Particle systems for visualizing the connection between math and anatomy

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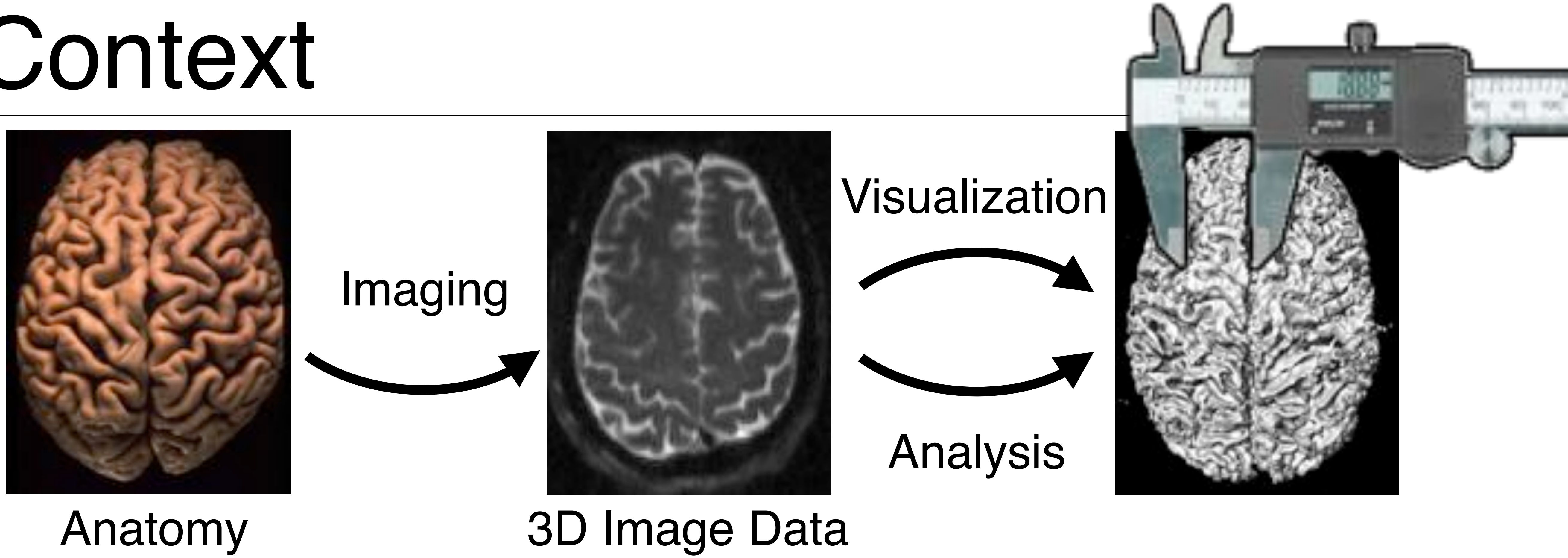
Context



- Goal: geometric models of **anatomic features** for quantitative analysis of large image datasets
- Strategy: **generality** WRT feature co-dimension
- Method: particle systems for feature **sampling**
- Question: how do I know if a given **mathematical** feature is plausible as an **anatomic** feature?

Outline

- Features
- Particles
- Scale-space
- Stability with respect to scale
- Results
- Discussion

What I mean by mathematical "feature"

- In a continuous & differentiable field $f(\mathbf{x})$ created by convolution:
- Feature = positions $\mathbf{x}$ satisfying feature equation
  - Isocontours: $f(\mathbf{x}) = f_0$
  - Laplacian zero-crossing edges: $\nabla^2 f = 0$
- (No models, priors, blend of data & smoothing terms ...)
- ($\mathbf{g} = \nabla f$; $\mathbf{H} = \nabla \otimes \nabla f$; $\mathbf{H} = \sum_i \lambda_i \mathbf{e}_i \otimes \mathbf{e}_i$; $\lambda_1 \geq \lambda_2 \geq \lambda_3$)
- Combinations of maxima and minima WRT $\mathbf{e}_i$
  - Ridge Surface (Eberly '96): $\mathbf{g} \cdot \mathbf{e}_3 = 0$; $\lambda_3 < 0$
  - Ridge Line: $\mathbf{g} \cdot \mathbf{e}_3 = \mathbf{g} \cdot \mathbf{e}_2 = 0$; $\lambda_3 < 0; \lambda_2 < 0$
  - Various valleys and connectors possible (Damon '98)
  - $\exists$ iterative update scheme (Newton optimization) to move closer to feature if near it: we can sample features

