Sampling and Visualizing Creases with Scale-Space Particles [Kindlmann-VIS-2009]

Please interrupt me with questions!

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Outline

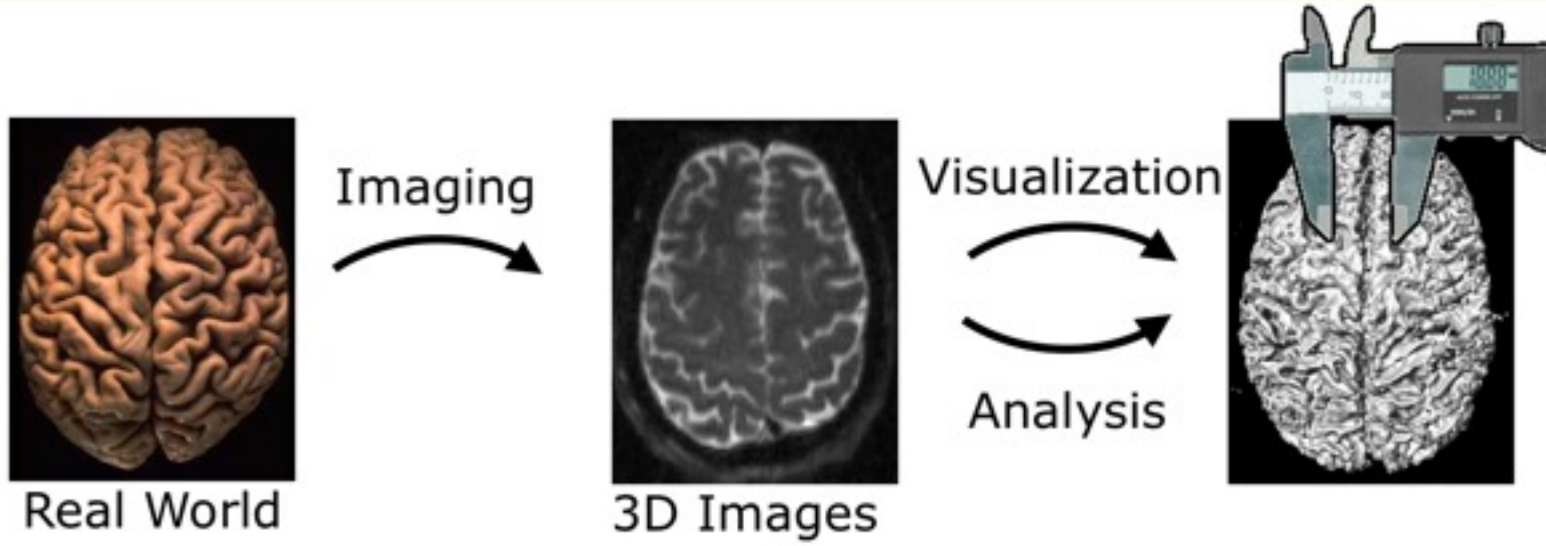
Introduction: motivation, example, contributions

Method: interpolation, features, energy, visualization, computation

Results: lung CT, brain DTI, more

Discussion: scale, particles, analysis, future

Context & Motivation



Medical imaging is for measuring

Need to extract image features

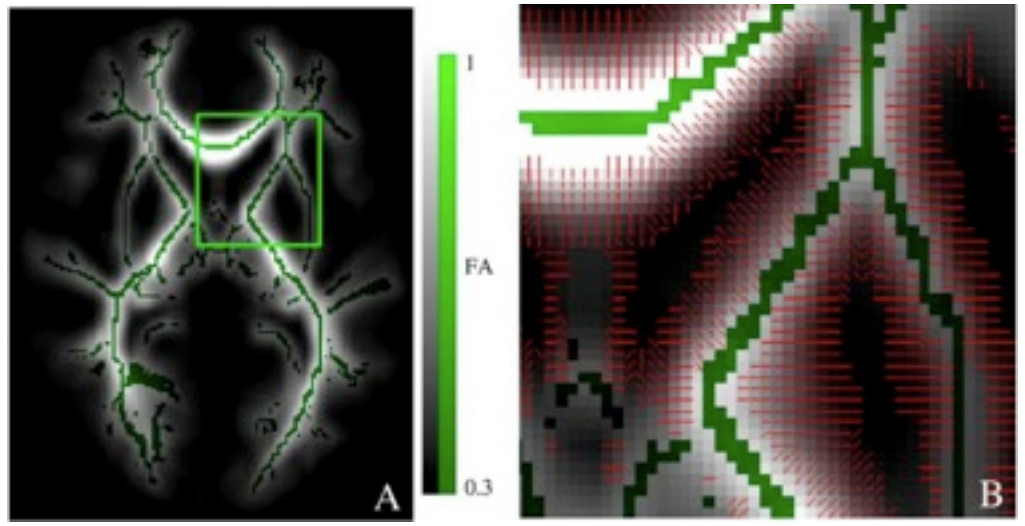
Feature geometry is a computational proxy for anatomy

Lung CT: find airways, measure radii, study emphysema

Brain DTI: find major WM tracts, measure Fractional Anisotropy (FA), study psychiatric disorders

DTI not just for tractography

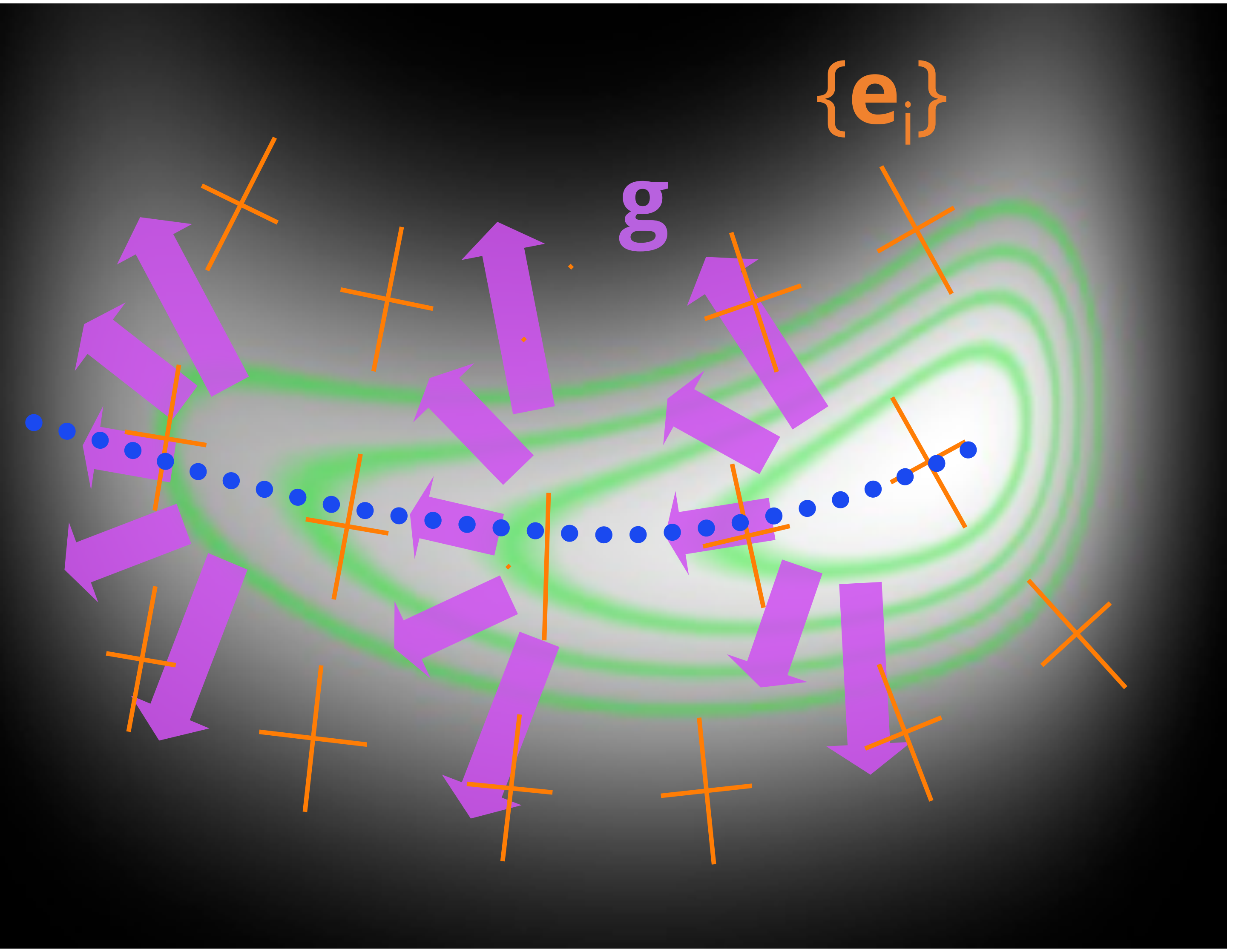
Much of it for FA studies



[Smith-TBS-2006]

Want to create general way to detect and sample image features

Ridges & Valleys = Creases [Eberly-JMIV-1994] [Eberly-1996]



Constrained extremum

Gradient $\mathbf{g}$

Hessian eigensystem $\mathbf{e}_i, \lambda_i$

λ_3

Crease: $\mathbf{g}$ ortho to one or more $\mathbf{e}_i$

Eigenvalue gives **strength**

e.g.: Ridge surface: $\mathbf{g} \cdot \mathbf{e}_3 = 0$; $\lambda_3 <$ thresh

Ridge line: $\mathbf{g} \cdot \mathbf{e}_3 = \mathbf{g} \cdot \mathbf{e}_2 = 0$; $\lambda_3, \lambda_2 <$ thresh

Valley surface: $\mathbf{g} \cdot \mathbf{e}_1 = 0$; $\lambda_1 >$ thresh

Feature sampling examples

Using Visible Human, Female CT hand

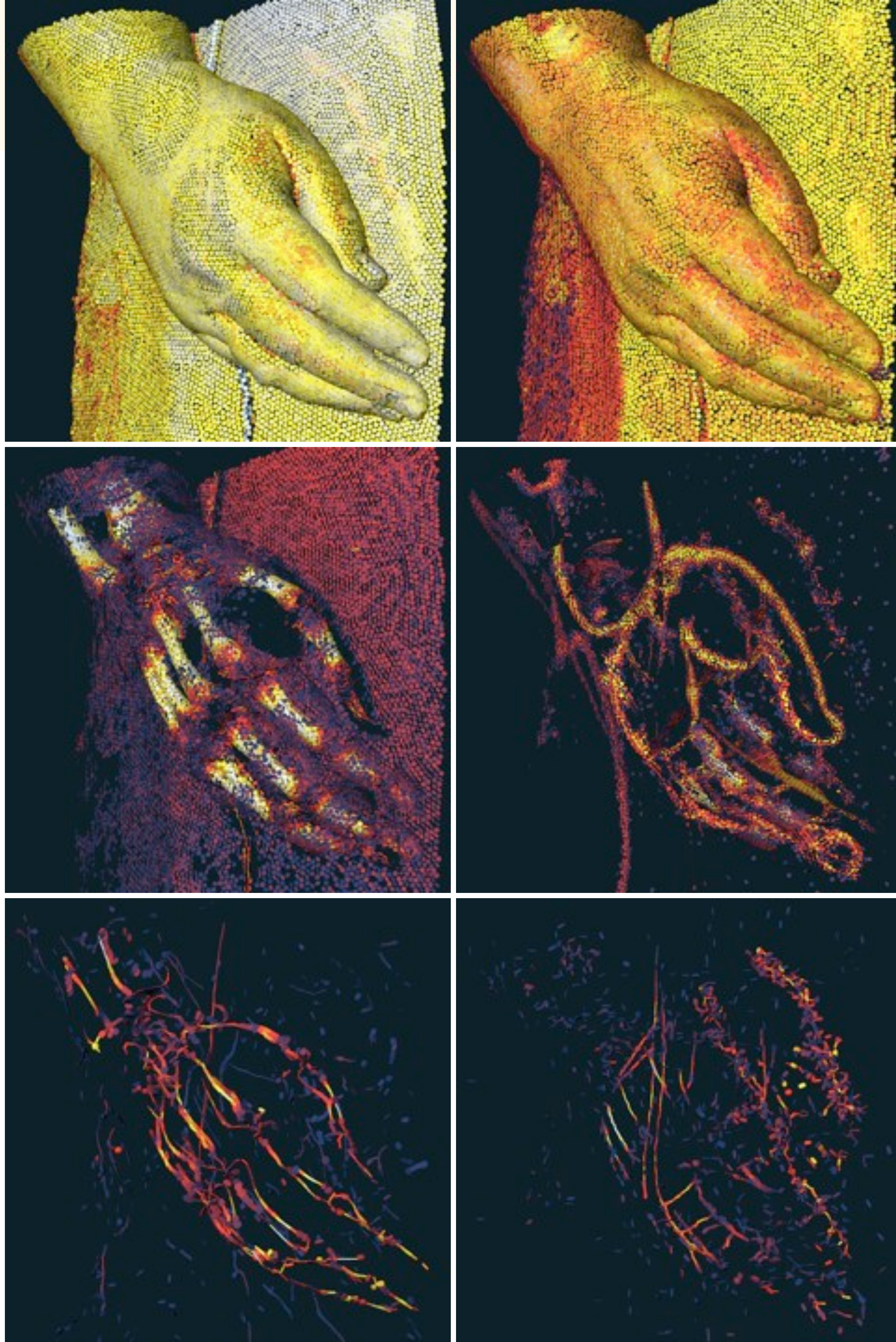
The different features:

- Isosurface
- Laplacian zero-crossing
- Ridges & Valleys (“creases”)
  - surfaces or lines

