensor glyphs. In Proceedings of IEEE TVCG/EG Symposium on Visualization 2004, pages 147–154, May 2004
- G Kindlmann and C-F Westin. “Diffusion Tensor Visualization with Glyph Packing.” Transactions on Visualization and Computer Graphics, 12(5):1329–1336 (2006)
- T Delmarcelle and L Hesselink. Visualizing second-order tensor fields with hyper streamlines. IEEE Computer Graphics and Applications, 13(4):25–33, 1993.