Symmetry and Continuity in Visualization and Tensor Glyph Design

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Topic

- **Symmetry** and **Continuity**
  - General: for colormaps, scalar vis ...
  - Specific: glyphs for symmetric tensors
- Experiences with superquadric tensor glyphs (VisSym '04)
- What we at Dagstuhl can do together:
  - Design a more general tensor glyph
  - Formulate principles of Vis design & evaluation
- I'm junior; tell me what I'm missing

Symmetry

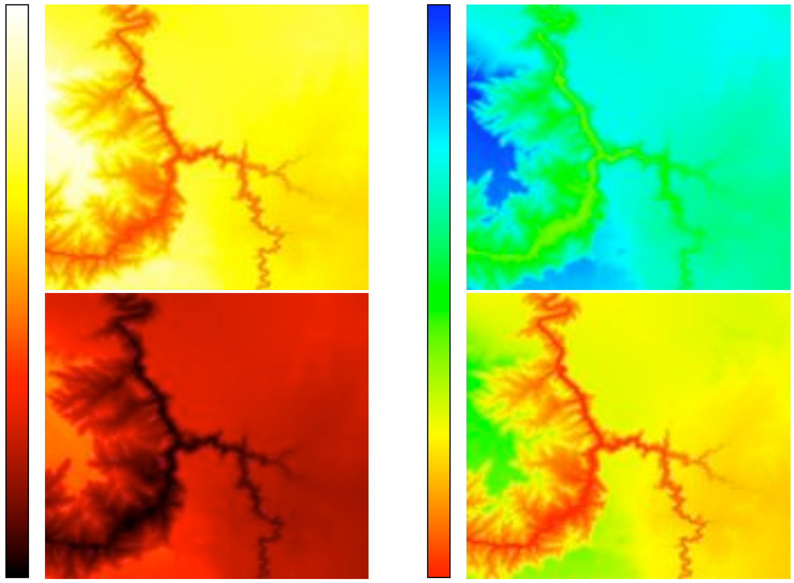
$$T(D) \approx D \Rightarrow V(T(D)) \approx V(D)$$

- Extending notation from van Wijk "Value of Visualization" Vis '05
- Symmetry: transformation *T* leaves thing unchanged
- Vis has same symmetries as data, no more, no less
- Tufte "Chart junk": avoid showing **structure** that does not reflect actual **information**
- Stevens 1946 "Scales of Measure": nominal, ordinal, interval, ratio

Symmetry

Coloring (or effectiveness of coloring) of **nominal** values should be symmetric under permutation

Symmetry



$V(D + c) \approx V(D)$ $V(D + c) \neq V(D)$

Coloring of **interval** values should be symmetric under addition of constant (like a translation)

Symmetry

- MRI samples measured in frequency space ⇒ location of spatial pixels is **arbitrary** (translation = phase)

Glyph packing invariant WRT translation of sample location

Anti-symmetry

$$T(D) \approx -D \Rightarrow V(T(D)) \approx -V(D)$$

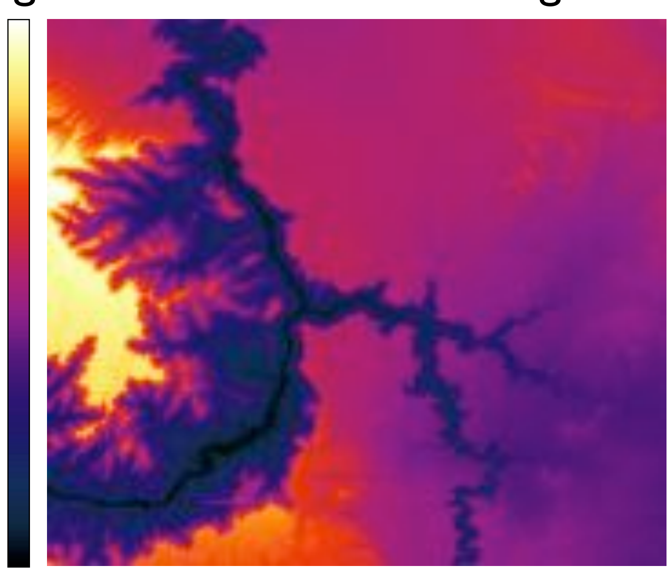
- Coloring of **ratio** values should be anti-symmetric under negation (like a reflection)

- Example of coloring divergence of vector field

Continuity

$$V(D + \epsilon) \approx V(D)$$

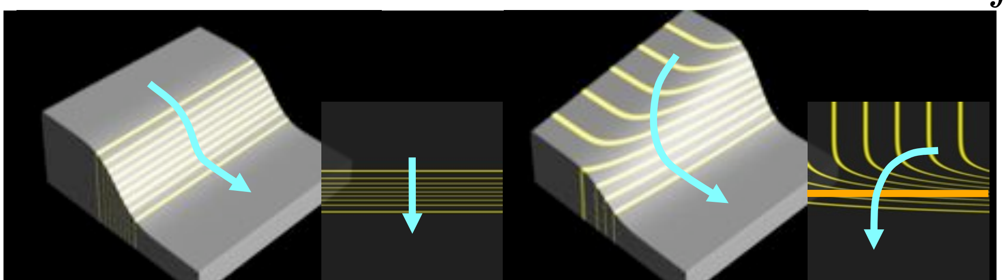
- Small changes in **data** → small changes in Vis