Same code, different optimization

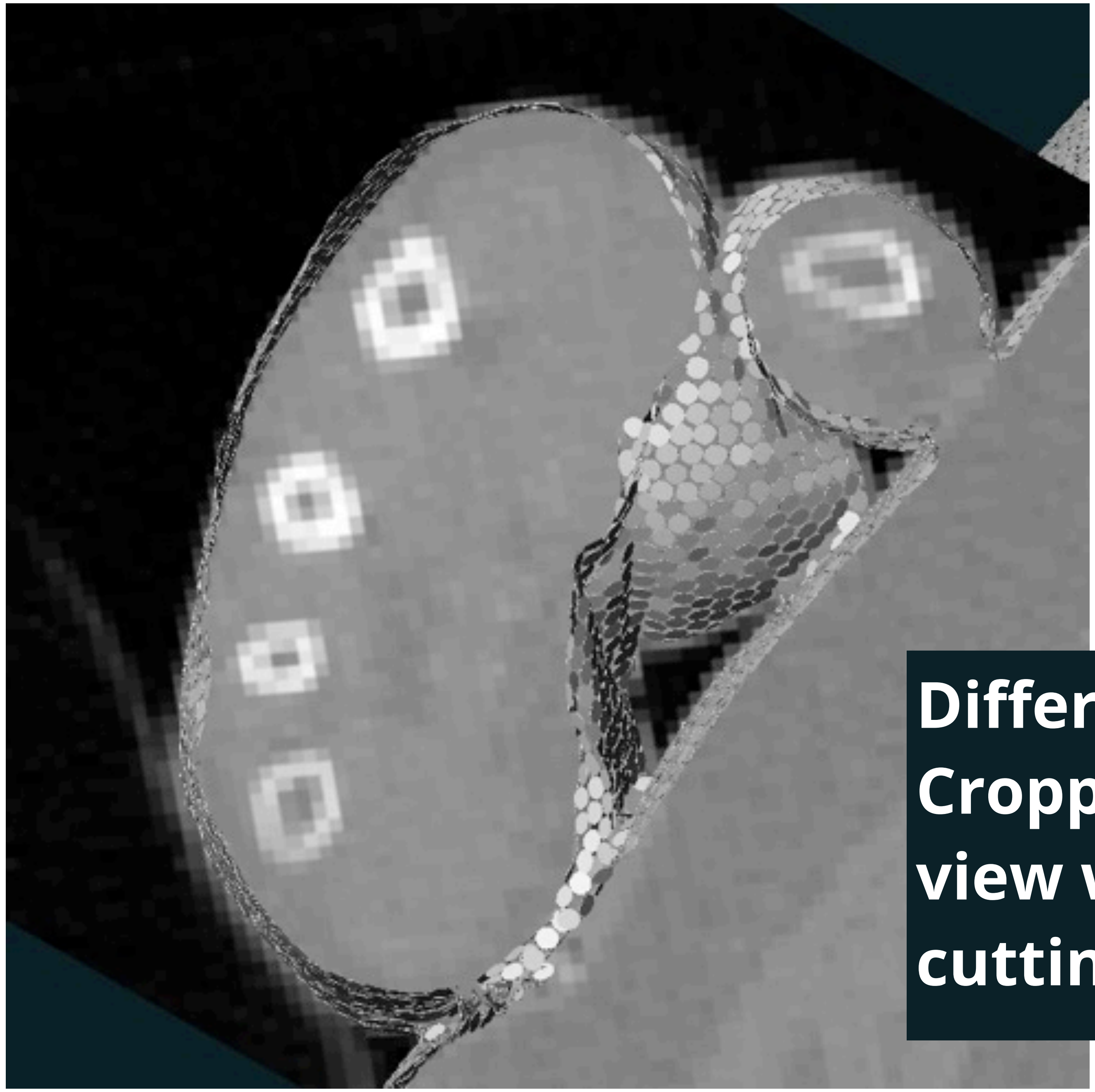
- each little glyph = one particle
- show local feature ingredients