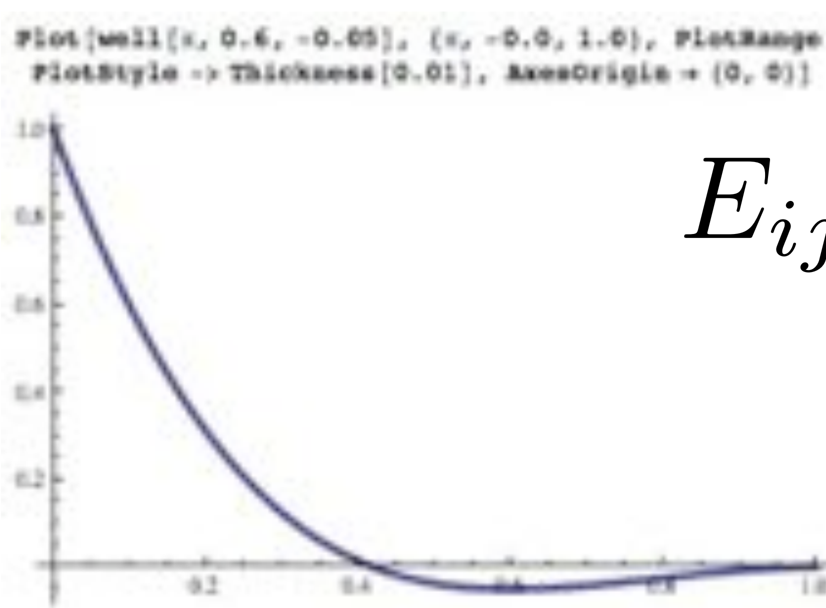
Dynamic Particle Systems

- G. L. Kindlmann, R. San Jose Estepar, S. M. Smith, C.-F. Westin, Sampling and Visualizing Creases with Scale-Space Particles. IEEE Trans. Vis. Comp. Graph, 15(6):1415-1424 (2009)
- Set of points subject to (particle-image) feature constraint and (interparticle) energy minimization

$$\underset{\mathbf{x}_i, N}{\operatorname{argmin}} \mathcal{E} = \underset{\mathbf{x}_i, N}{\operatorname{argmin}} \sum_{i,j=1}^{N} E_{ij}$$

- Hard constraint of particles to feature; no energy

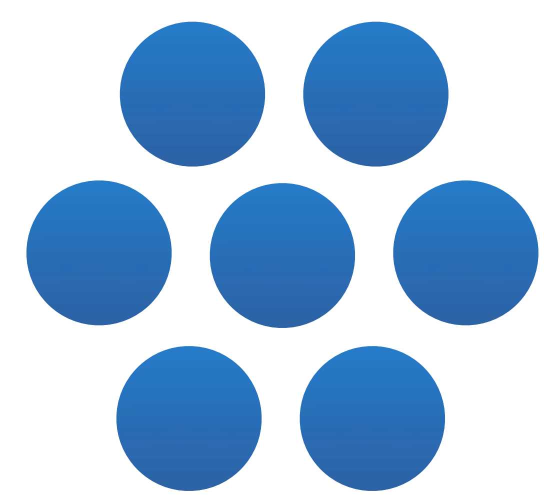
φ



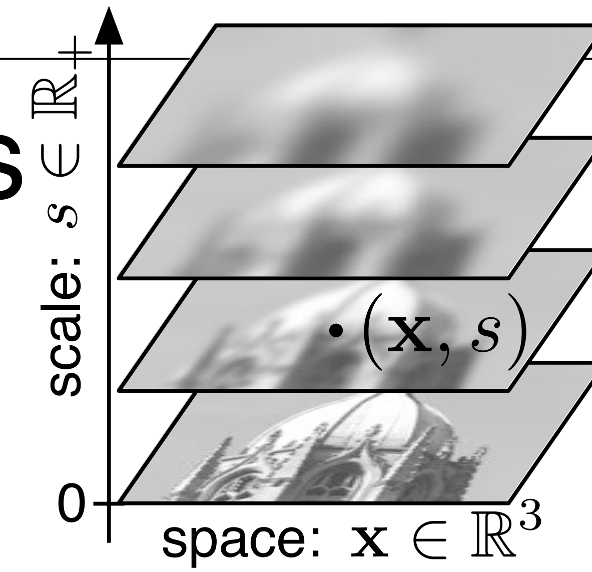
r

$$E_{ij} = \Phi\left(\frac{|\mathbf{x}_i - \mathbf{x}_j|}{\sigma_r}\right)$$

(demo)