Continuity

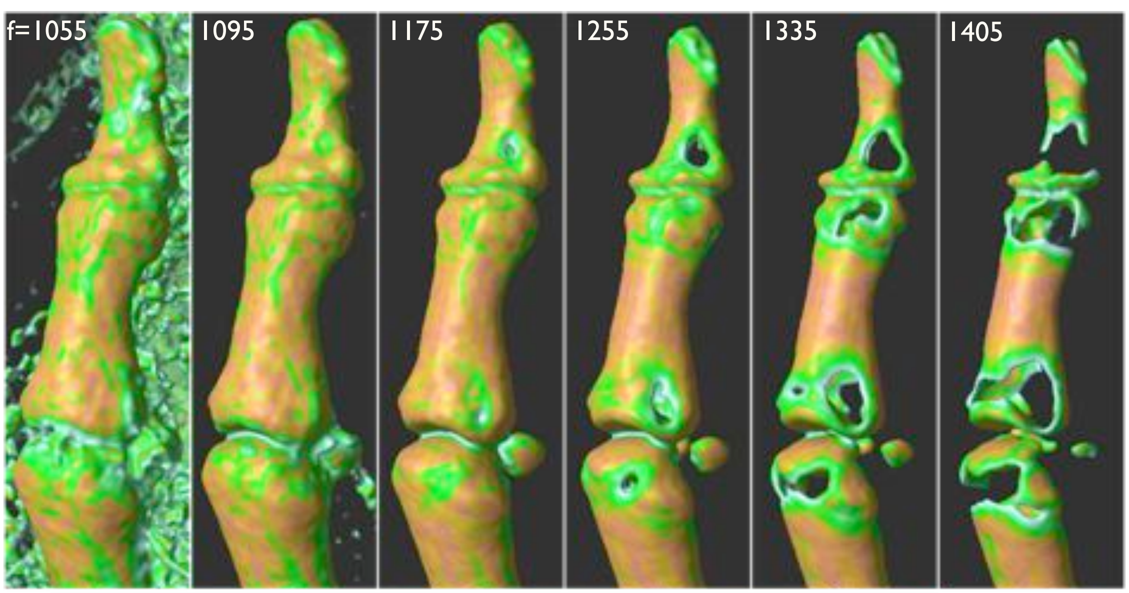
$$V(D, S + \epsilon) \approx V(D, S)$$

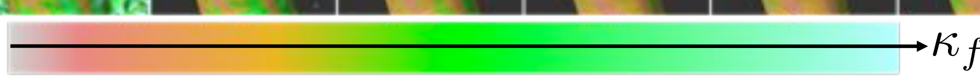
- Small changes in **parameters** → small changes in Vis
- Failures of continuity:
  - Gimbal lock in rotations or fixed up-vectors
  - Small changes in isovalue cause large changes in isosurface orientation: high flowline curvature κ_f



$\kappa_f = 0$ $\qquad$ $\kappa_f > 0$

Continuity





Flowline Curvature κ_f highlights "discontinuity" (or at least uncertainty) in isosurface visualization

Symmetric Tensors

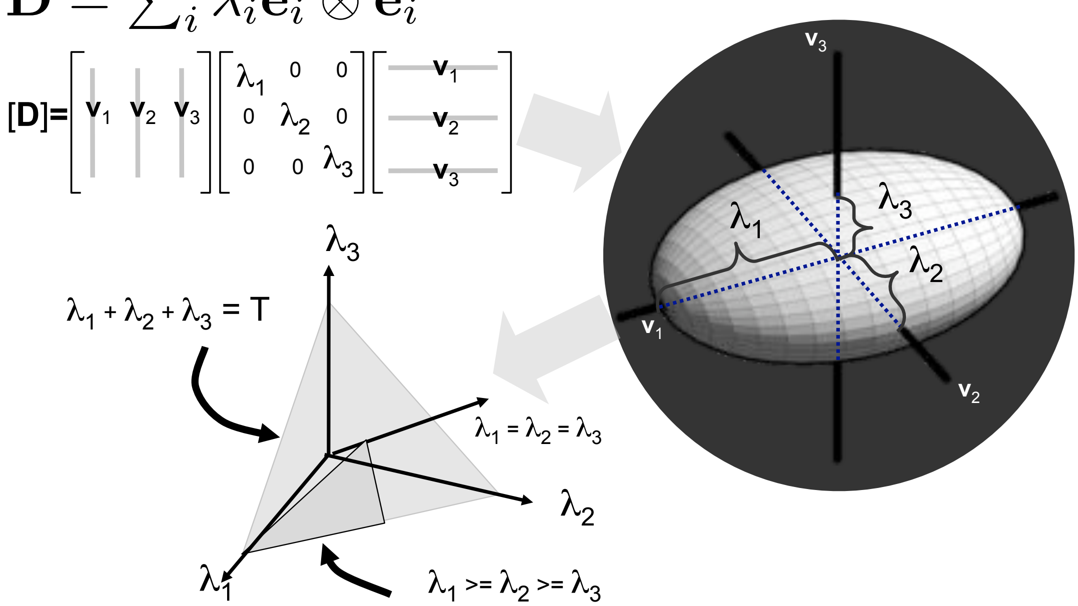
$$\mathbf{D}^{\mathsf{T}} = \mathbf{D}$$

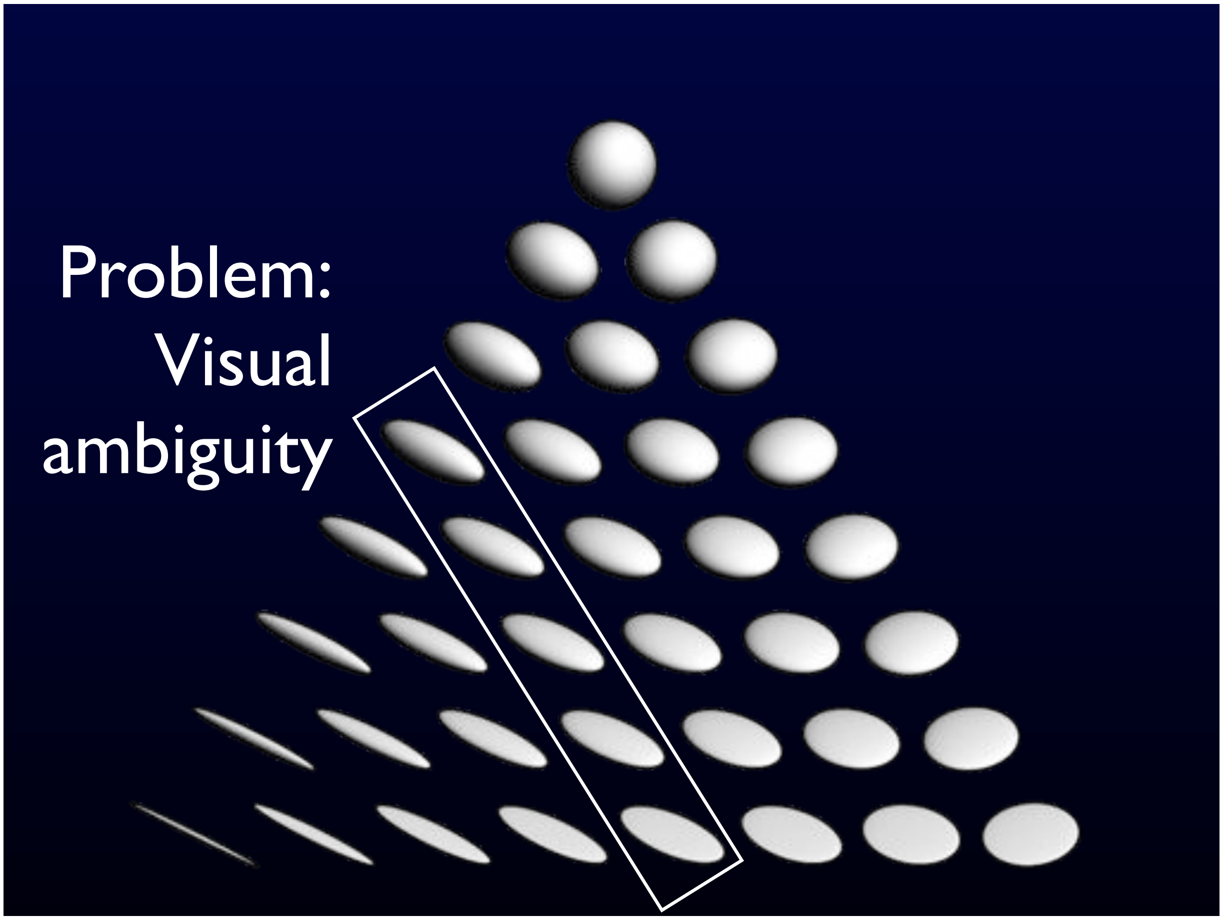
- (different meanings of "symmetry")
- Applications: Diffusion tensors
  - Only positive eigenvalues, glyphs exist
- Hessian, Rate of Strain, Strain/Stress
  - Positive and negative eigenvalues, no glyphs!
  - Missed opportunities for vis applications
- This is a basic, unsolved vis problem