Sample features a **single scale**



Isosurface

Outside 3D view

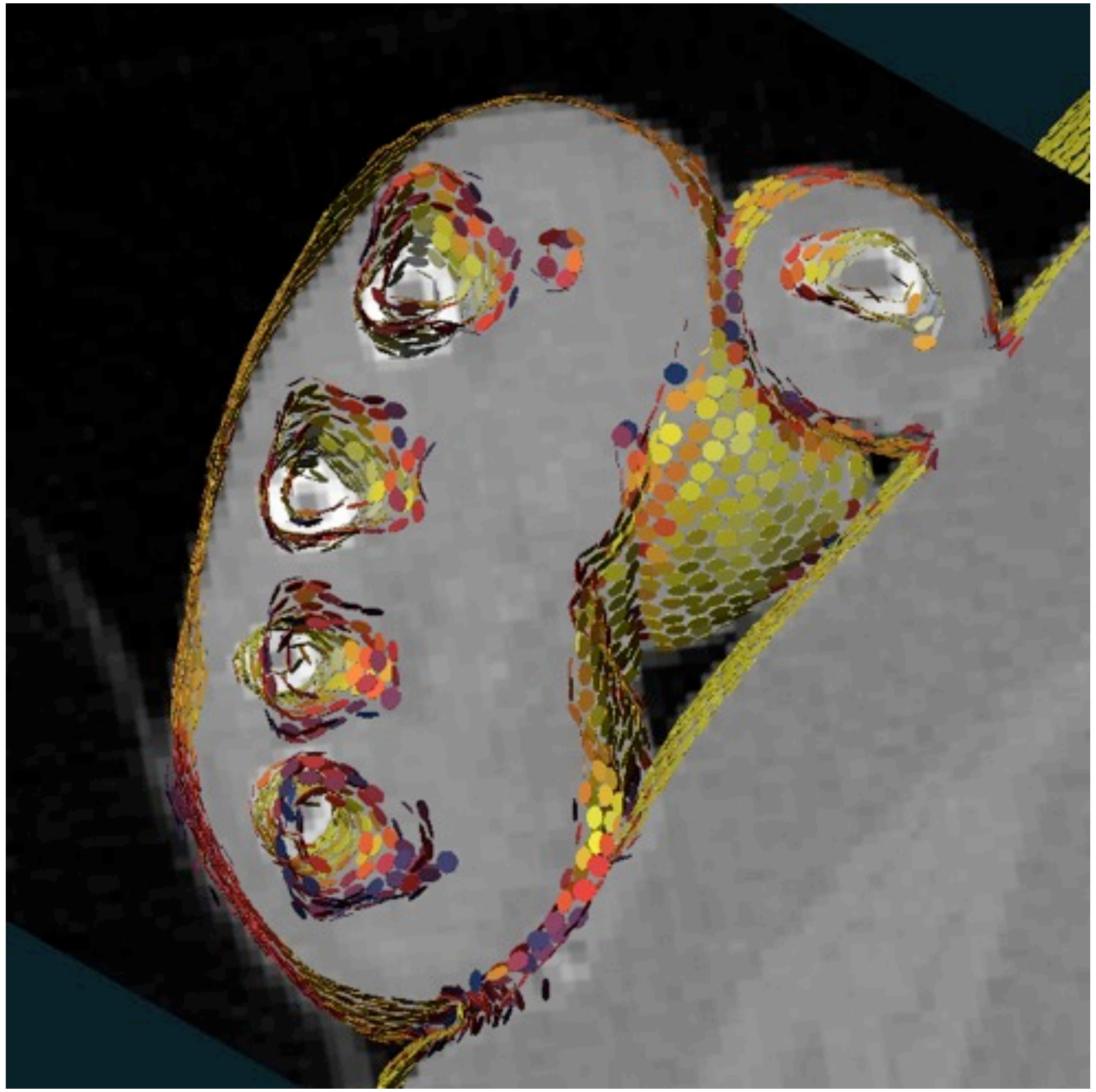


Different, Cropped 3D view with 2D cutting plane

AKA isocline, isophote, isocontour, level set

$$f(\mathbf{x}) = v_0$$

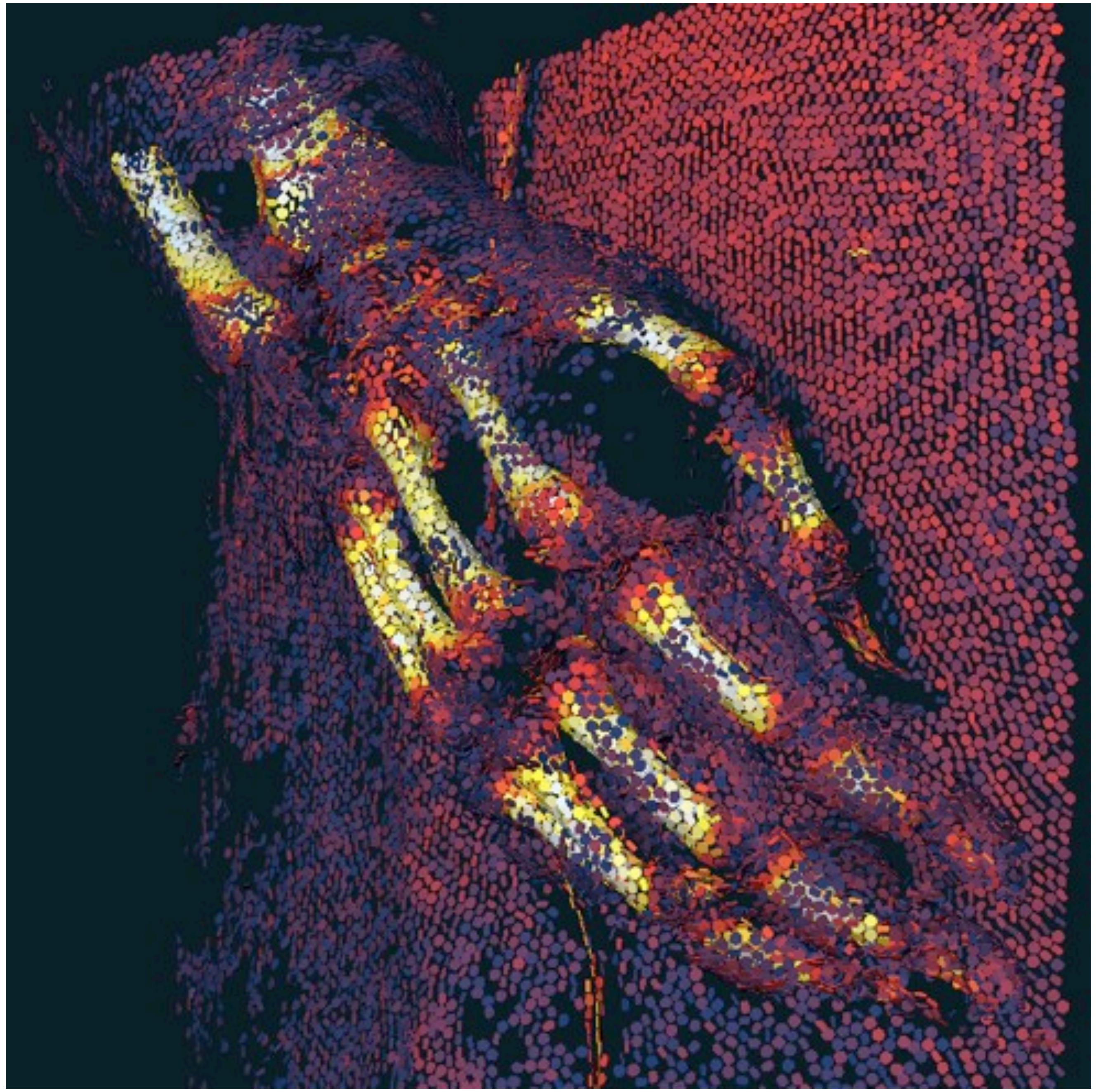
Laplacian 0-crossing

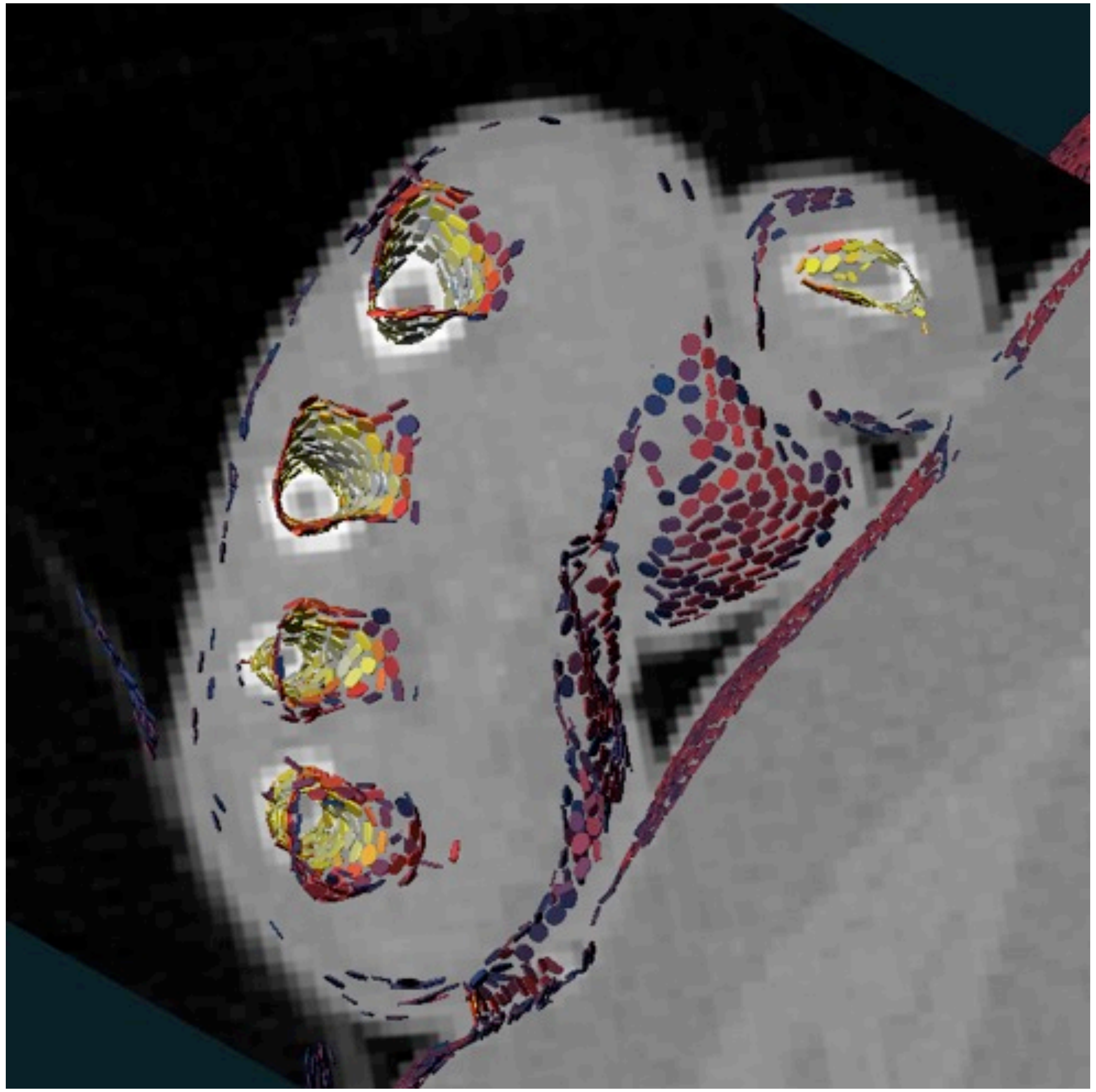


Classical definition of edge

$\nabla^2 f(\mathbf{x}) = 0$; strength = $|\nabla f(\mathbf{x})|$

Ridge Surface

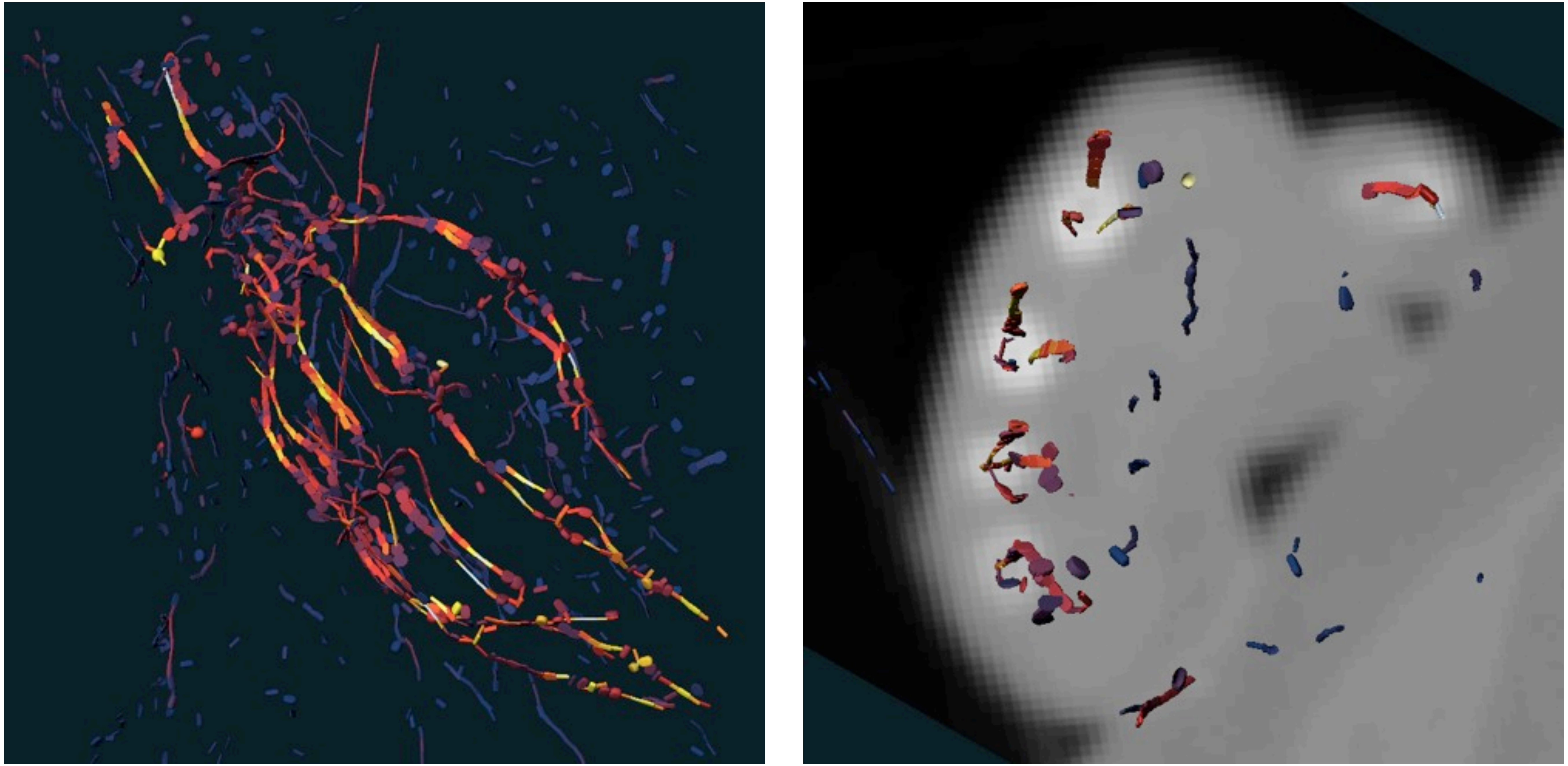




Maximal surface wrt Hessian minor eigenvector $\mathbf{e}_3$

$\nabla f(\mathbf{x}) \cdot \mathbf{e}_3(\mathbf{x}) = 0$; strength $= -\lambda_3$

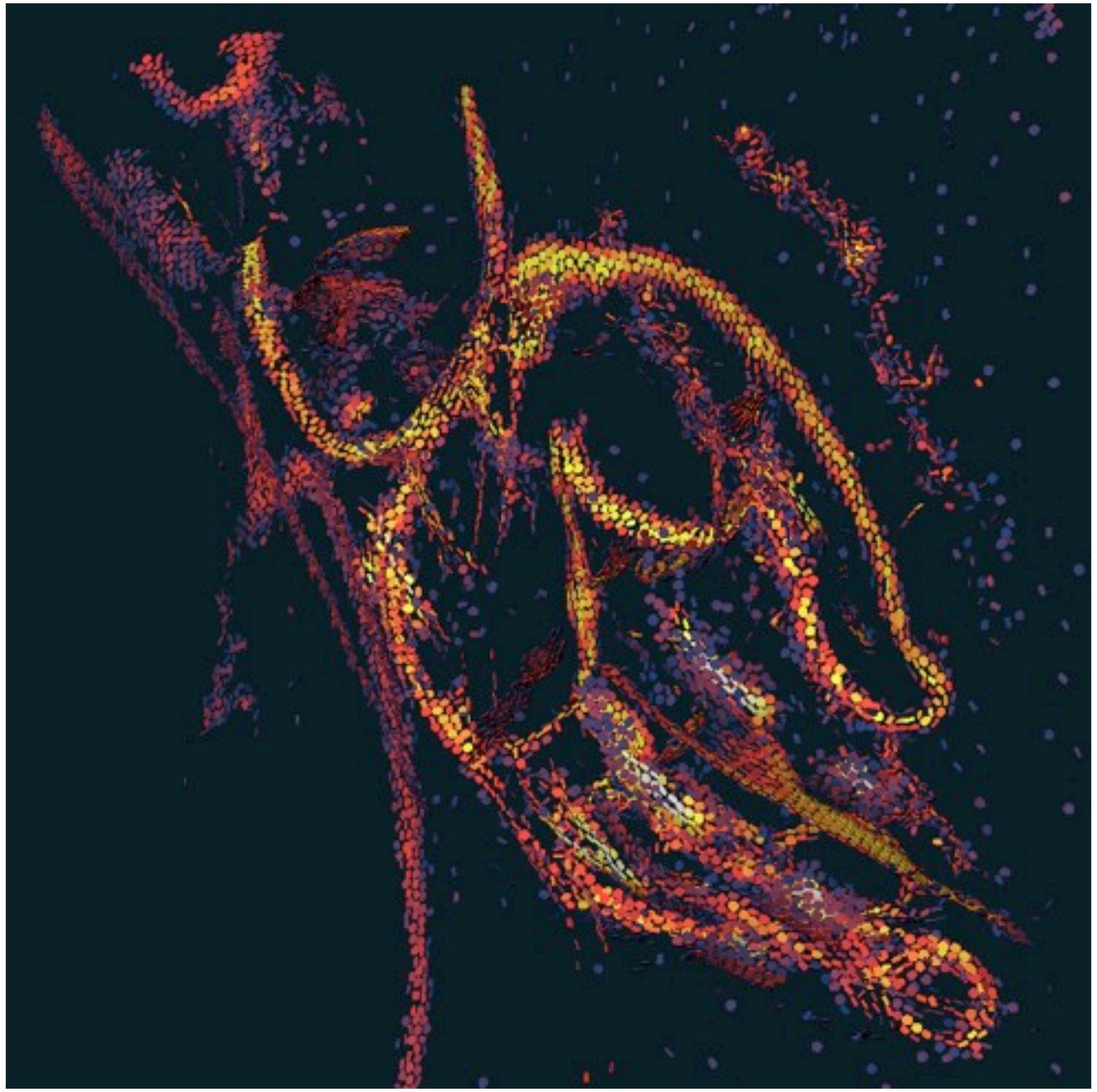
Ridge Line

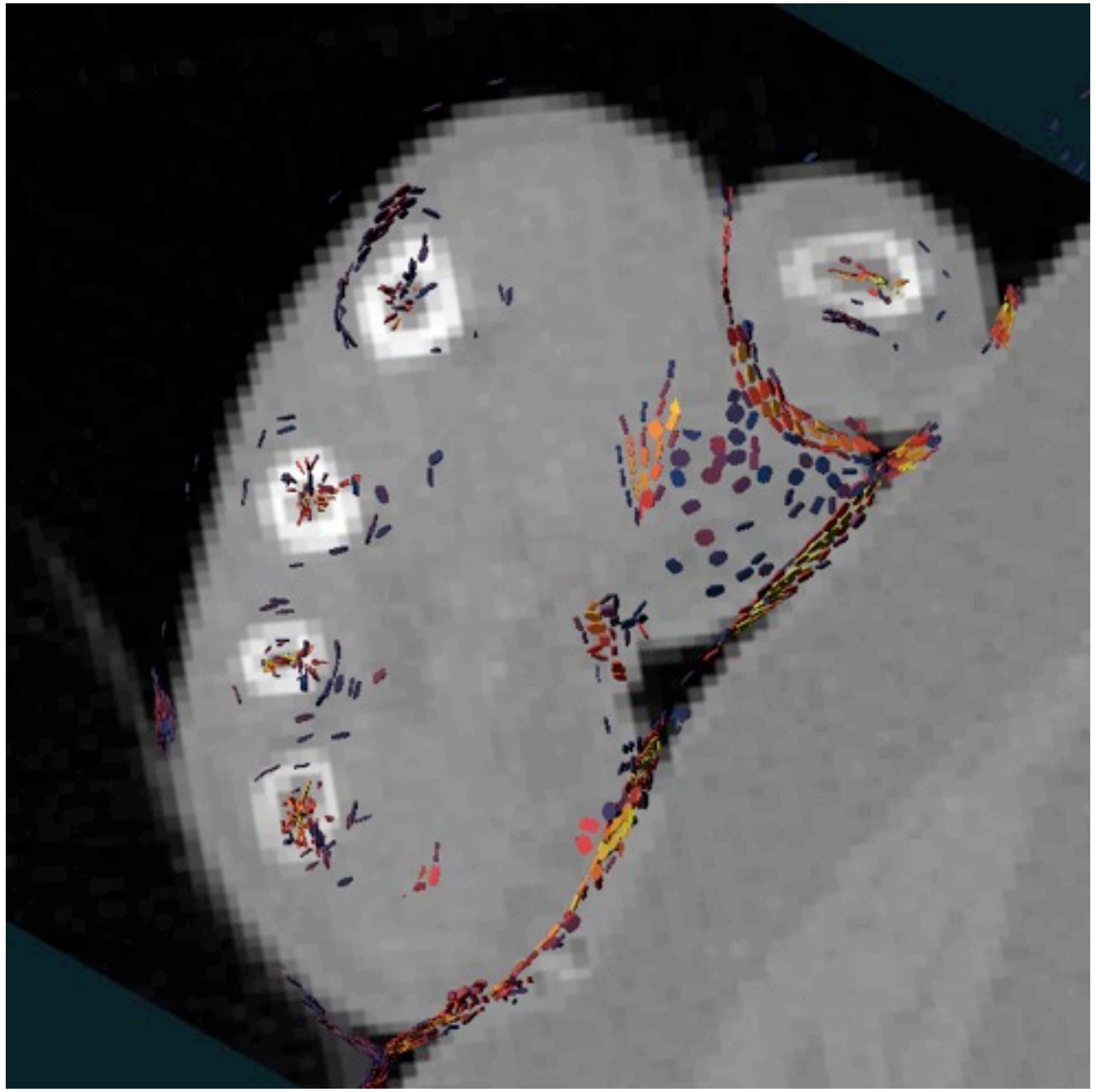


Maximal curve wrt Hessian minor,medium eigenvecs

$\nabla f(\mathbf{x}) \cdot \mathbf{e}_3(\mathbf{x}) = 0$, $\nabla f(\mathbf{x}) \cdot \mathbf{e}_2(\mathbf{x}) = 0$; strength = $-\lambda_2$