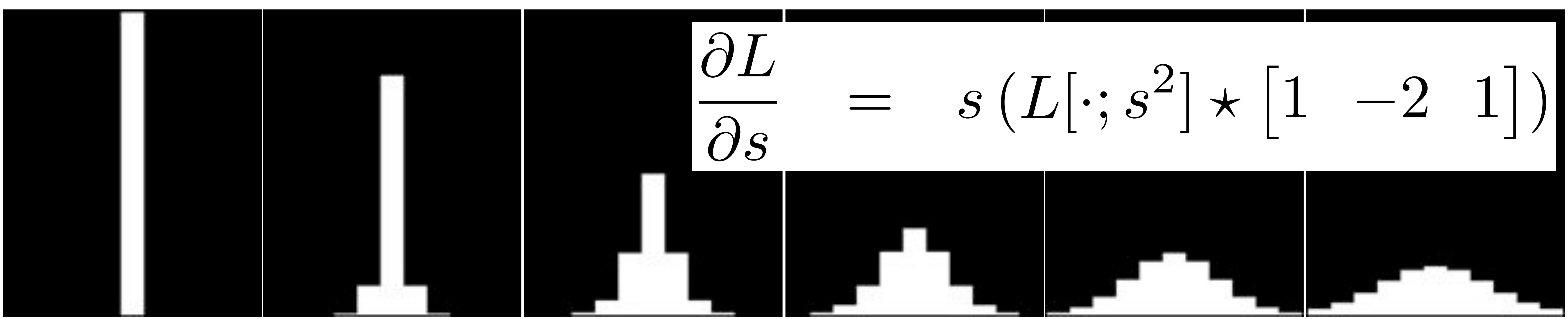


Scale Space



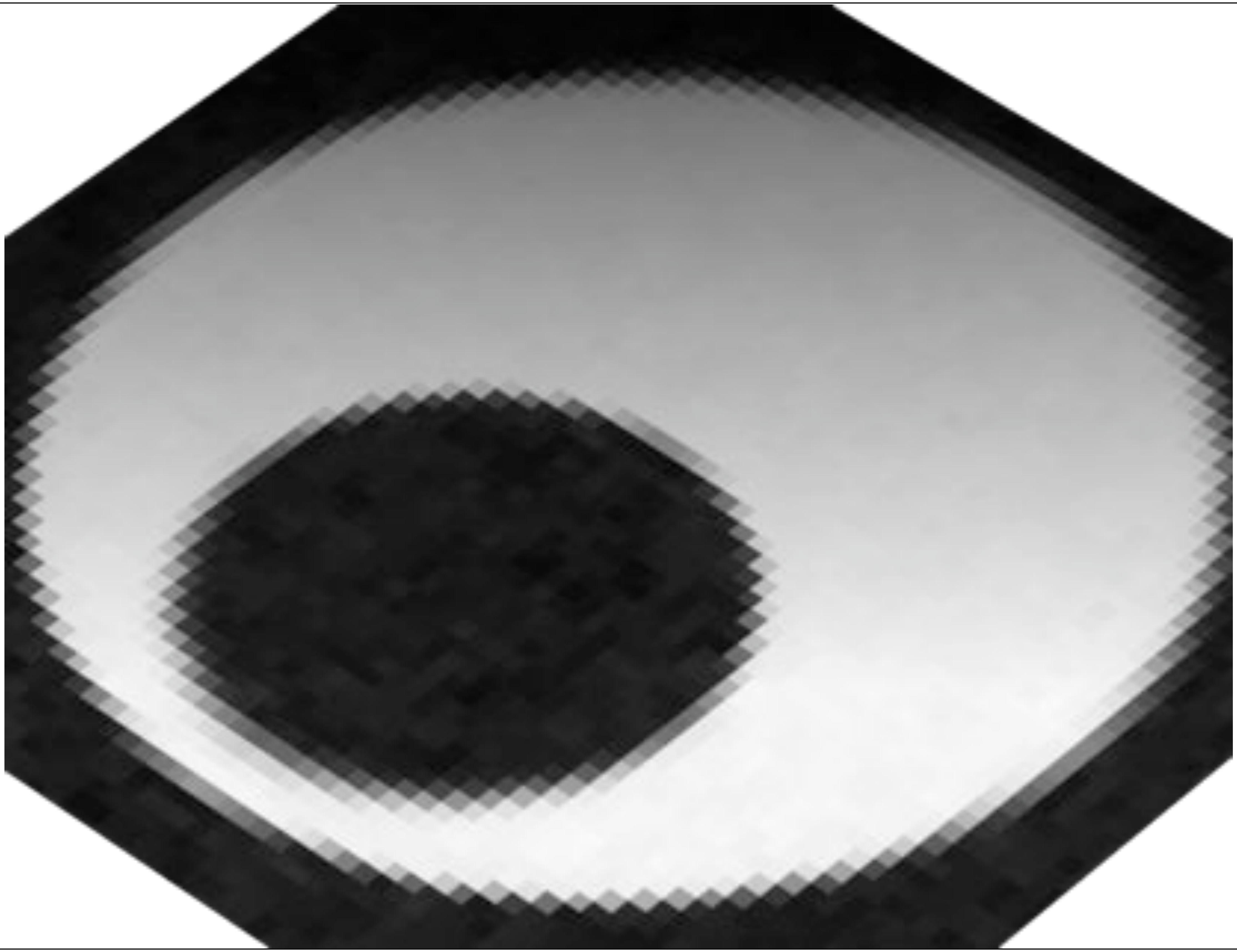
- Image & continuous family of blurrings
- Practical requirements
  - Probe at arbitrary points in scale space
  - Efficiently handle real-world 3D datasets (minimize memory usage and resulting cache misses)
- Scale interpolation based on Lindeberg's "Gaussian" (soln. to heat eq. in discrete domain)

$$\frac{\partial L}{\partial s} = s\left(L[\cdot; s^2] \star \begin{bmatrix}1 & -2 & 1\end{bmatrix}\right)$$