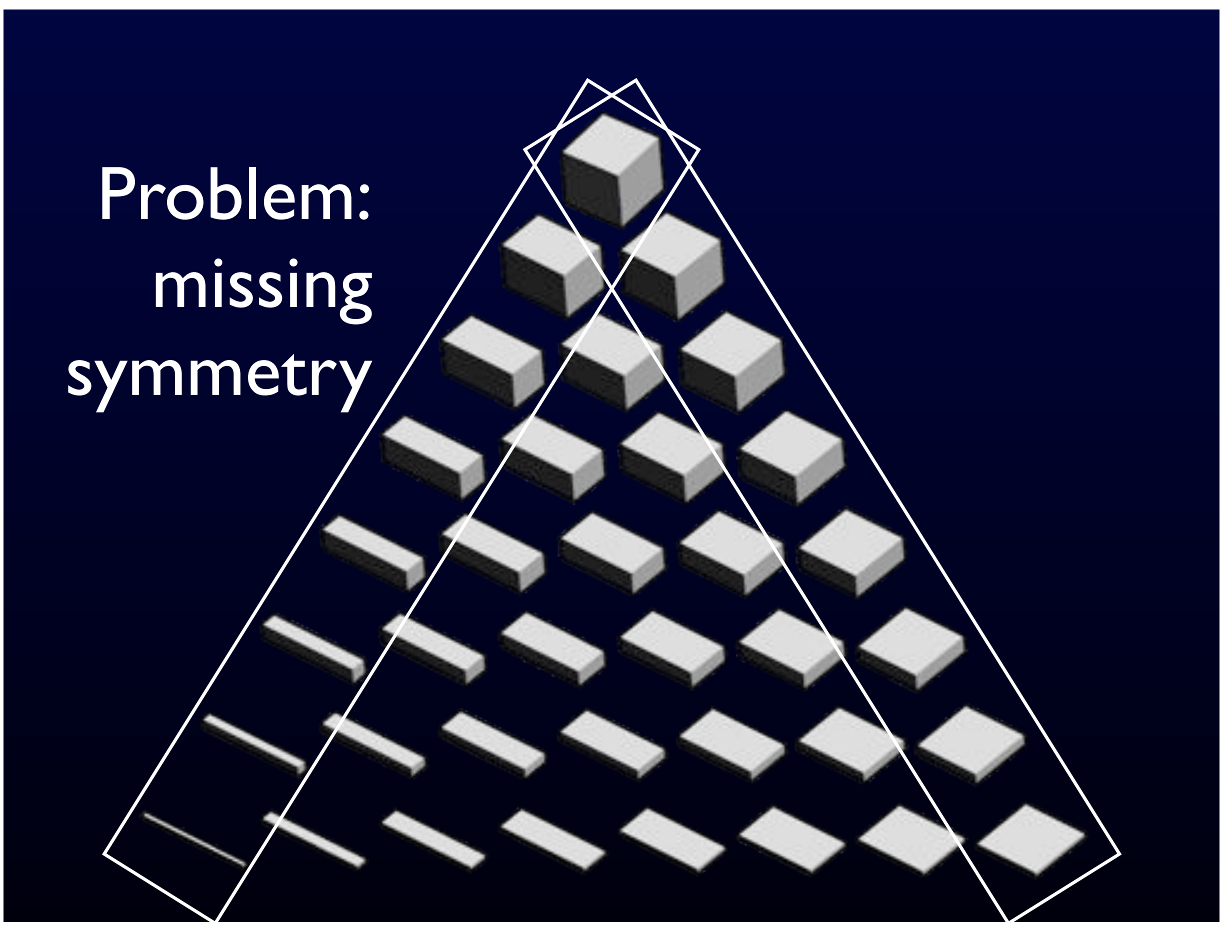
Symmetric Tensors

- Real eigenvalues, orthogonal eigenvectors

$$\mathbf{D} = \sum_i \lambda_i \mathbf{e}_i \otimes \mathbf{e}_i$$




Problem:
Visual
ambiguity


Problem:
missing
symmetry

$\lambda_1 = \lambda_2$

$\Downarrow$

2-D eigenspace(λ_1)