Valley Surface

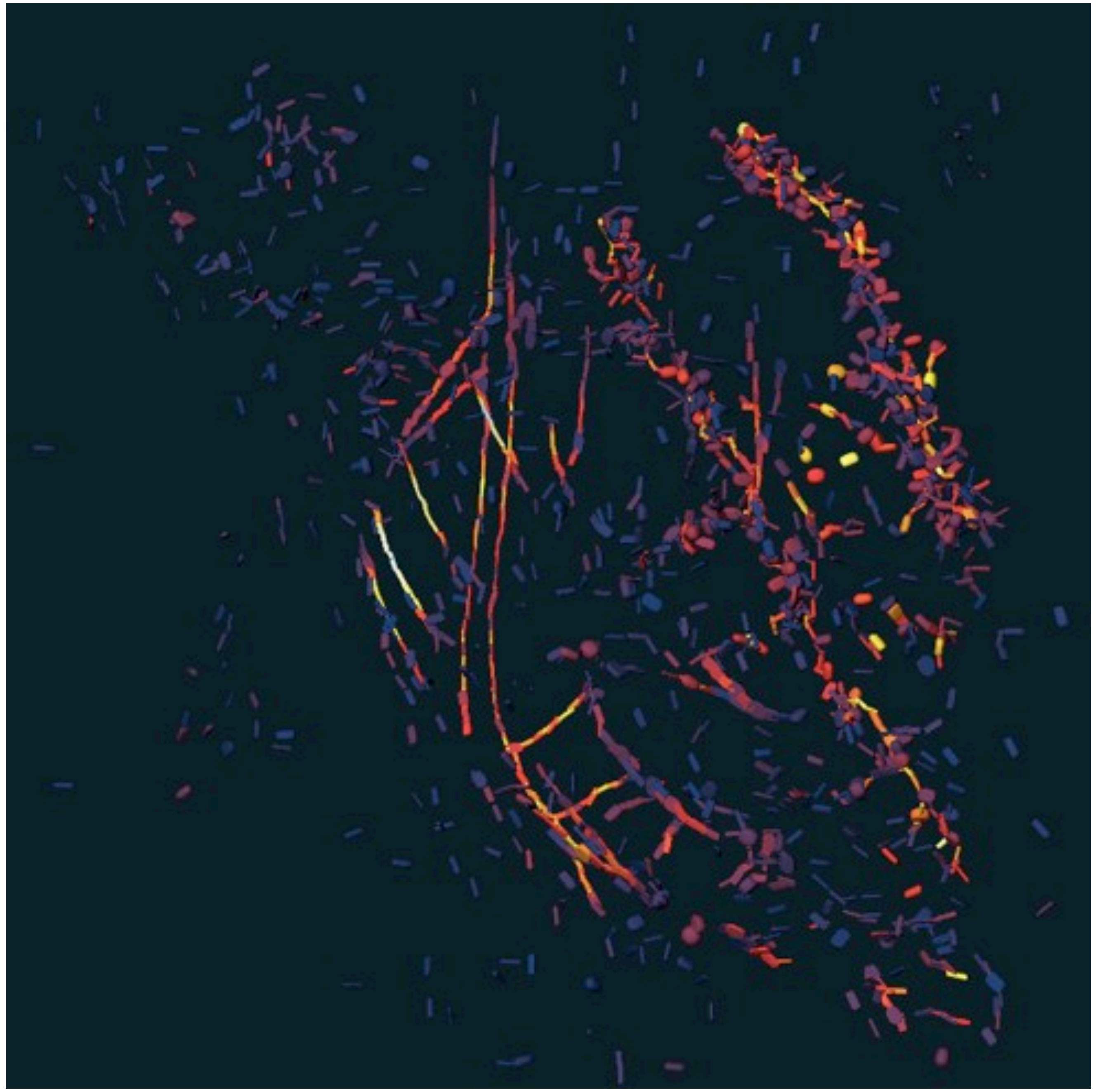


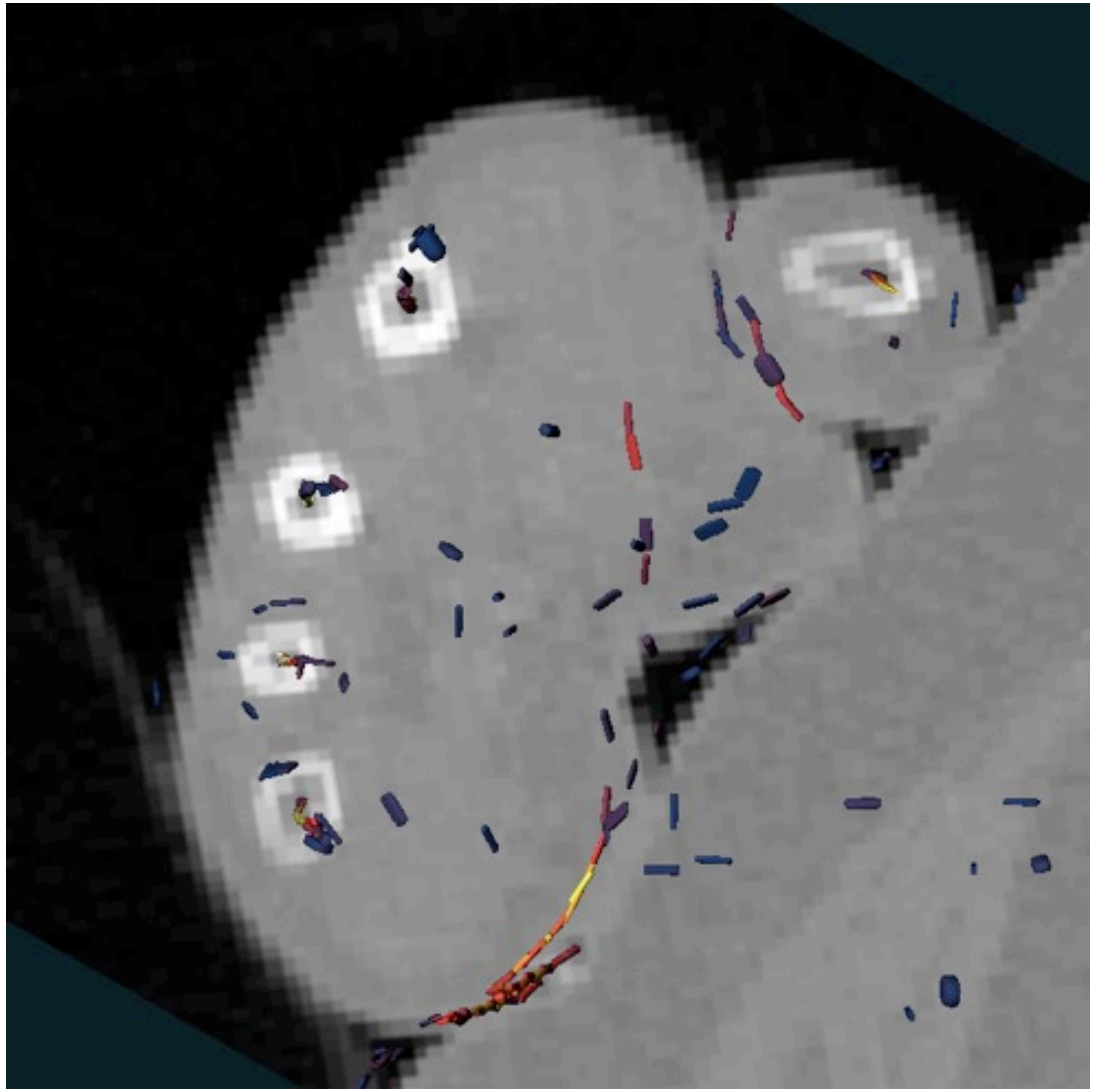


Minimal surface wrt Hessian major eigenvector $\mathbf{e}_1$

$$\nabla f(\mathbf{x}) \cdot \mathbf{e}_1(\mathbf{x}) = 0;\ \text{strength} = \lambda_1$$

Valley Line



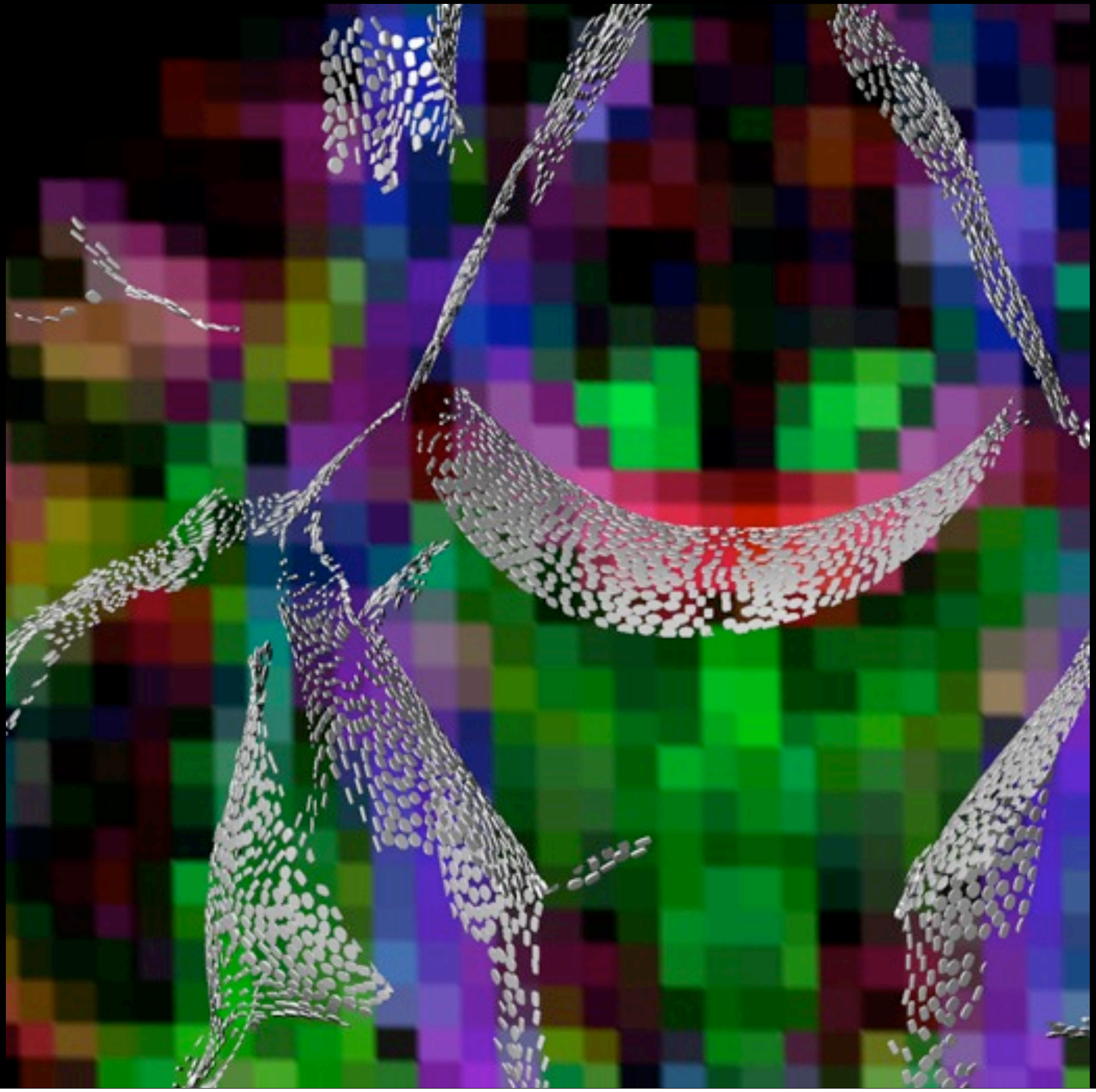


Minimal curve wrt Hessian major,medium eigenvectors

$\nabla f(\mathbf{x}) \cdot \mathbf{e}_1(\mathbf{x}) = 0$, $\nabla f(\mathbf{x}) \cdot \mathbf{e}_2(\mathbf{x}) = 0$; strength = λ_2

Two uses of glyphs …

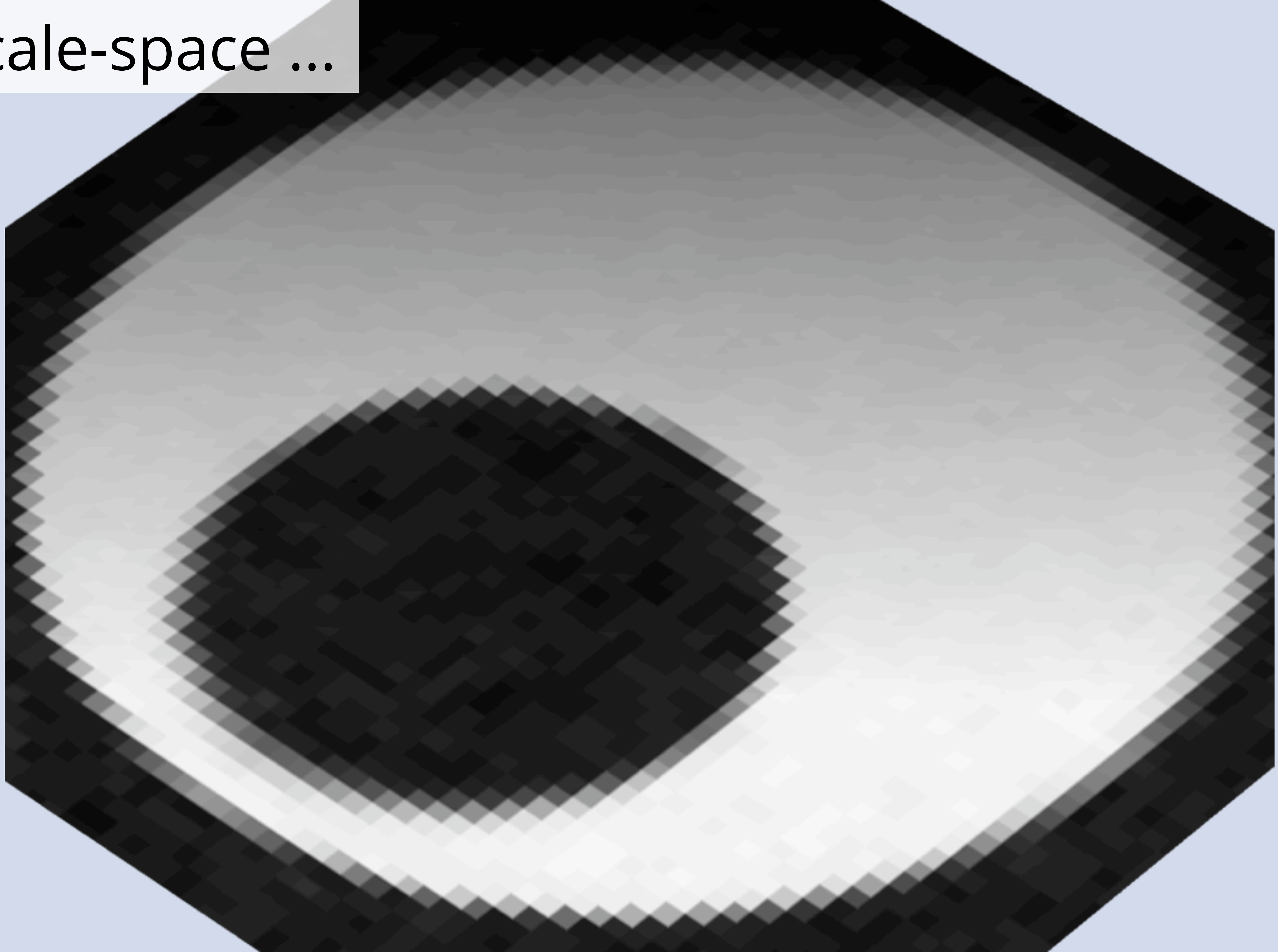
[Kindlmann-VisSym-2004]



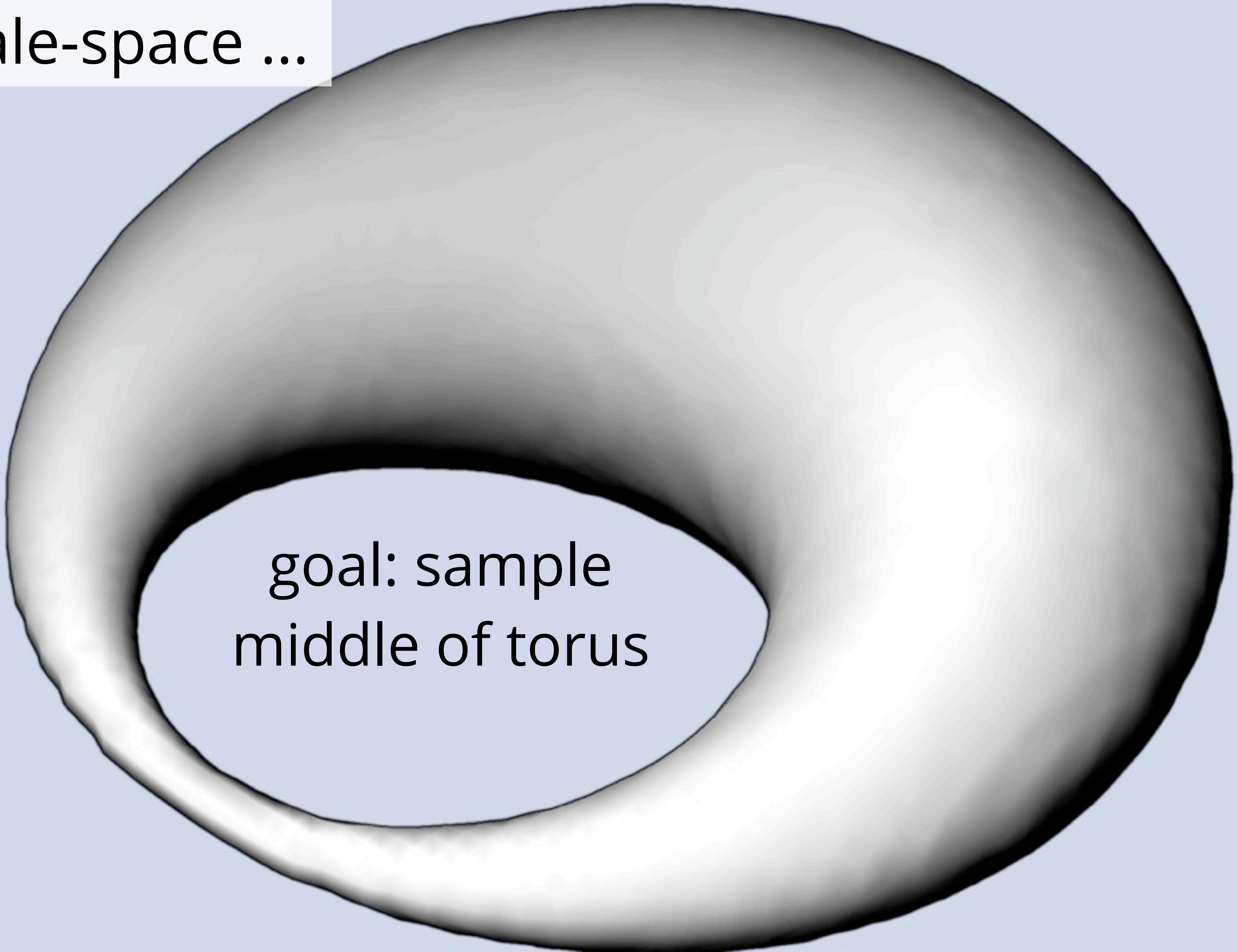
Particles for DTI **visualization**
(earlier “Glyph Packing” [Kindlmann-VIS-2006])
Glyphs for diffusion tensors
Sampling whole field
Image for humans to look at