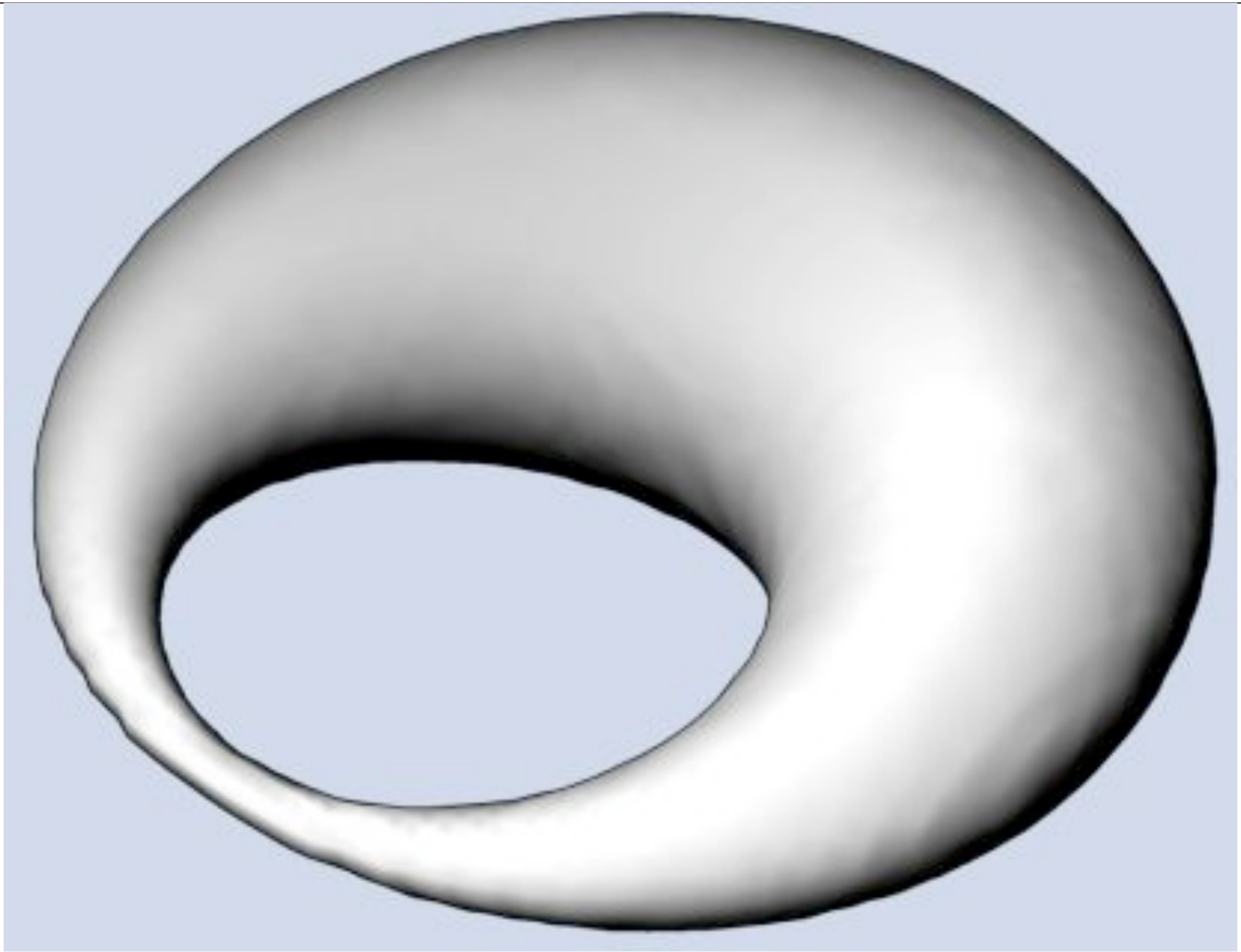


s=0 s=0.5 s=1.0 s=1.5 s=2.0 s=2.5

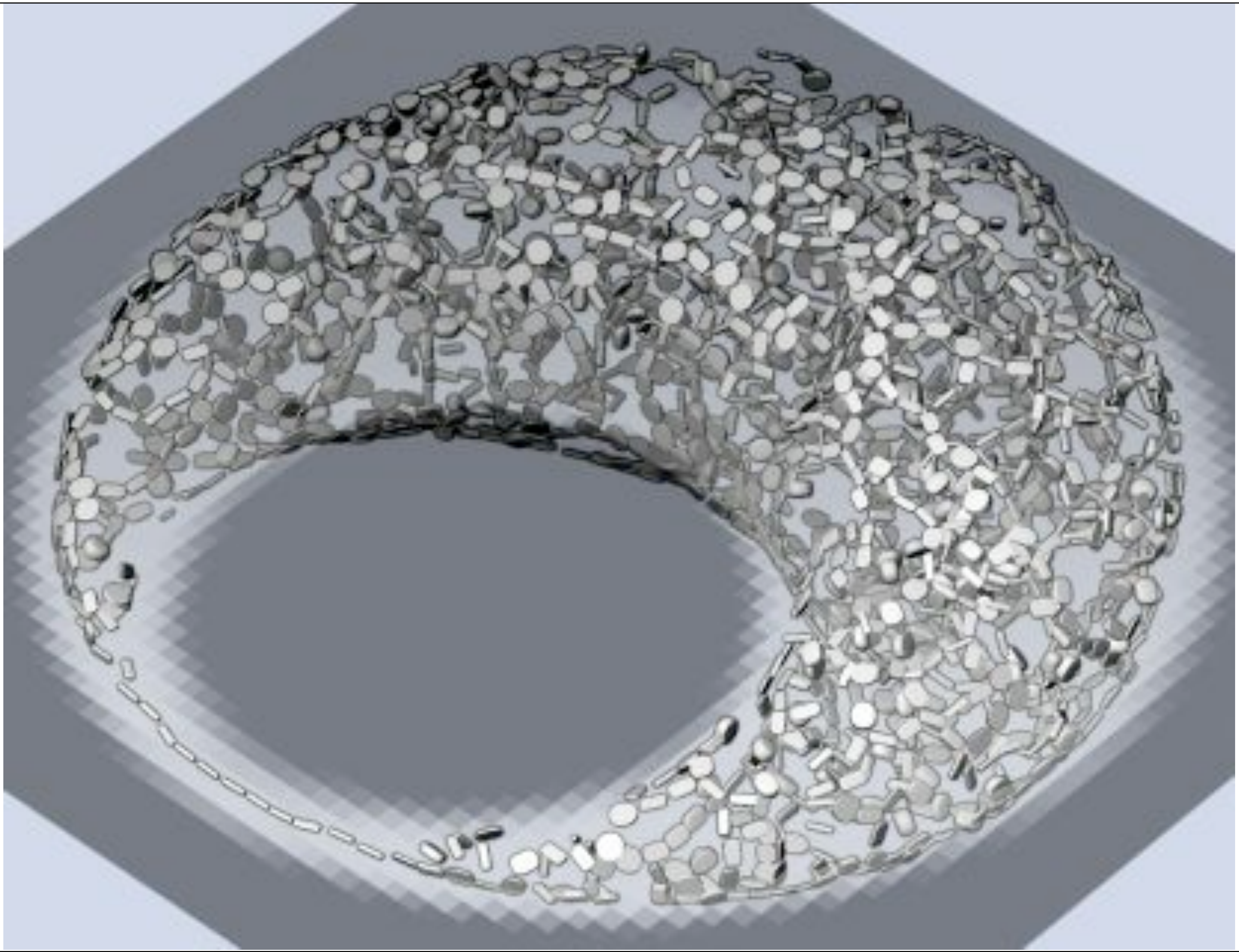
Sampling scale-space feature



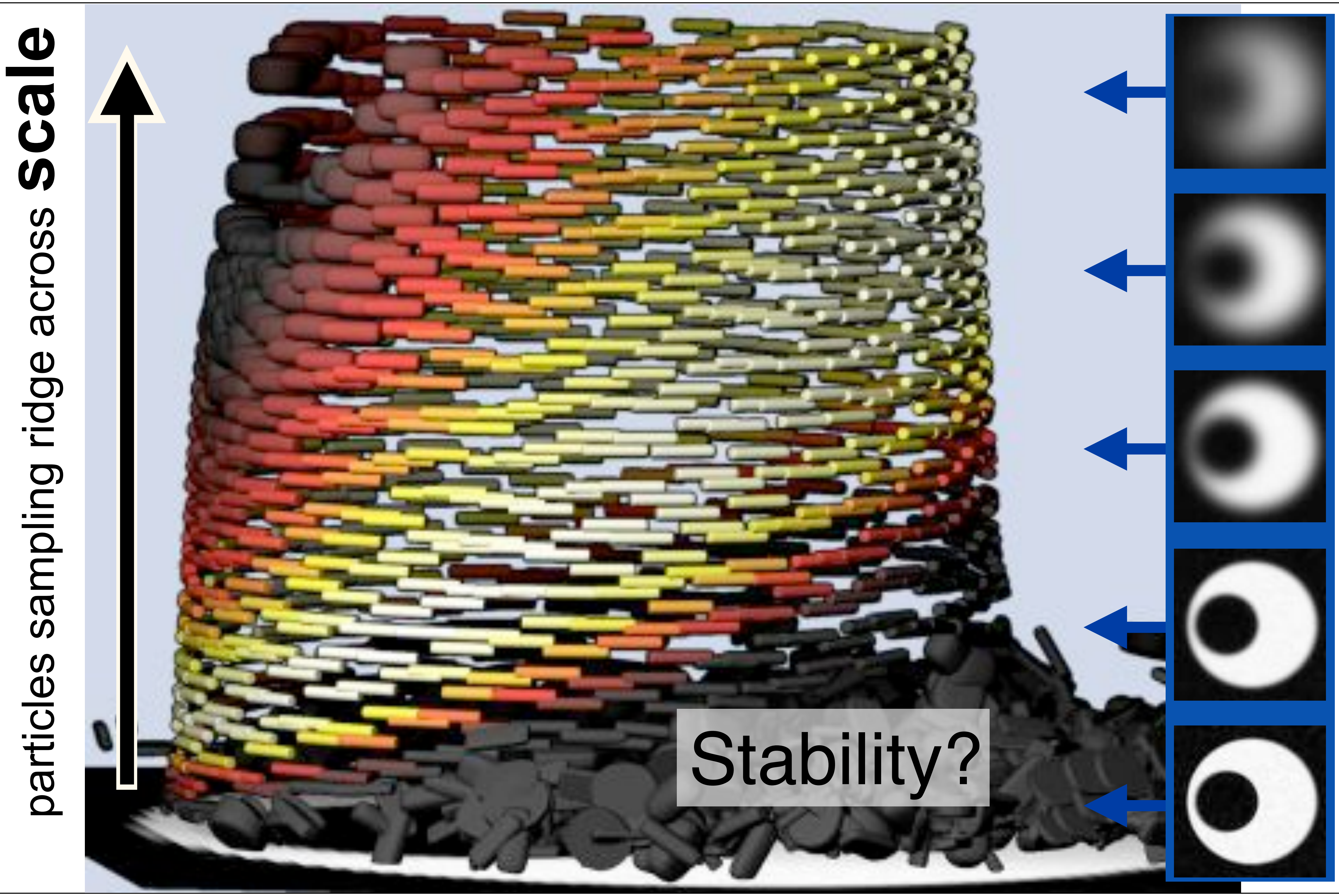
Sampling scale-space feature



Sampling scale-space feature

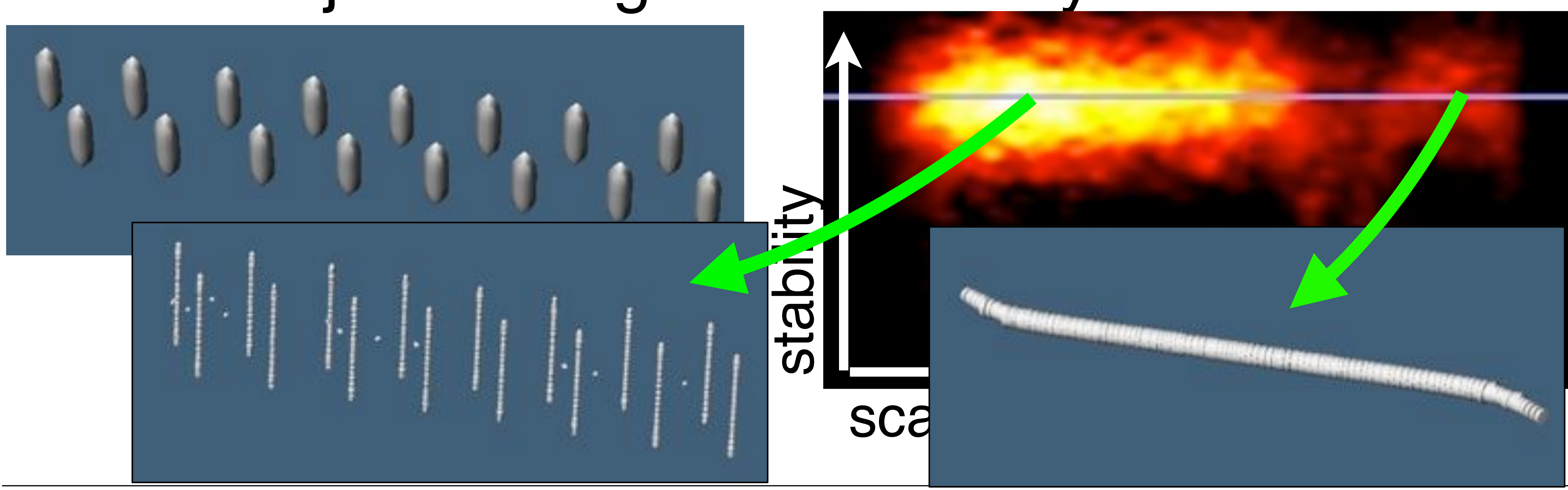