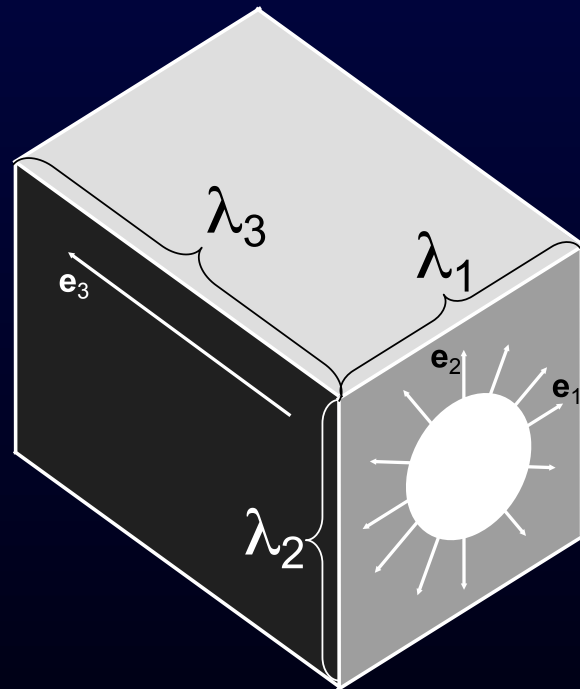
$\Downarrow$

$\mathbf{e_1}$, $\mathbf{e}_2$ not unique

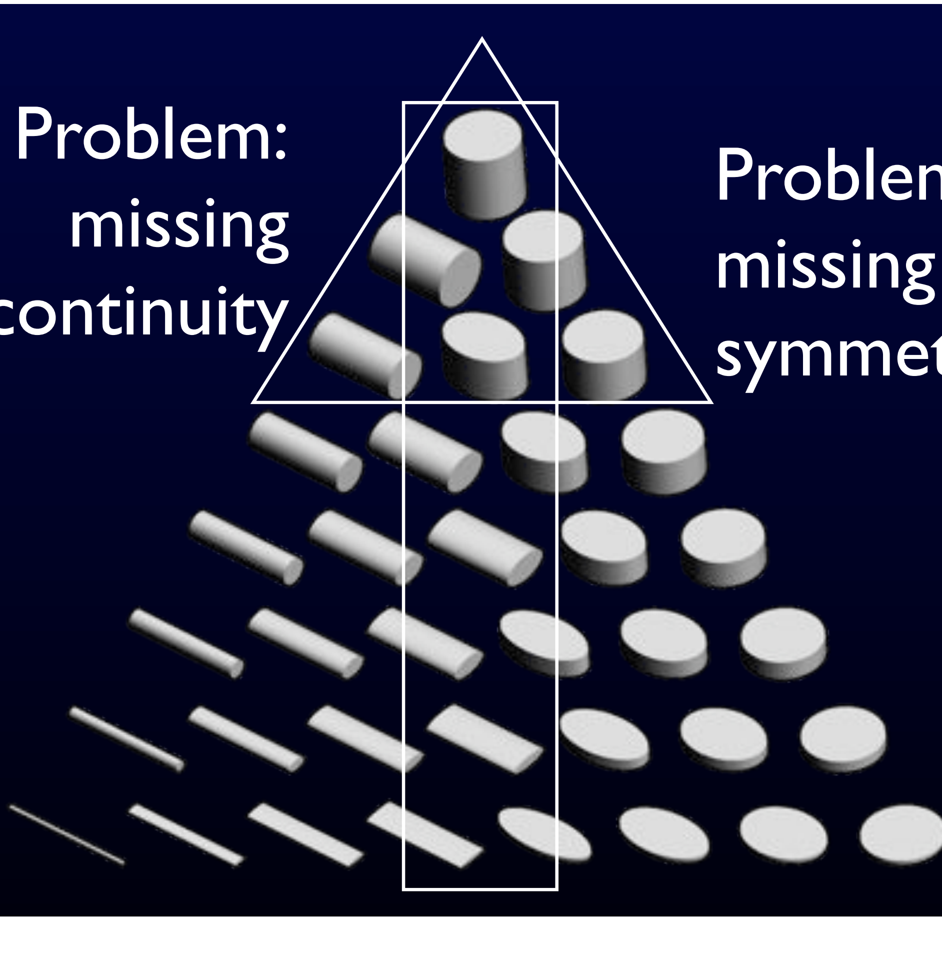
(continuous rotational symmetry)