Particles for DTI **analysis**
(more recent [Kindlmann-VIS-2009])
Glyphs for Hessians of FA
Sampling crease features
Geometry for measurement

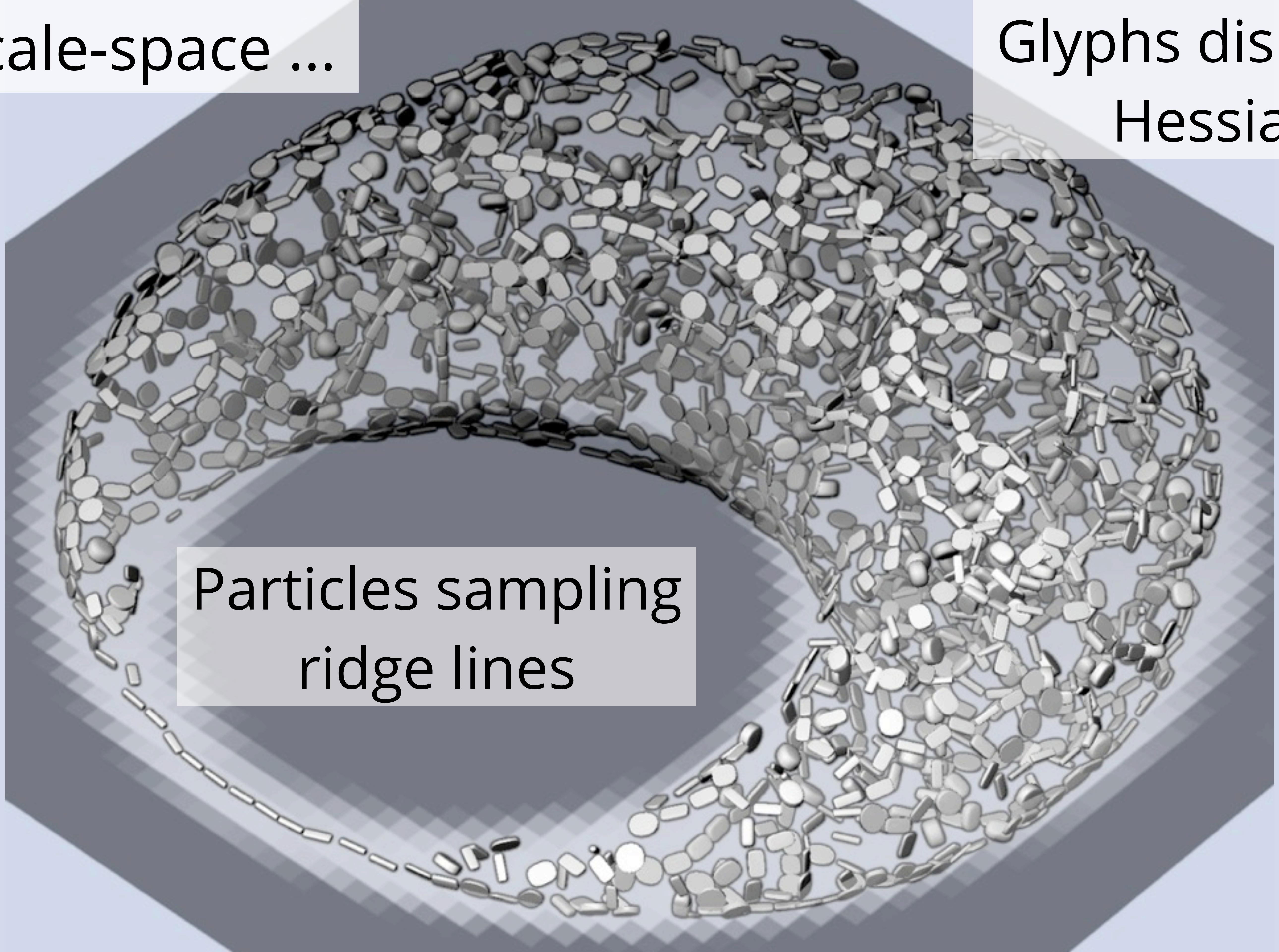
Why scale-space ...

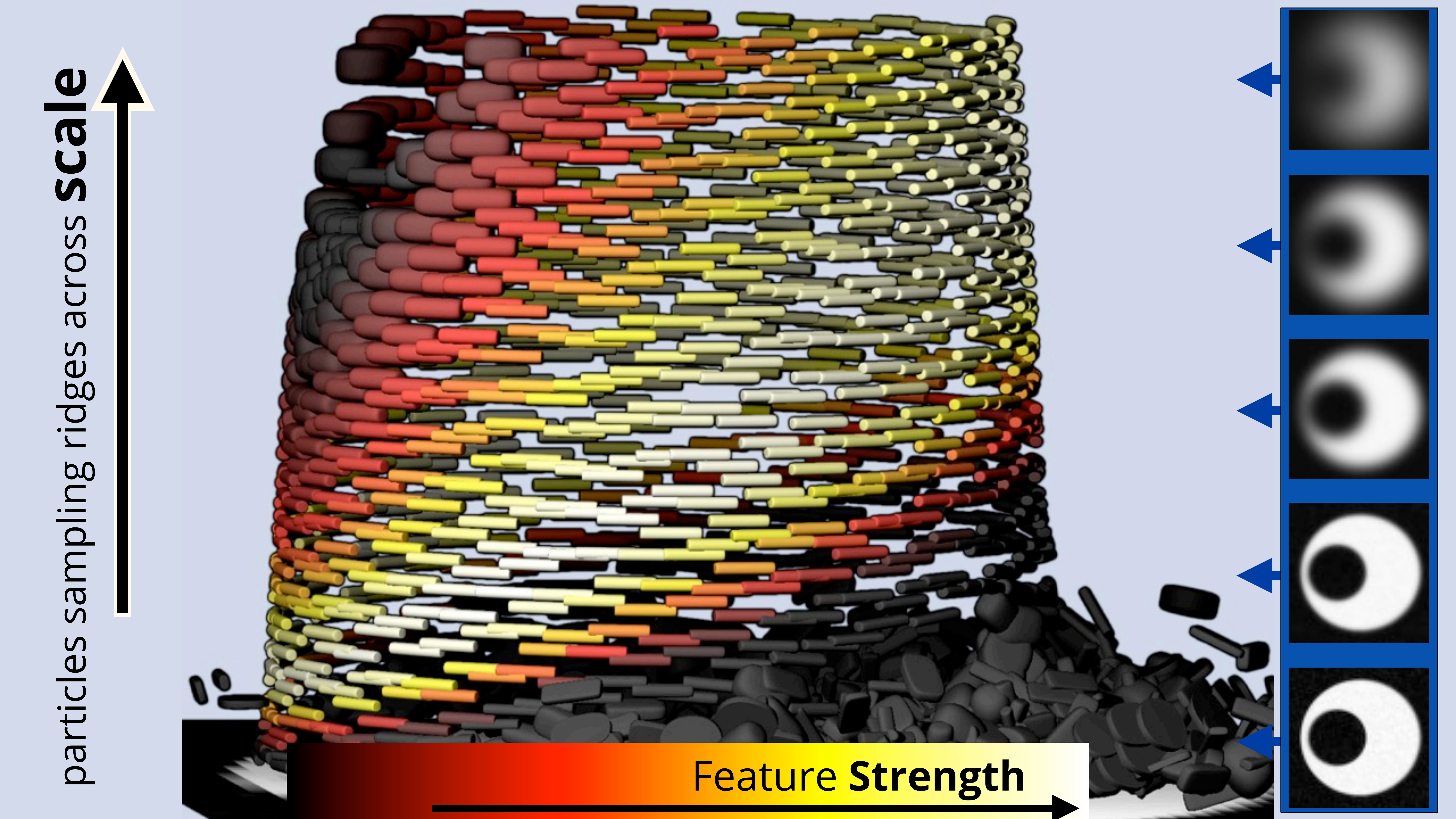


Why scale-space …

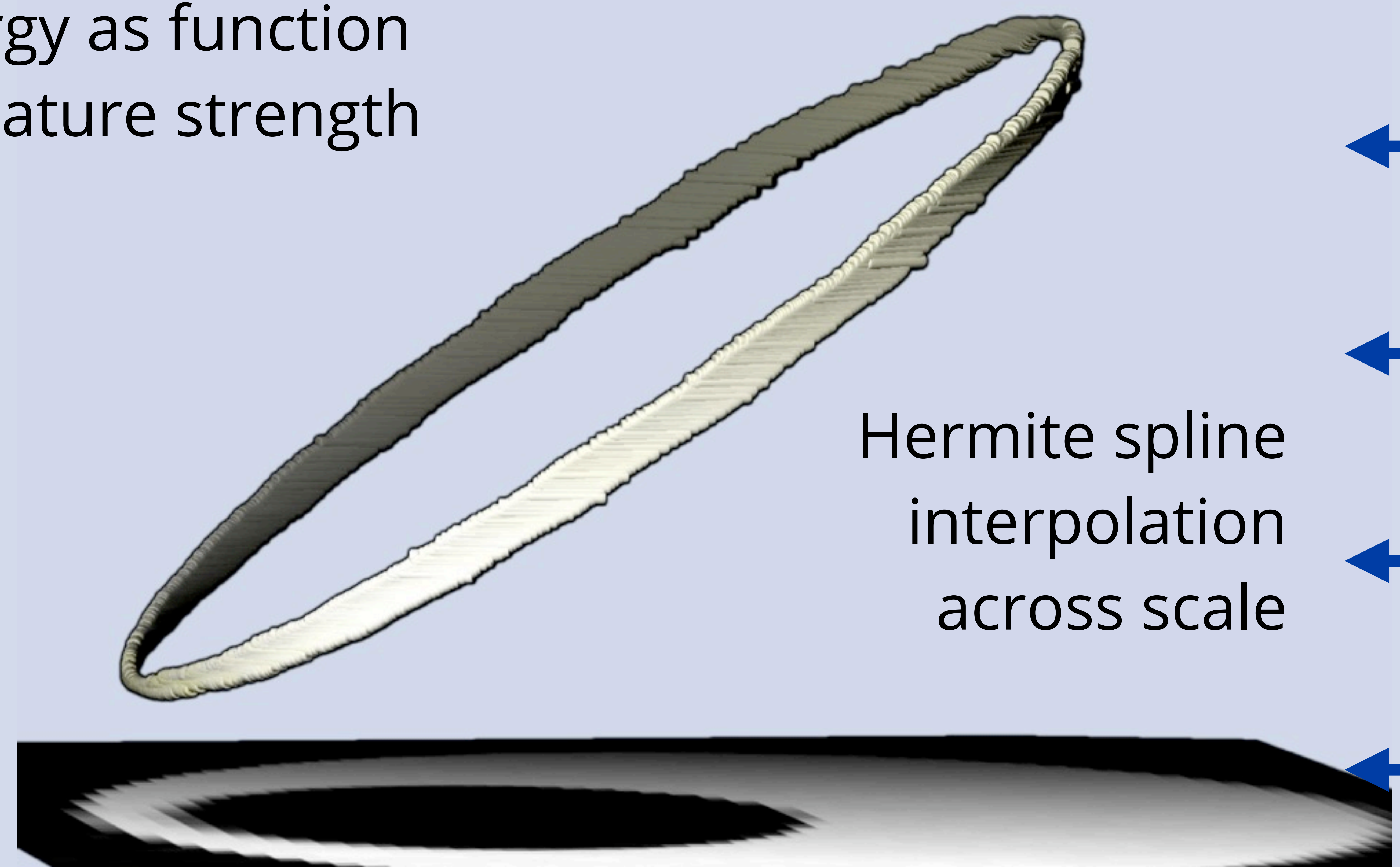


Why scale-space ...