Sampling scale-space feature

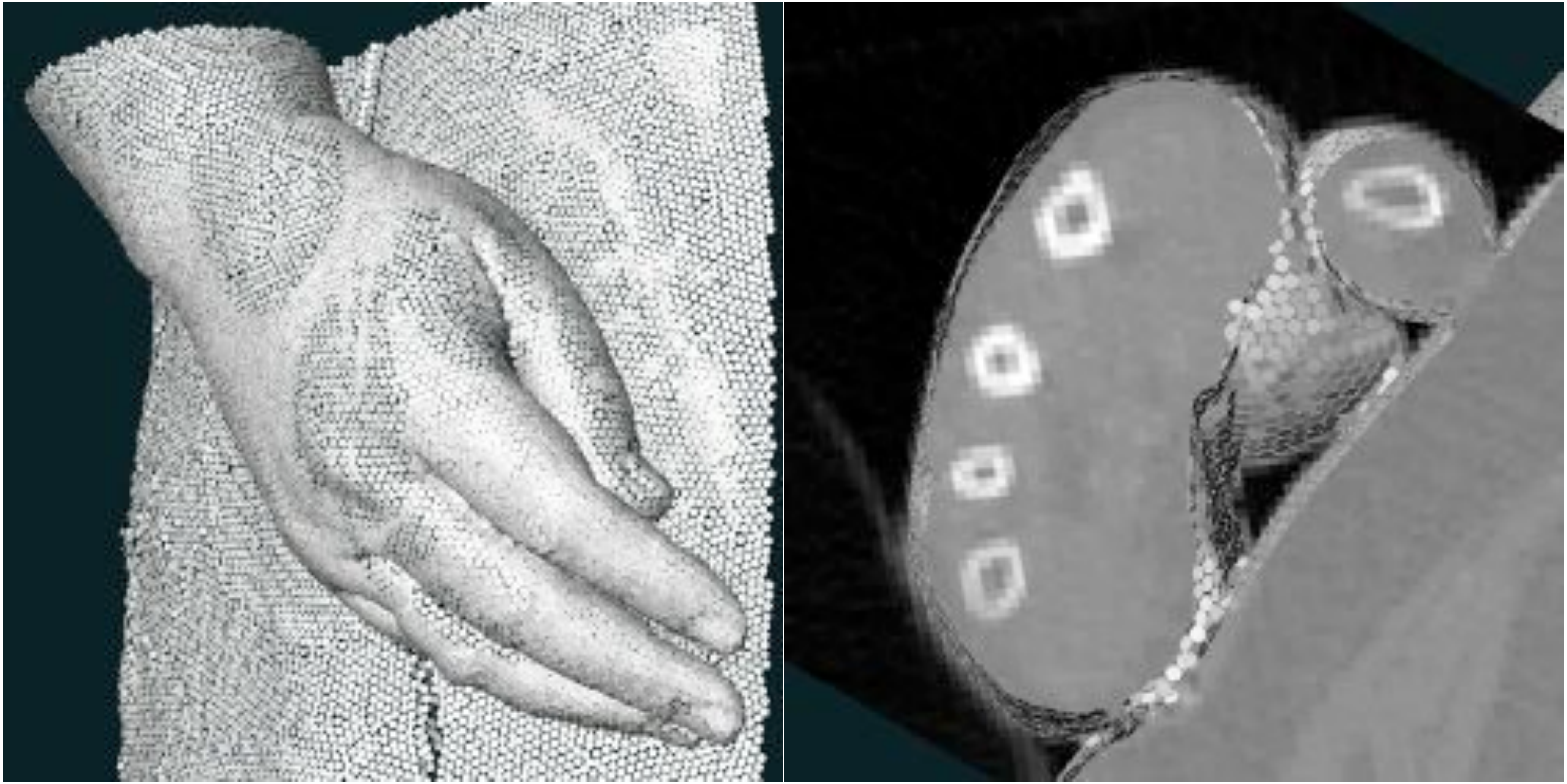


Stability with respect to scale

- One possible measure of feature significance: feature doesn’t move with small additional blur
  - e.g. SIFT for image stitching
- Recover scale stability at $\mathbf{p}_i$ from covariance tensor $\boldsymbol{\Sigma}$ of vectors $\mathbf{p}_j$ - $\mathbf{p}_i$ to interacting particles
  - stability $\approx \Sigma_{ss} * (1 + \text{codim}) / \text{trace}(\boldsymbol{\Sigma})$
- Look at joint histogram of stability and scale