Conversely,

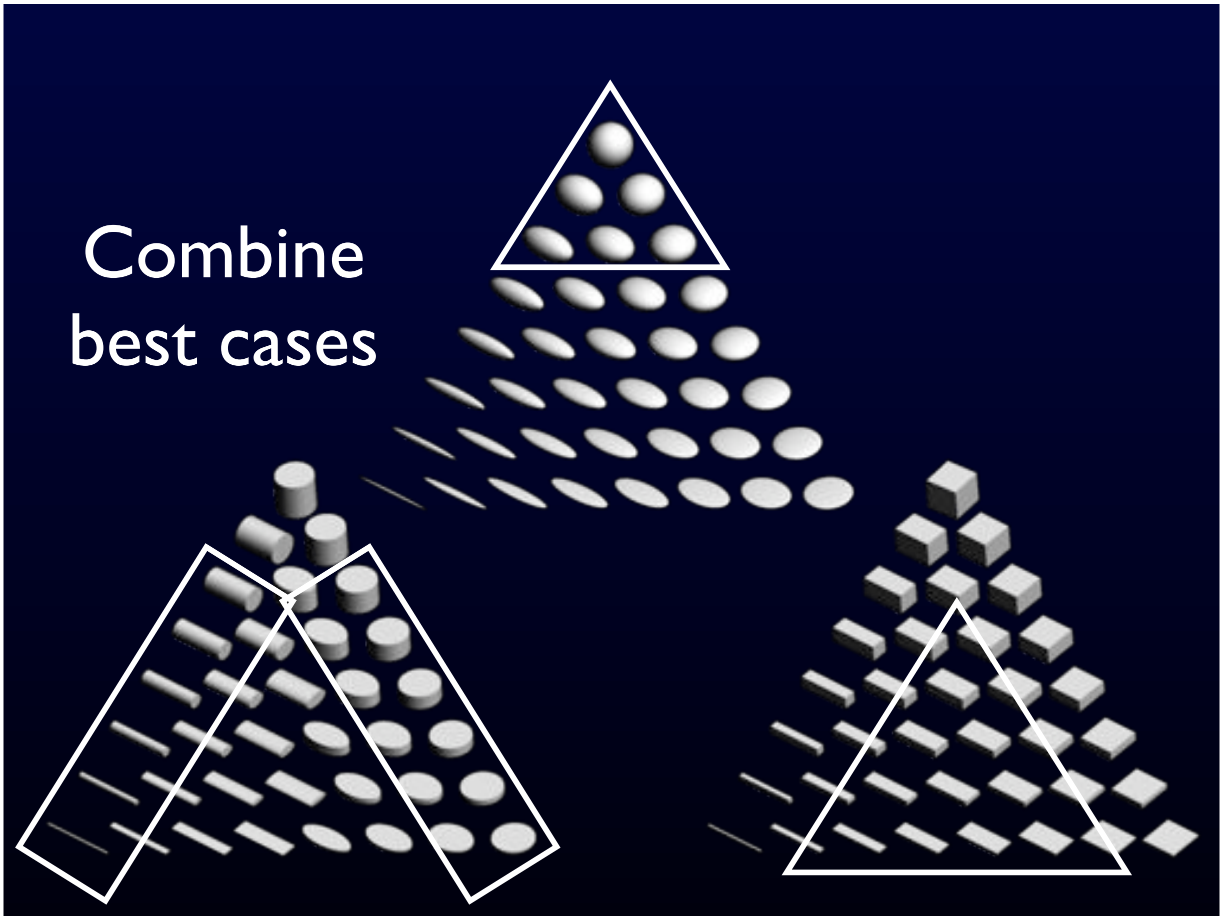
$\mathbf{e_1}$, $\mathbf{e}_3$ distinct

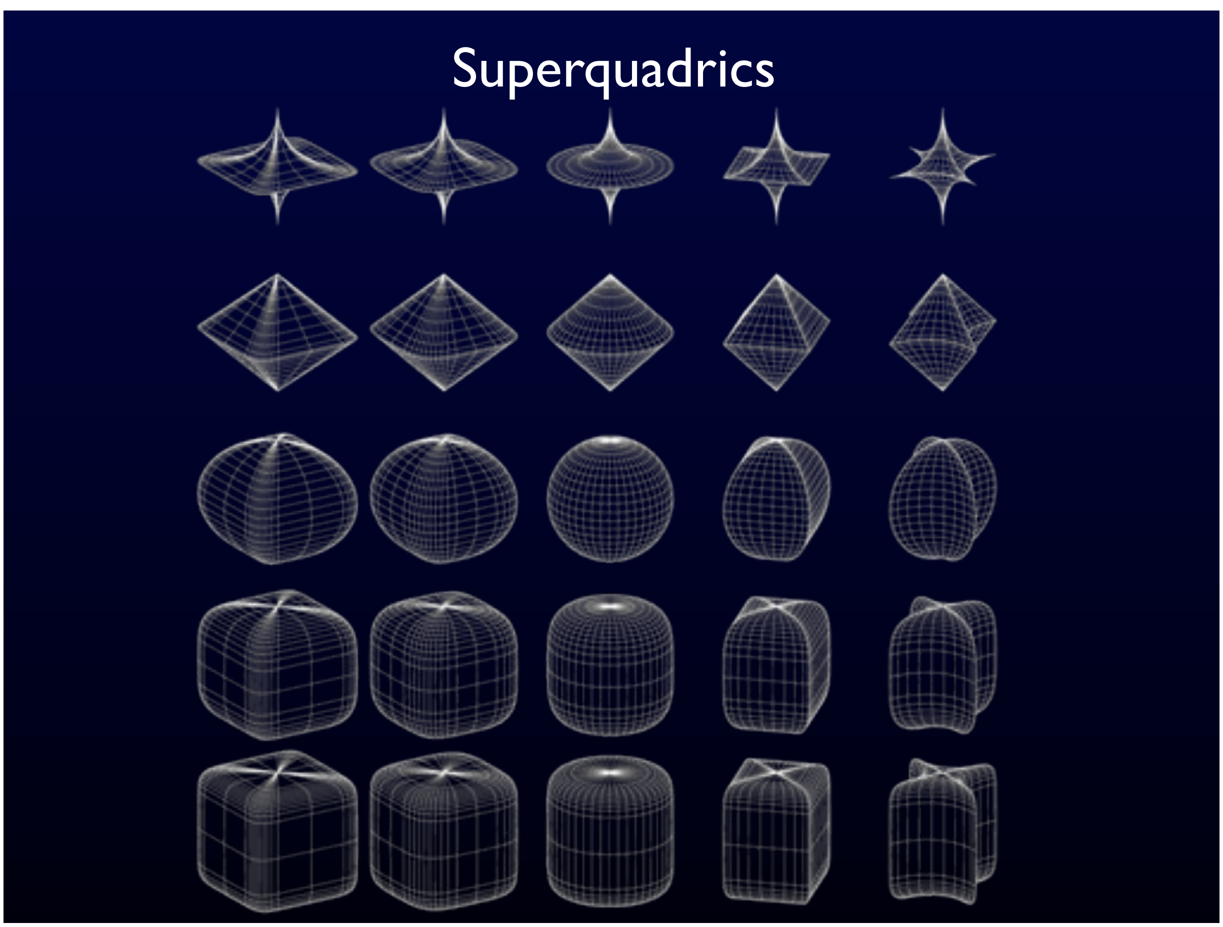


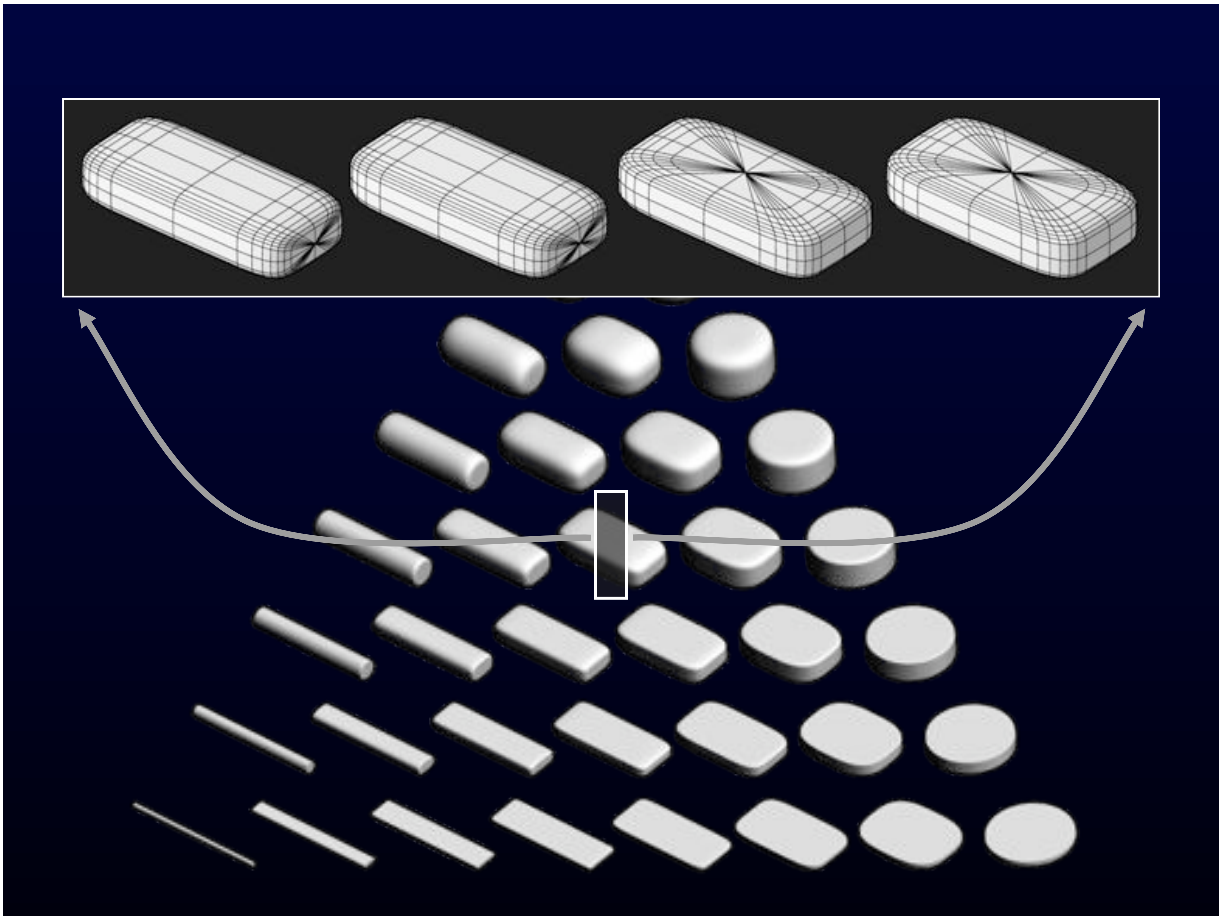
Problem:
missing
continuity



Problem:
missing
symmetry


Combine
best cases


Superquadrics



Symmetric Tensors

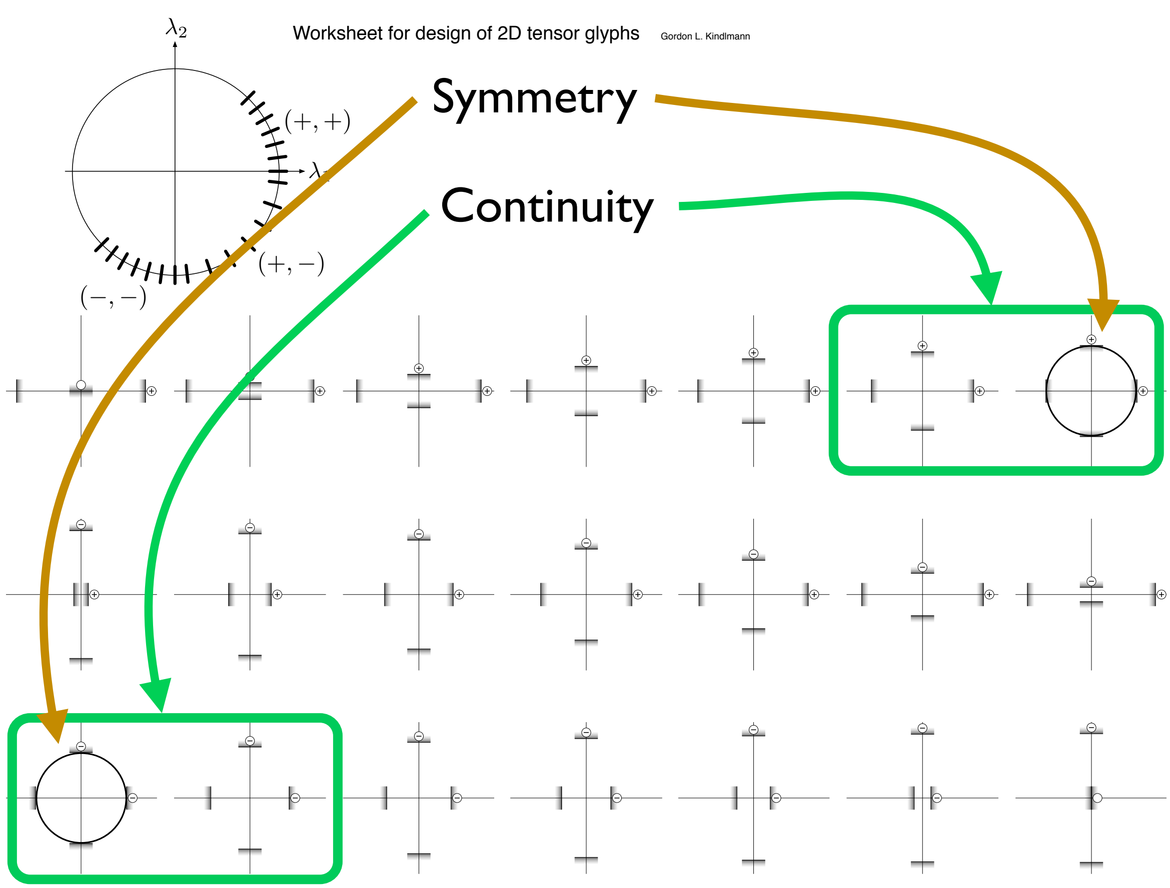
$$\lambda_1 = \lambda_2 = \lambda \Rightarrow$$

$$\begin{aligned} \mathbf{D} &= \lambda_1 \mathbf{e}_1 \otimes \mathbf{e}_1 + \lambda_2 \mathbf{e}_2 \otimes \mathbf{e}_2 \\ &= \lambda(\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2) \\ &= \lambda \mathbf{I} \end{aligned}$$

- Everything is an eigenvector, tensor has no intrinsic orientation ⇒ glyph must be circle or sphere
- Regardless of $\lambda > 0$ or $\lambda < 0$ ⇒ can't use shape to indicate sign of λ in isotropic case $\mathbf{D}=\lambda\mathbf{I}$