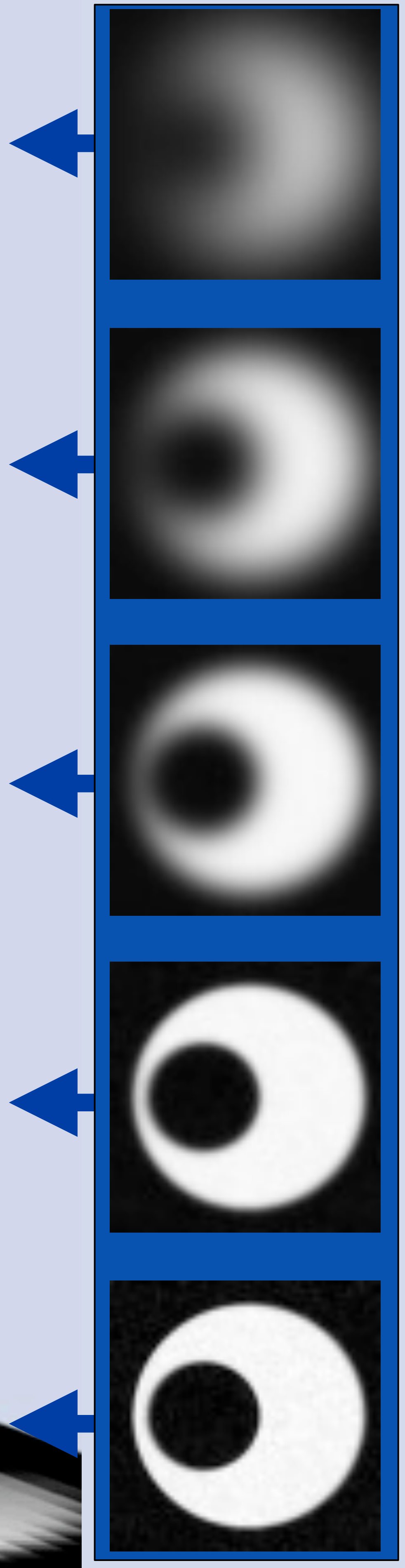


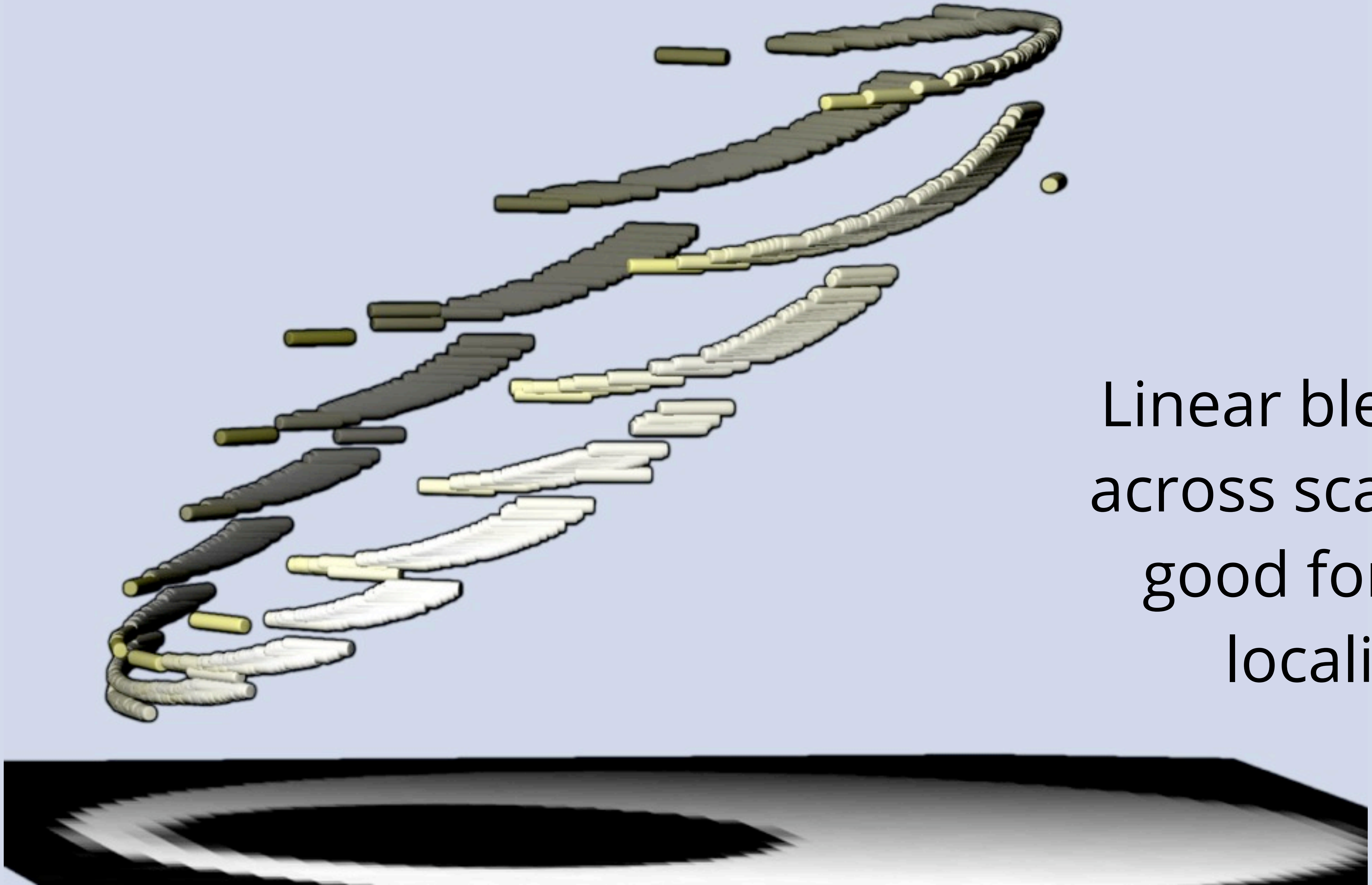

particles sampling ridges across **scale**
Feature **Strength**

Particle-Image energy as function of feature strength



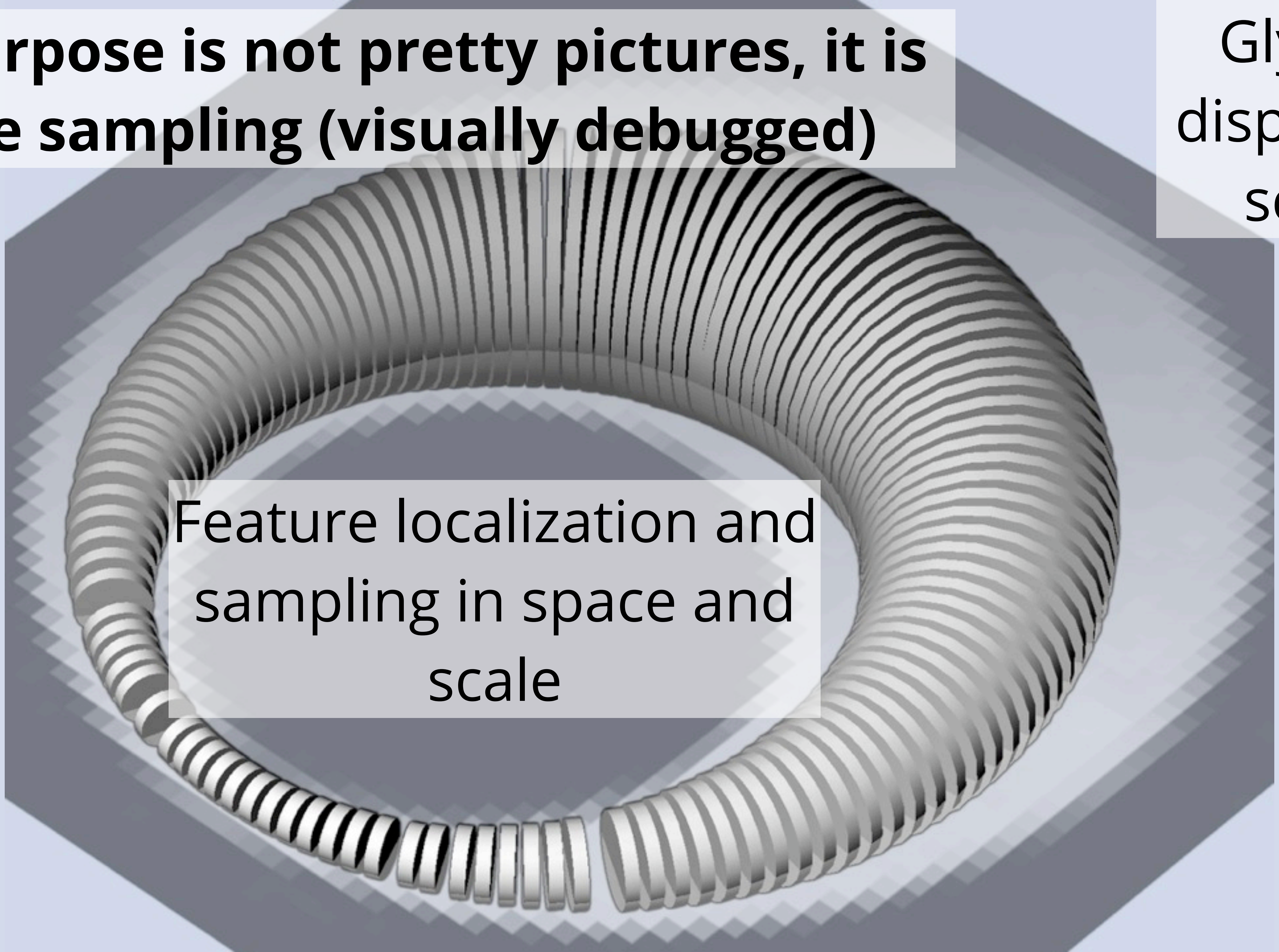
Hermite spline interpolation across scale





Linear blending across scale not good for scale localization

The purpose is not pretty pictures, it is feature sampling (visually debugged)



Contributions

Efficient interpolation of scale for 3D images

Particle-based sampling of ridges and valleys, in scale-space (vs. implicit surfaces, at single scale)

- Energies that implement scale-localization

Glyphs for depicting Hessians and/or scale

Scale-space feature extraction in DTI

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Method Overview

Governing equation

Interpolation: discrete to continuous, scale & space

Crease feature constraints and strengths

Particle-Image energy

Inter-particle energy

System visualization and dynamics

Governing equation

$$\underset{\{(\mathbf{x}_i, s_i)\}, N}{\operatorname{argmin}} \; \mathcal{E} = \underset{\{(\mathbf{x}_i, s_i)\}, N}{\operatorname{argmin}} \; (1-\alpha) \sum_{i=1}^{N} E_i + \frac{\alpha}{2} \sum_{i,j=1}^{N} E_{ij}$$

- $\{(\mathbf{x}_i, s_i)\}$: Particle positions
- N: Particle number
- $\sum_{i=1}^{N} E_i$: Particle-image energy: localizes feature **scale**
- $\sum_{i,j=1}^{N} E_{ij}$: Inter-particle energy: induces uniform **spatial** sampling

Feature constraints: enforced without contributing to energy

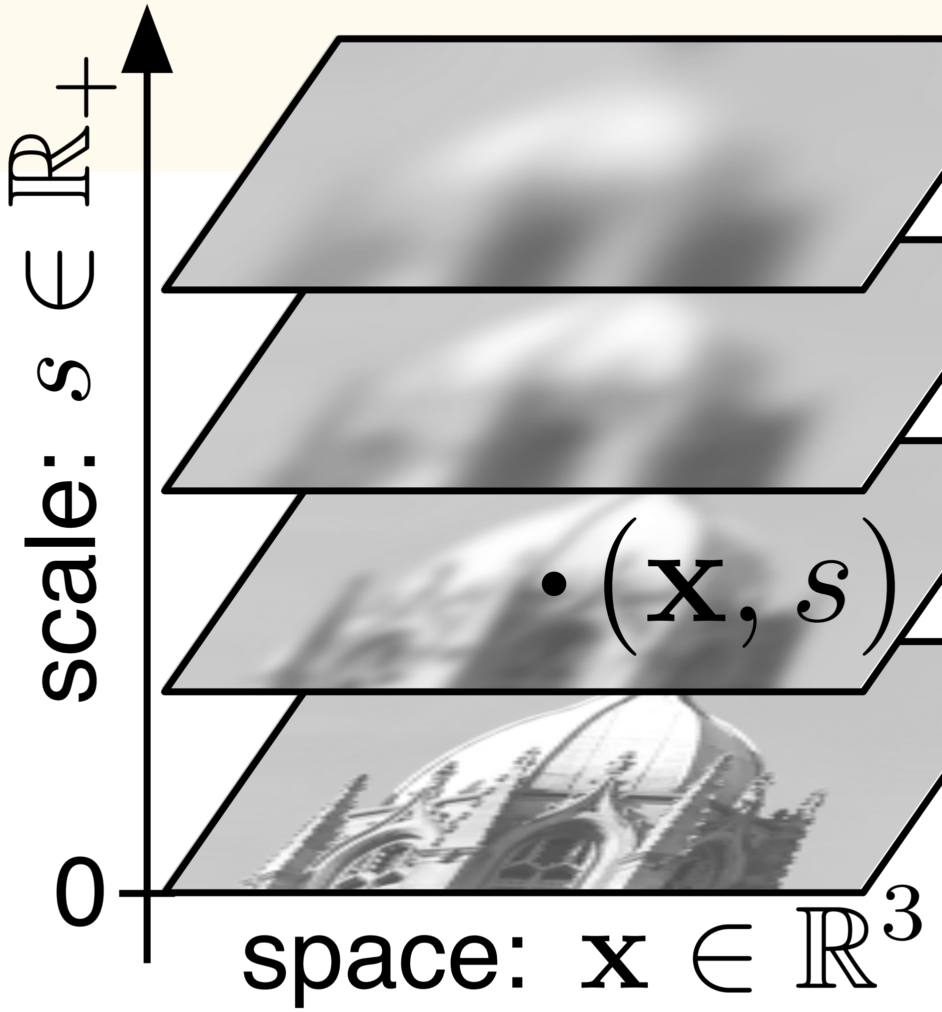
Population control (finding N) becomes side-effect of energy minimization

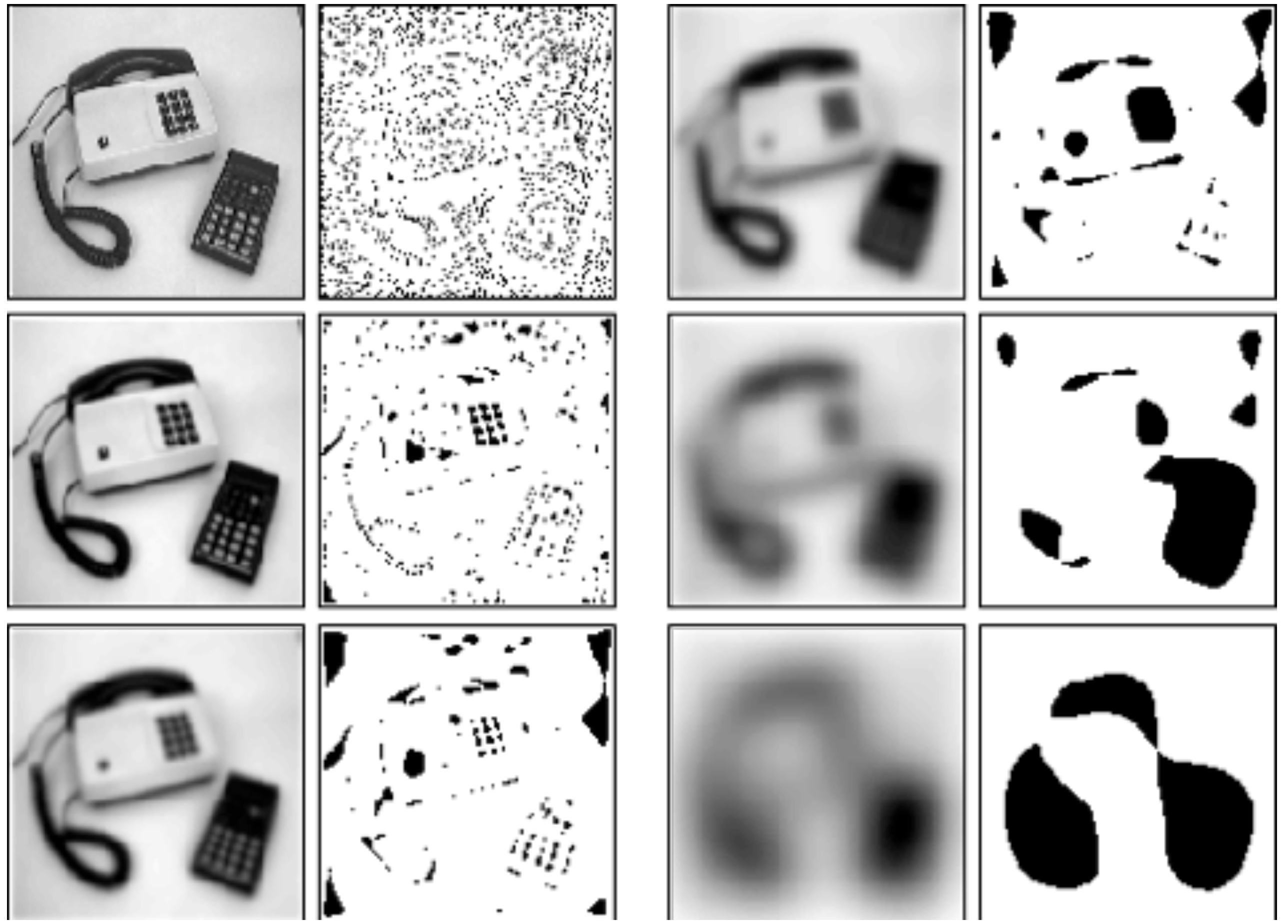
Scale-Space

From ‘90s computer vision

Image and all possible **blurrings**

More general than multi-resolution methods: scale is **continuous**





Structures of different sizes are naturally extracted at different scales [Lindeberg-IJCV-1998]

Scale-Space And Diffusion

Various considerations lead to blurring with a **Gaussian**

Image L(*x*) diffuses for time *t* →continuum of images L(*x*;*t*)

$$
\begin{aligned}
L(x;t) &= (L(\cdot) \star g(\cdot;t))(x) = \int g(\xi;t)L(x-\xi)d\xi \\
g(\xi;t) &= \exp(-\xi^2/2t)/\sqrt{2\pi t}
\end{aligned}
$$

Heat equation: 1st deriv. in time → 2nd deriv. in space

$$
\frac{\partial L(x;t)}{\partial t} \propto \frac{\partial^2 L(x;t)}{\partial x^2}
$$

What's the analog in the **discrete** (implementation) domain?