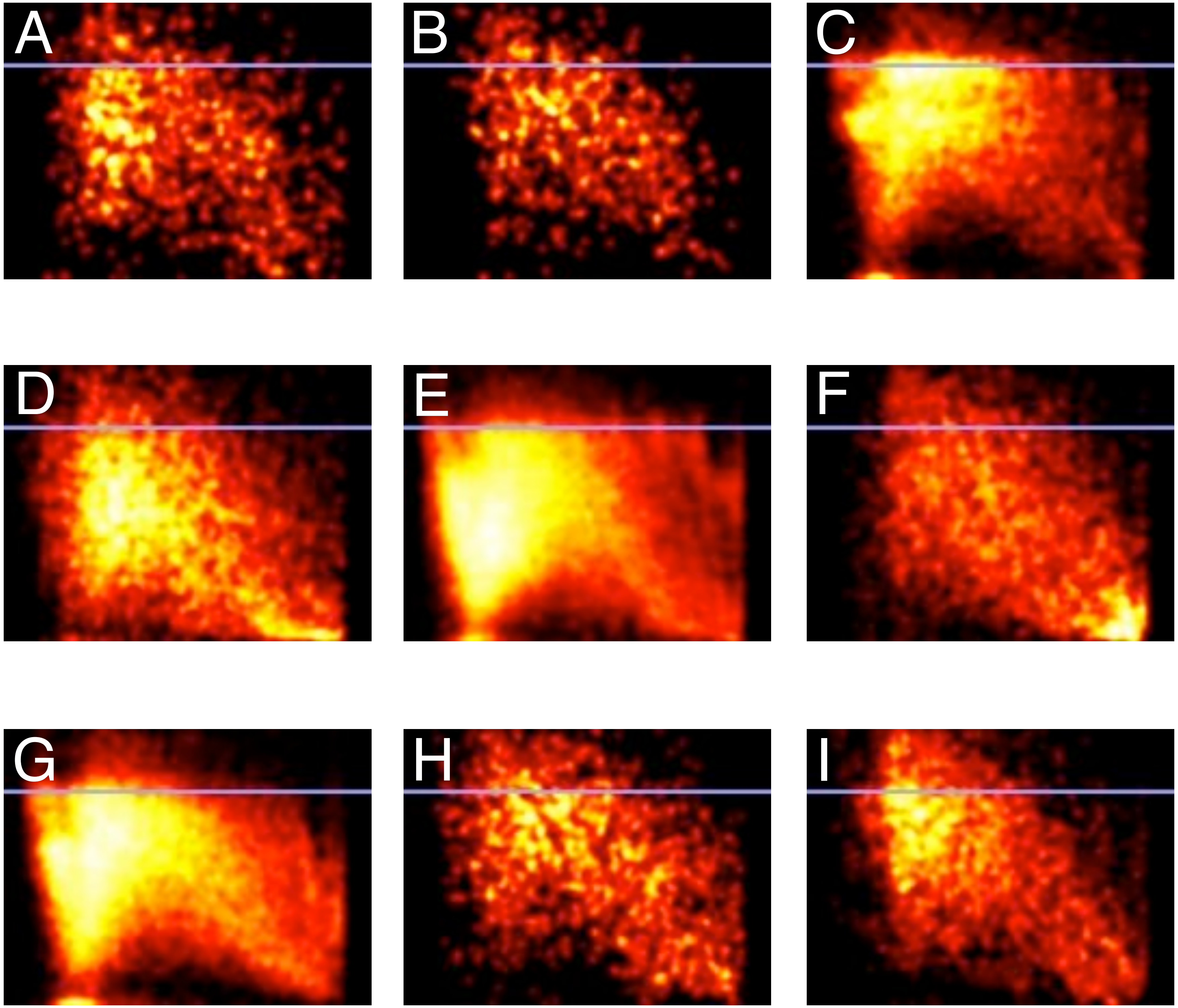


Results for codim-2 features in hand

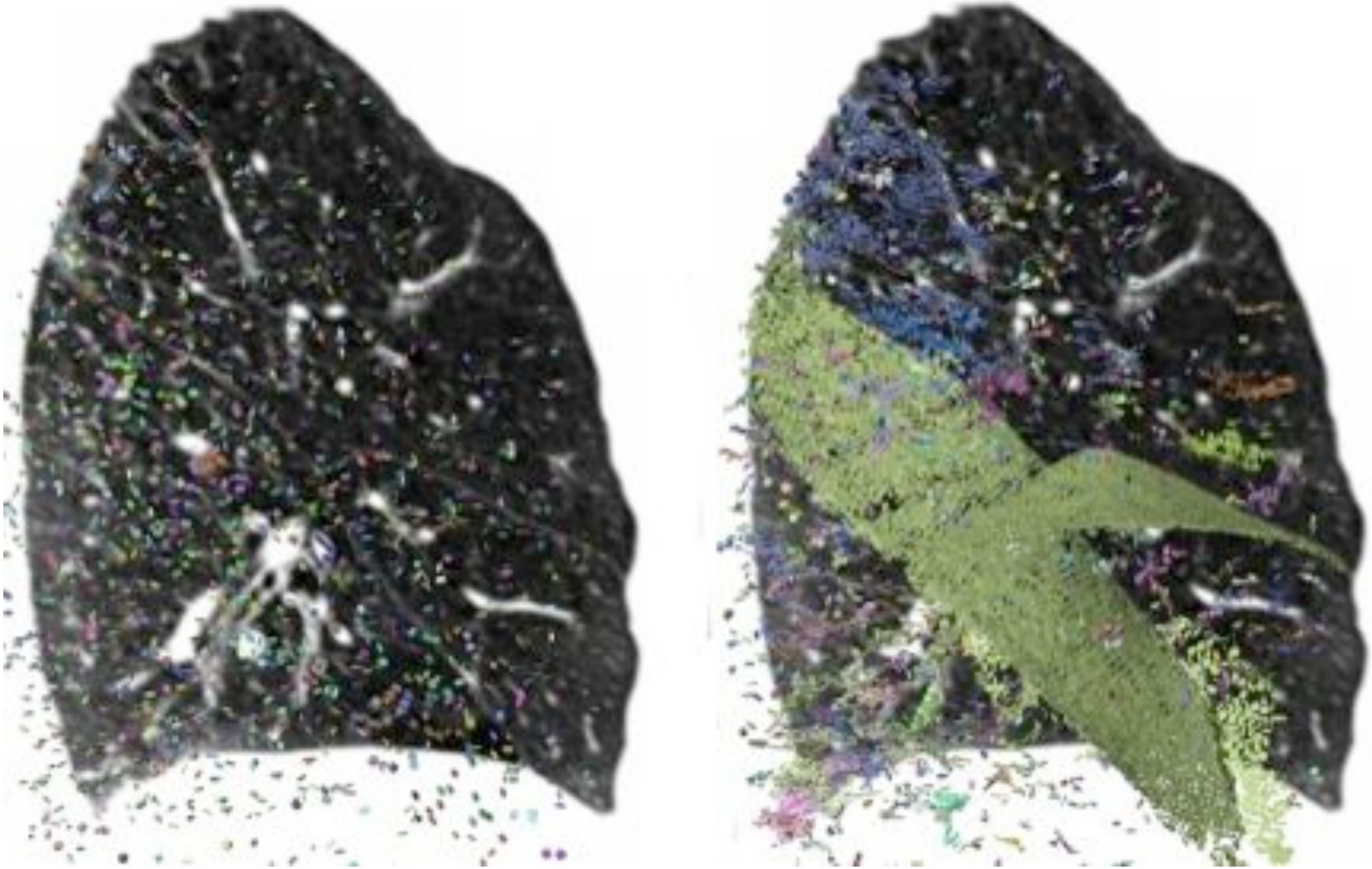


Results for codim-2 features in hand

	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$
A		min	min
B	min		min
C	min	min	
D		min	MAX
E	min		MAX
F	min	MAX	
G		MAX	MAX
H	MAX		MAX
I	MAX	MAX	



Lung lobes from clinical CT



J Ross, RSJ Estepar, G Kindlmann, A Diaz, C-F Westin, E Silverman, G Washko, "Automatic Lung Lobe Segmentation Using Particles, Thin Plate Splines, and Maximum a Posteriori Estimation", Proceedings MICCAI 2010, pp 163-171

Discussion

- New imaging modalities, contrast mechanisms, and their combination => proliferation of possibly useful features; this may help show the way
- “Finding” features in 2 senses: where, and which
- From image samples to feature samples
- Future work & possible collaborations:
  - Points into polyline trees, polygonal meshes
  - Shape modeling and statistics
- Why not more work on **applied** 3D scale-space?
- Thanks for your attention; questions?