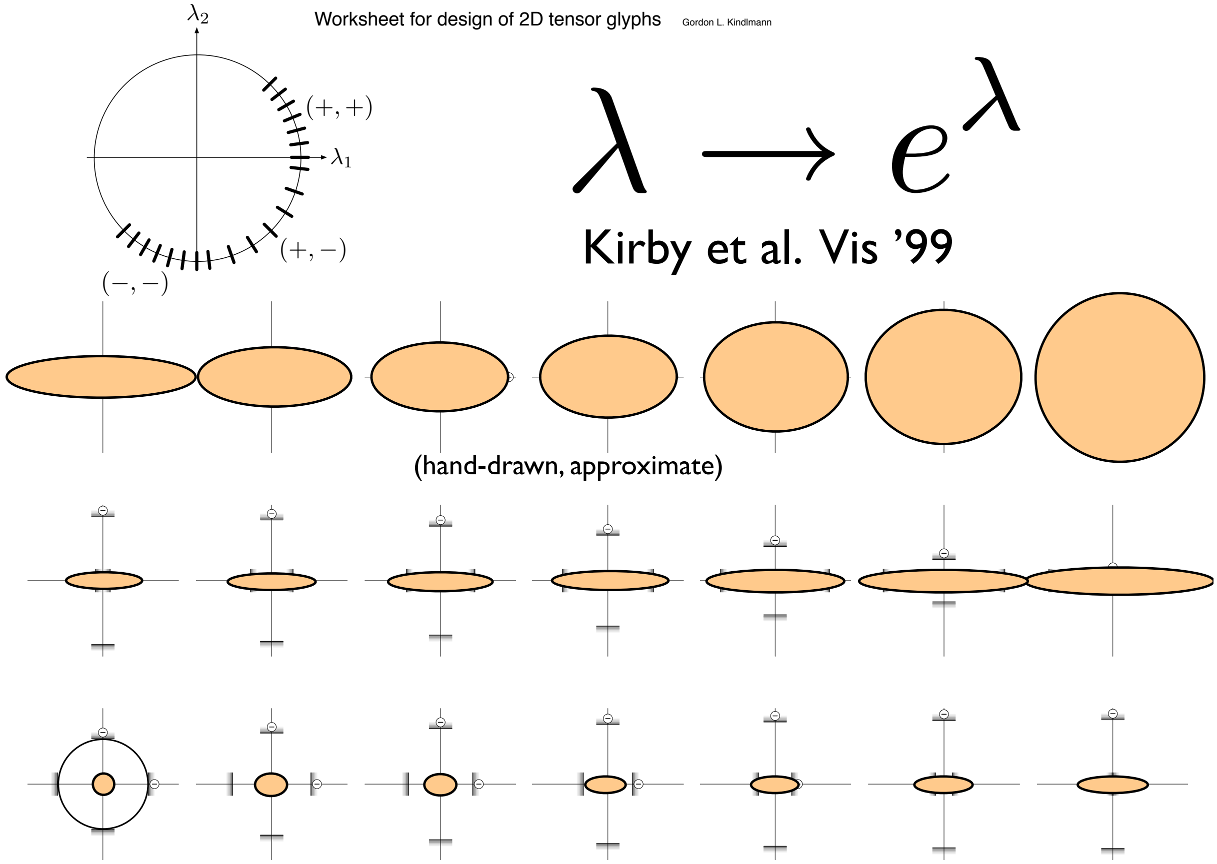
Tensor Glyph Design

- Want glyph for (symmetric) tensors with all necessary **symmetry** and **continuity** properties
- How to show eigenvalue sign?
- Made worksheets for designing 2D tensor glyphs
    - Let's solve a visualization **modest** challenge
- Keep 3D extension in mind
    - 2D glyphs as cross-sections of 3D glyphs
- Keefe et al. TVCG 14(4) Scientific Sketching for Collaborative VR Visualization Design (2008)



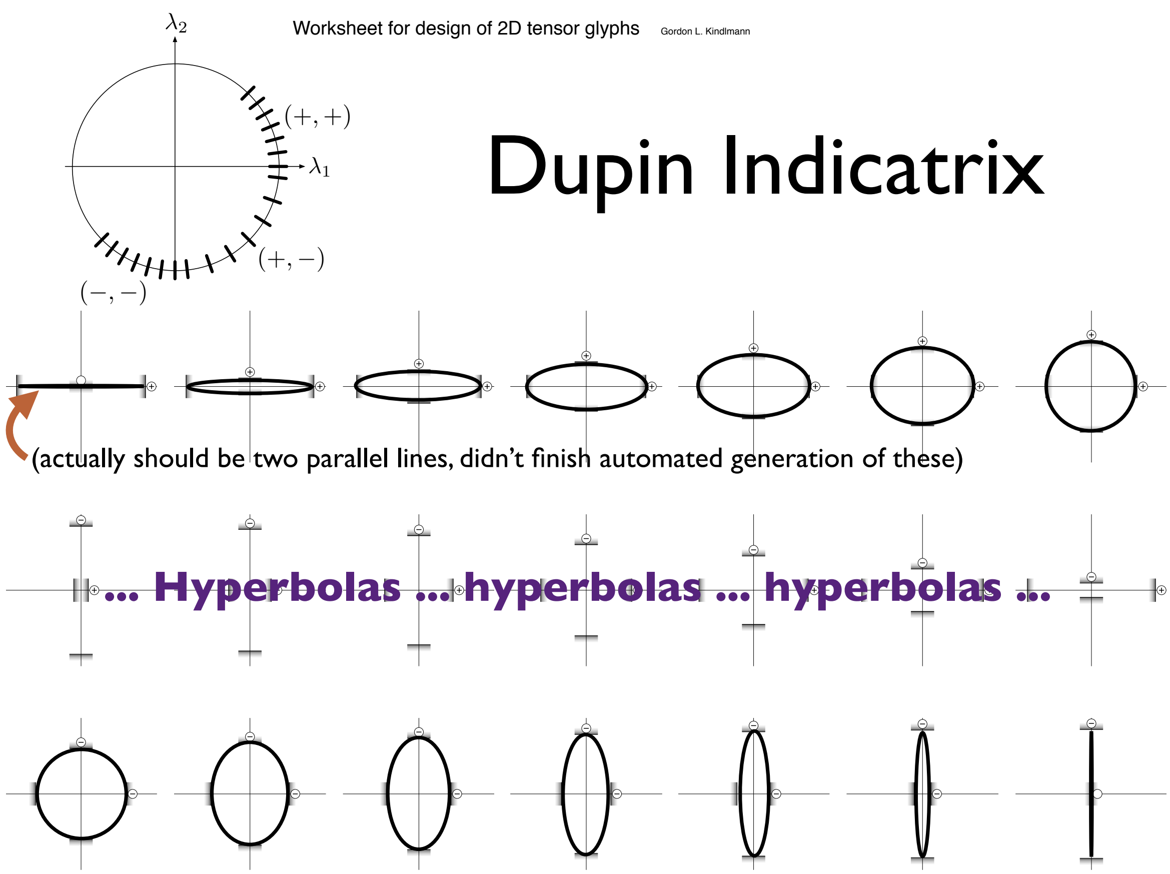
$$\lambda \rightarrow e^{\lambda}$$

Kirby et al. Vis '99



(hand-drawn, approximate)