Lindeberg's Discrete Gaussian

"Scale-Space for Discrete Signals" [Lindeberg-PAMI-1990]

Beautiful analog to continuous Gaussian

$$L[i;t] \quad = \quad (f \star K[\cdot;t])[i] = \sum_n K[n;t] f[i-n]$$

$$K[n;t] \quad = \quad \exp(-t) I_n(t)\,;\ s = \sqrt{t} = \text{``}\sigma\text{''}$$

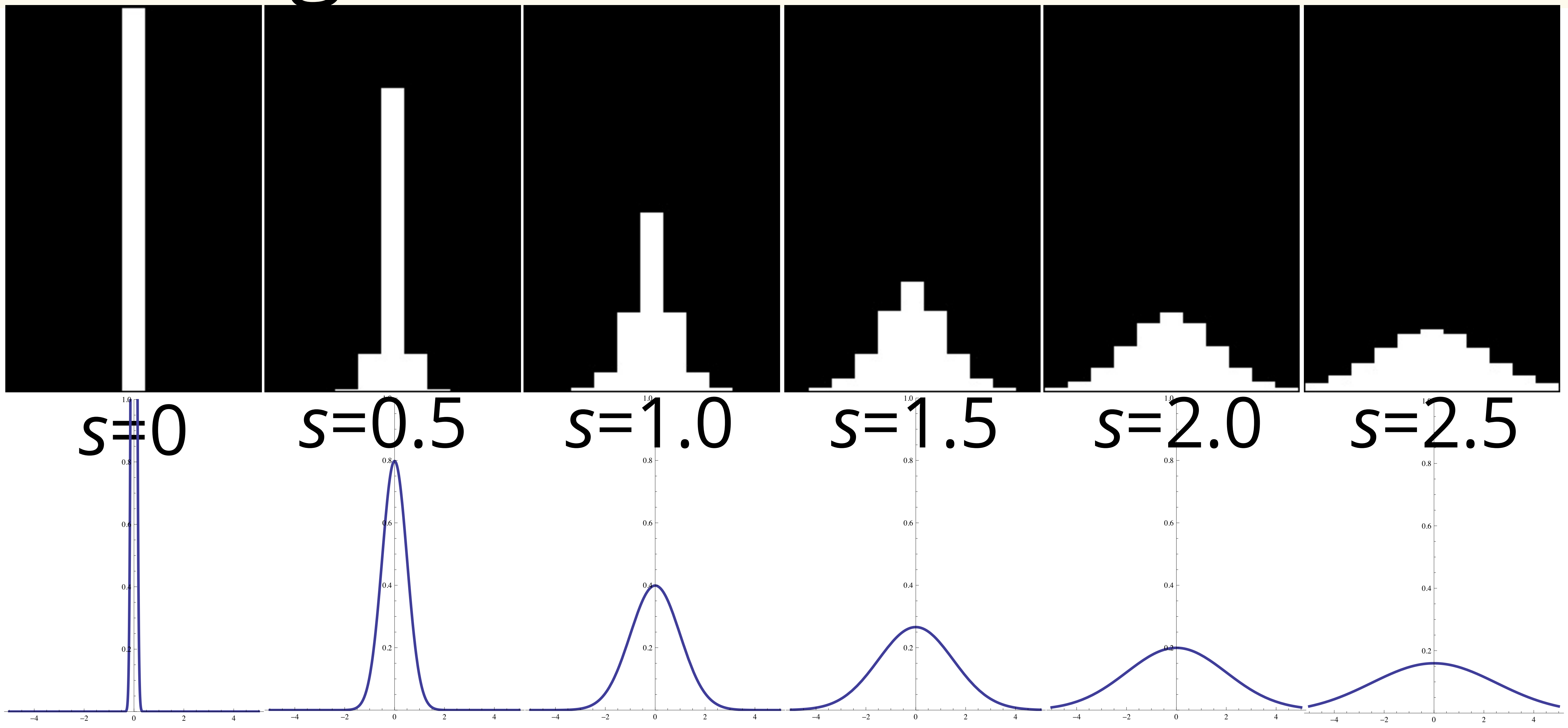
$I_n(t)$ = modified Bessel function of order *n*

$$\frac{\partial K[i;t]}{\partial t} \quad = \quad \frac{1}{2}(K[\cdot;t] \star \begin{bmatrix} 1 & -2 & 1 \end{bmatrix})[i]$$

$$\Rightarrow \frac{\partial L[i;t]}{\partial t} \quad = \quad \frac{1}{2}(L[\cdot;t] \star \begin{bmatrix} 1 & -2 & 1 \end{bmatrix})[i]$$

$$\Rightarrow \frac{\partial L[i;s^2]}{\partial s} \quad = \quad s\,(L[\cdot;s^2] \star \begin{bmatrix} 1 & -2 & 1 \end{bmatrix})[i]$$

Lindeberg's Discrete Gaussian



Not the same as sampling a Gaussian

Change **between** blurring levels important

Interpolate along scale

Want continuous scale, but hard with full-resolution volume images: have a memory limit

Sampling and reconstruction problem: how to pre-compute some blurrings and then accurately/efficiently interpolate?

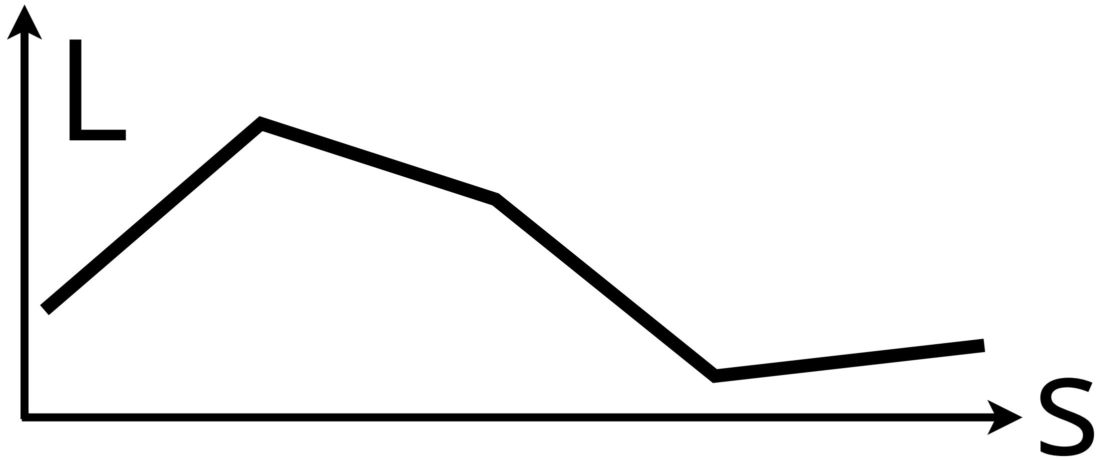
Leverage Lindeberg's Gaussian:

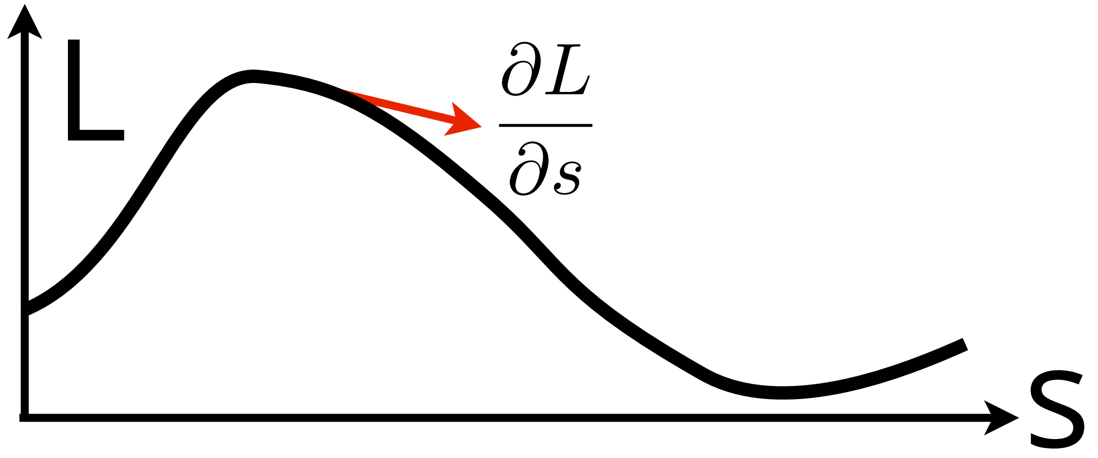
$$\frac{\partial L}{\partial s} = s\left(L[\cdot; s^2] \star \begin{bmatrix}1 & -2 & 1\end{bmatrix}\right)$$

Pre-compute blurrings of image L for discrete set of blurring levels

For intermediate scales, could linearly blend between, or

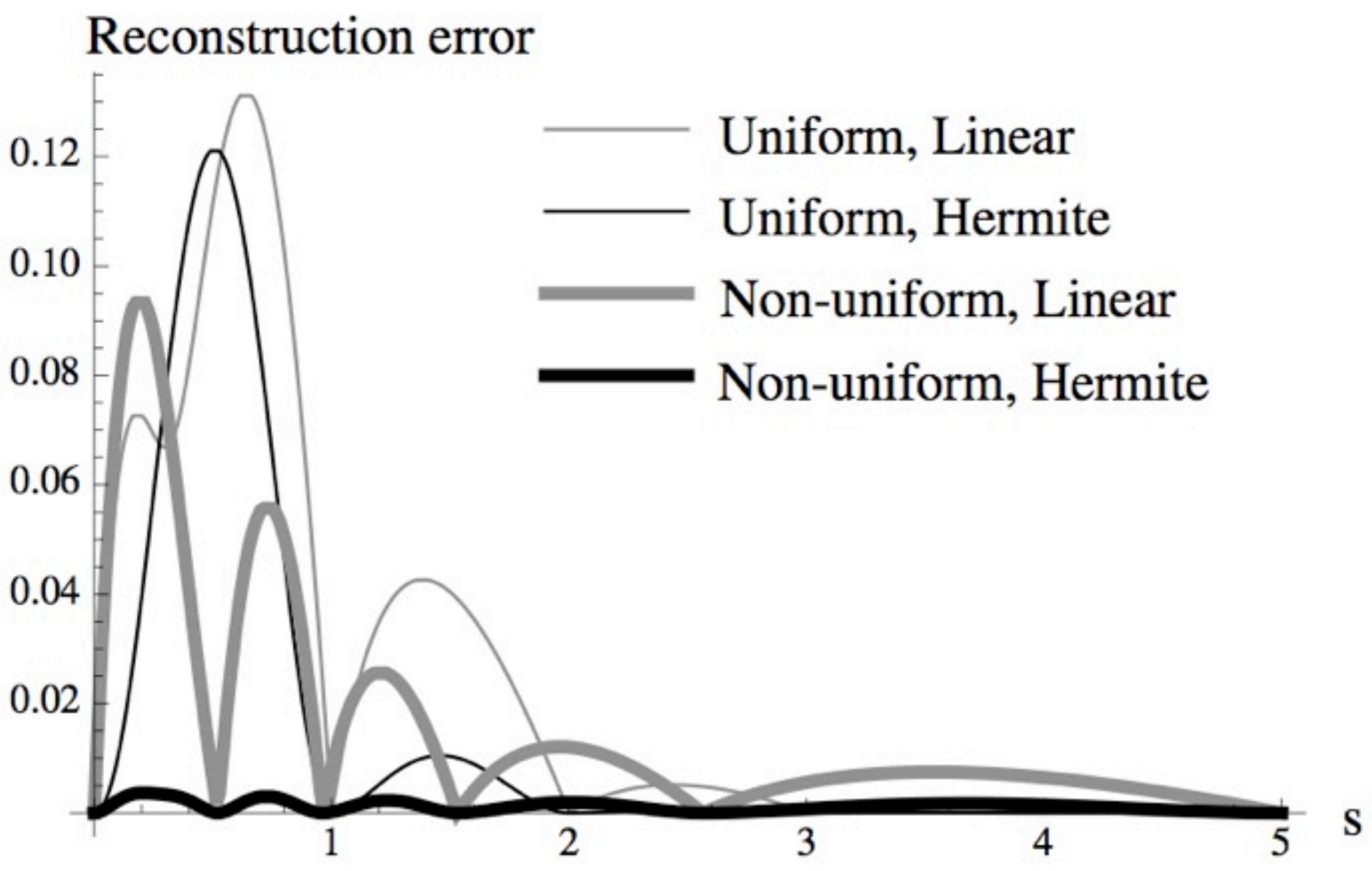
Knowing dL/ds at each scale, create cubic **Hermite spline**