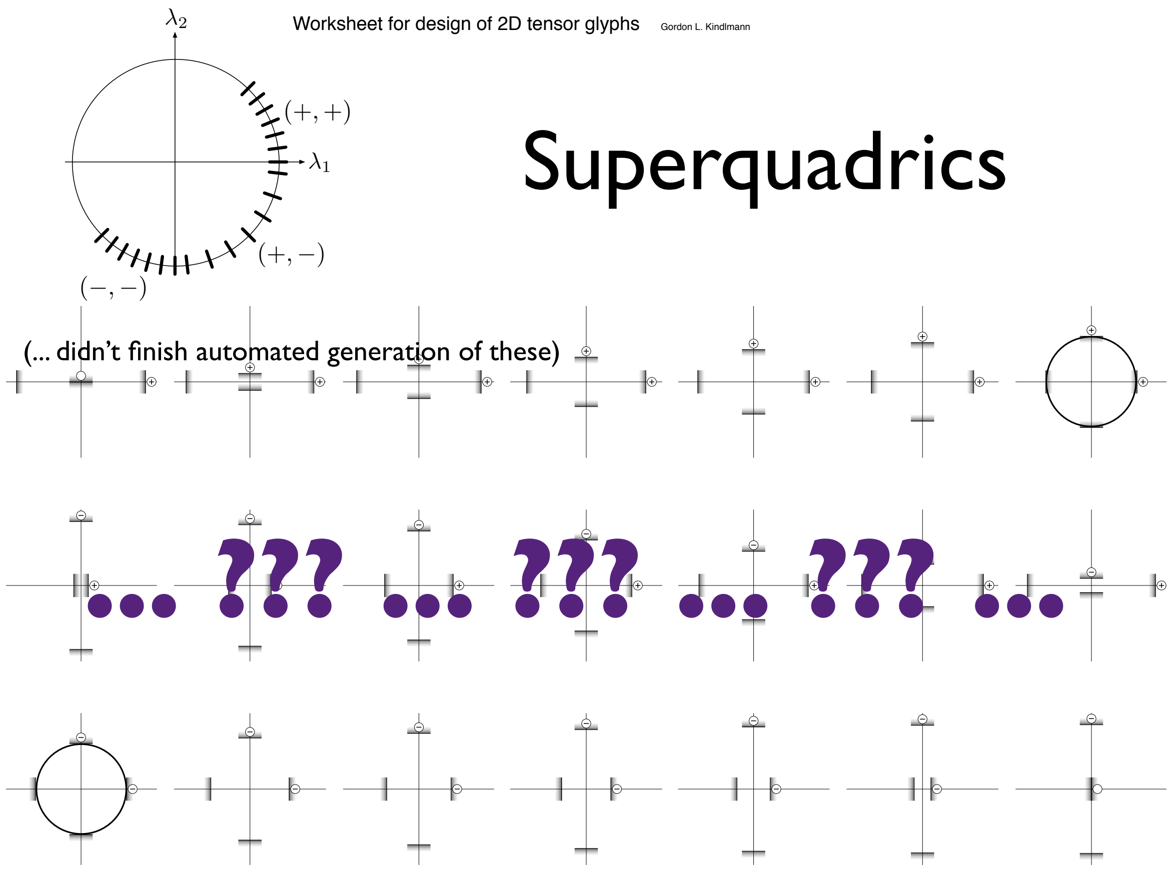
Dupin Indicatrix

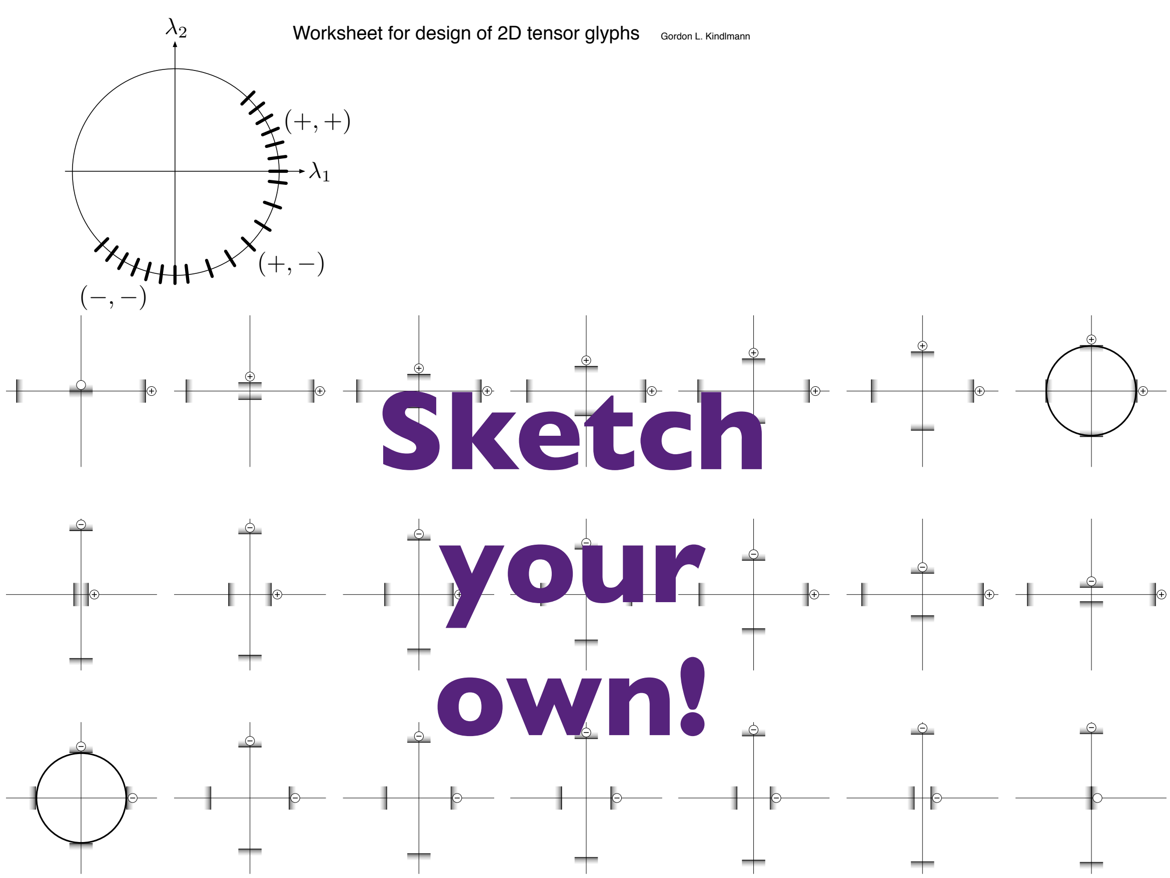


(actually should be two parallel lines, didn't finish automated generation of these)

... Hyperbolas ... hyperbolas ... hyperbolas ...

Superquadrics





Final points

- We have **basic** visualization research to do
  - Specific: Tensor glyphs in 2D and 3D
  - General: Formulating visualization principles: **symmetry**, **continuity**, ambiguity, what more?
- Visualization research not dead (Lorensen ’04)
  - Not only about bringing **existing** commodity visualization tech to customers/collaborators
  - Not only about showing **more** data (**quantity**)
  - Inviting new customers/collaborators by creating **new** visualization technology for different data **qualities** and different physical phenomena