Scale interpolation accuracy

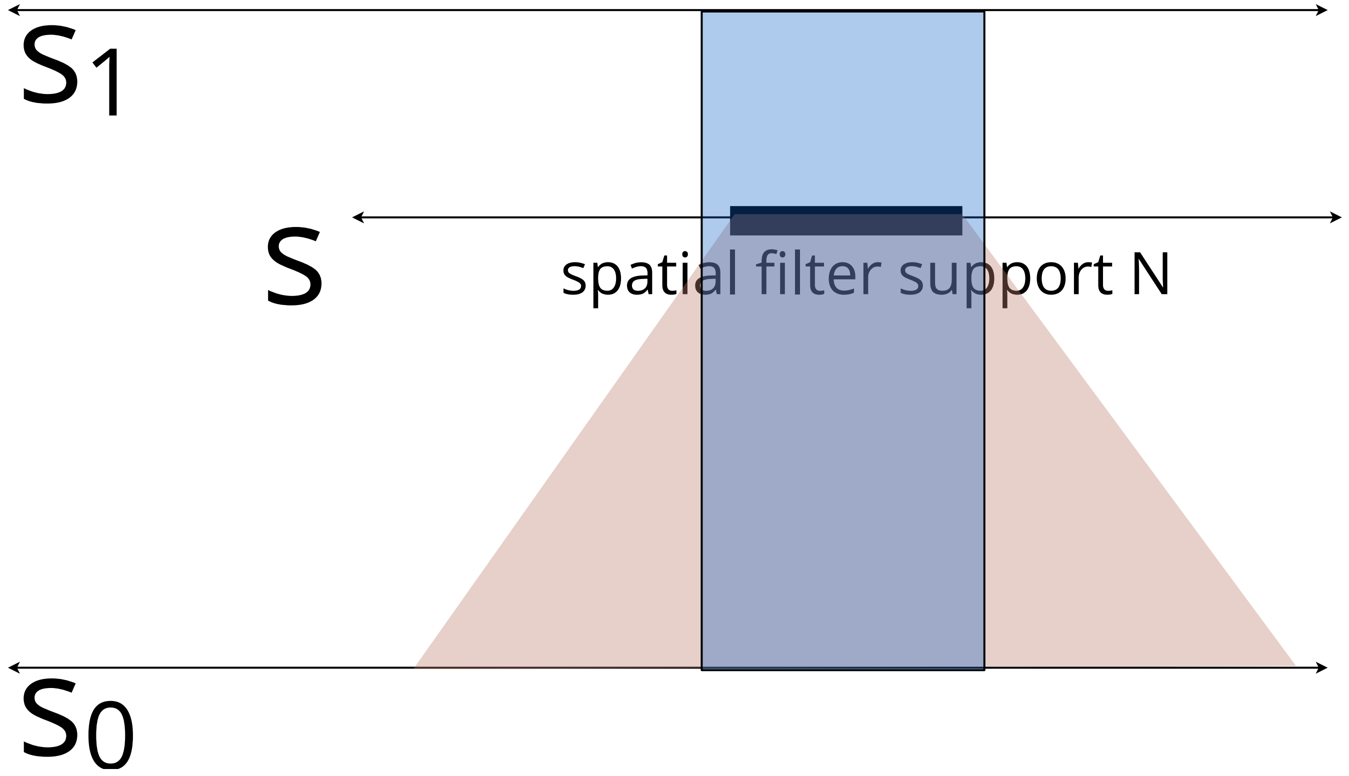
- Goal: best accuracy with minimum number of pre-blurring volumes
- Measure error as squared difference between interpolated K[] and true K[], summed over support
- Optimize non-uniform scale sample locations by gradient descent on error



Hermite-spline scale interpolation makes scale-space practical for real-world 3D volumes

Why not do it correctly?

I.e: reconstruct spatial image neighborhood around particle, at intermediate scale s (between pre-computed blurrings s_0, s_1)



Exact solution: blur from next-lowest scale

E.g. $s_0 = 4$; $s_1 = 8$; $s = 7$ ($t_0 = 16$; $t_0 = 64$; $t = 49$)

diffuse for 49 - 16 = 33: blur with s=5.74 needs support of ~30 samples

Exact solution needs $(30 + N)^3$ samples

Hermite-spline approximation needs $2(2 + N)^3$ samples

Memory is slowest part of a computer

Spatial Interpolation

At each particle location:

- Scale interpolation → discrete spatial support
- Separable convolution → values and derivatives

6-sample support

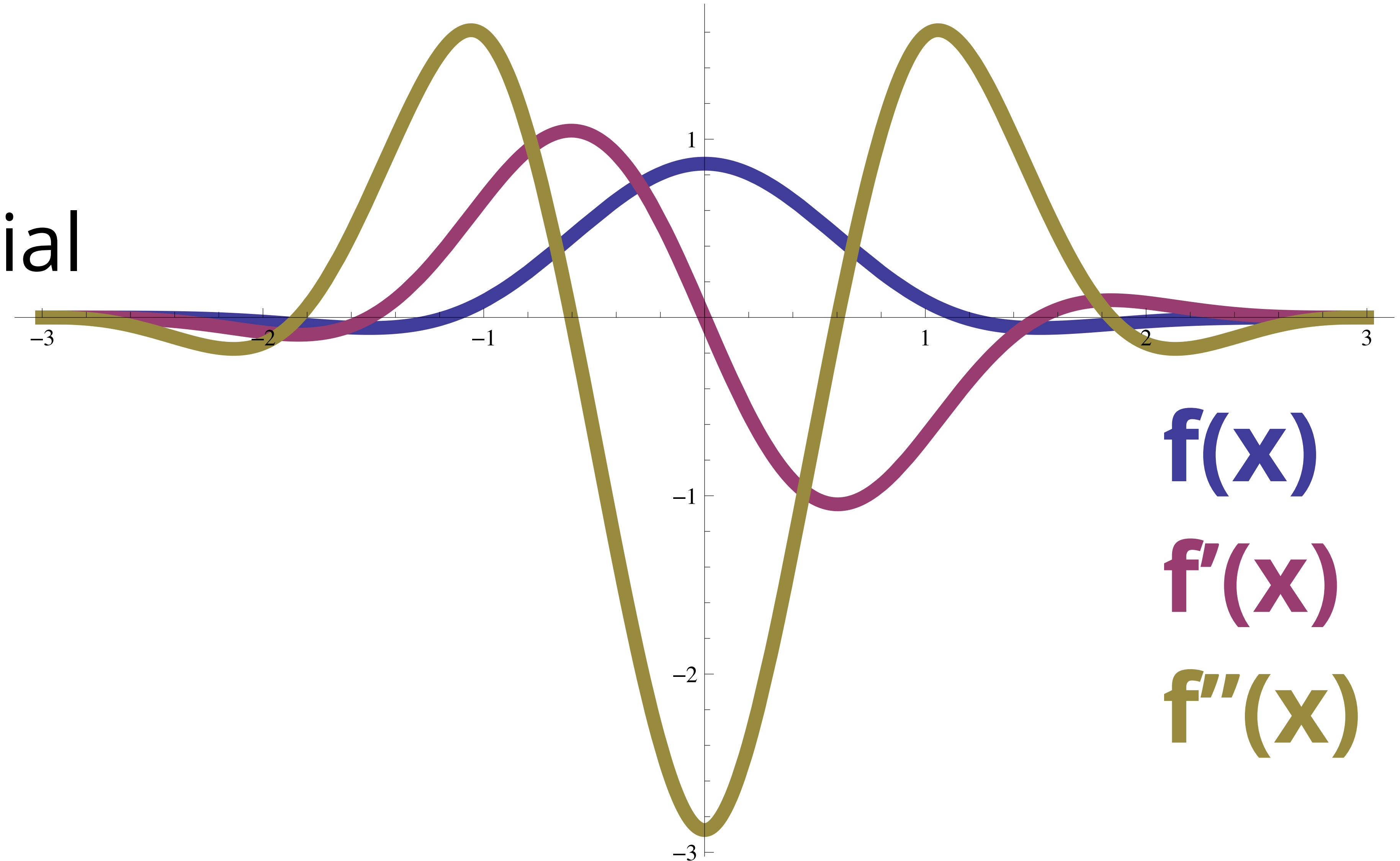
Piece-wise 6th-order polynomial

C^4 continuity

Reconstructs cubics

("c4hexic" in Diderot)

[Möller-TVCG-1997]



Particle-Image Energy E_i

Ridge (R) and valley (V) surfaces (S) and lines (L)

	R L	R S	V L	V S
Definition	$\mathbf{g}\cdot\mathbf{v}_2=0$ $\mathbf{g}\cdot\mathbf{v}_3=0$	$\mathbf{g}\cdot\mathbf{v}_3=0$	$\mathbf{g}\cdot\mathbf{v}_1=0$ $\mathbf{g}\cdot\mathbf{v}_2=0$	$\mathbf{g}\cdot\mathbf{v}_1=0$
λ sign	$\lambda_3 \le \lambda_2 < 0$	$\lambda_3 < 0$	$0 < \lambda_2 \le \lambda_1$	$0 < \lambda_1$
Strength h	$-\tilde{\lambda}_2$	$-\tilde{\lambda}_3$	$\tilde{\lambda}_2$	$\tilde{\lambda}_1$
Tangent $\mathbf{T}$	$\mathbf{v}_1 \otimes \mathbf{v}_1$	$\mathbf{v}_1\otimes\mathbf{v}_1$ $+\mathbf{v}_2\otimes\mathbf{v}_2$	$\mathbf{v}_3 \otimes \mathbf{v}_3$	$\mathbf{v}_2\otimes\mathbf{v}_2$ $+\mathbf{v}_3\otimes\mathbf{v}_3$

$$E_i = e(f, \mathbf{x}_i, s_i) = -\gamma h(\mathbf{x}_i, s_i)$$

Particles migrate to **scale of** maximal feature strength as part of energy minimization (not a constraint)

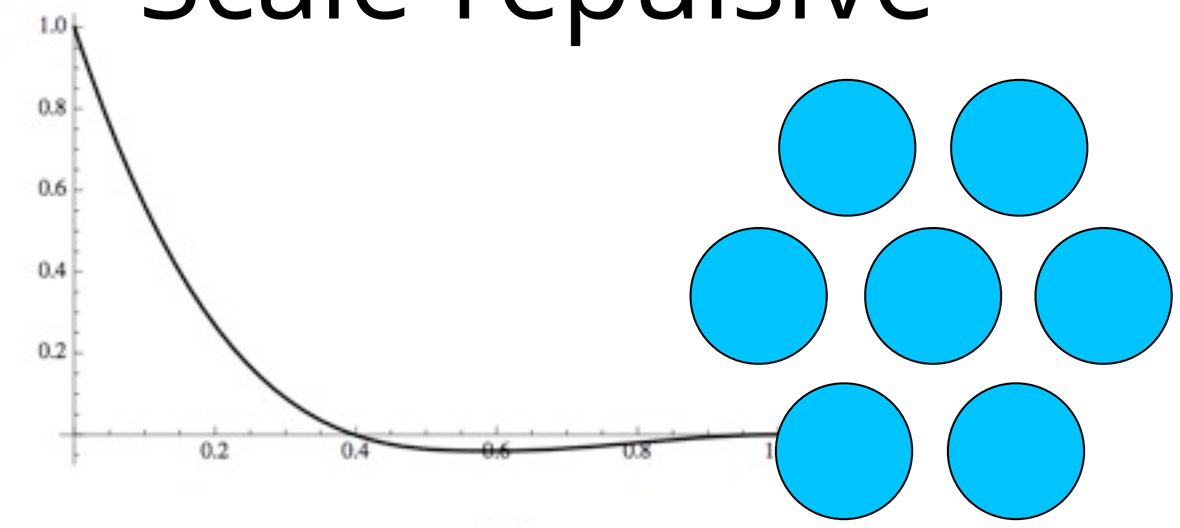
Inter-particle Energy E_{ij}

$$E_{ij} = \Phi(r_{ij}, s_{ij}) = \Phi\left(\frac{|\mathbf{x}_i - \mathbf{x}_j|}{\sigma_r}, \frac{s_i - s_j}{\sigma_s}\right)$$

No intrinsic orientation to particles' potential

Have user-set "radii" in space σ_r and scale σ_s

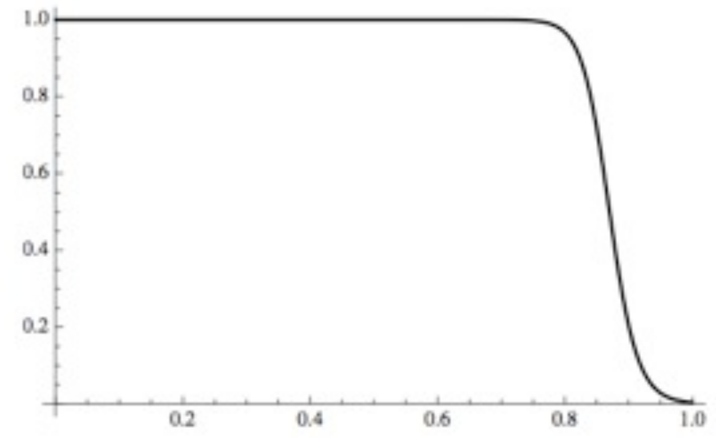
Scale-repulsive



$\phi_c(r)$

$$\Phi_1(r,s) = \phi_c(\sqrt{r^2 + s^2})$$

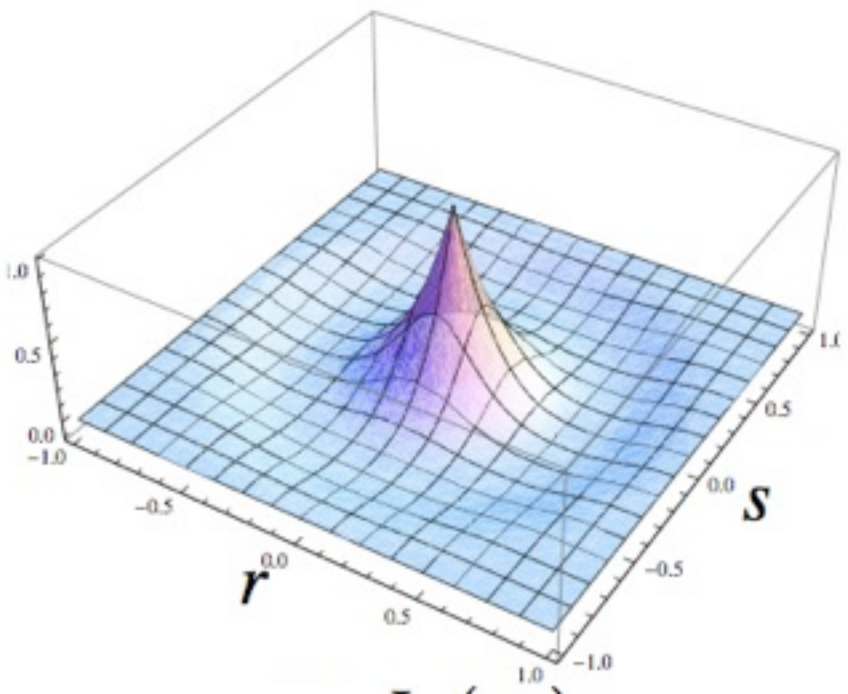
Scale-attractive



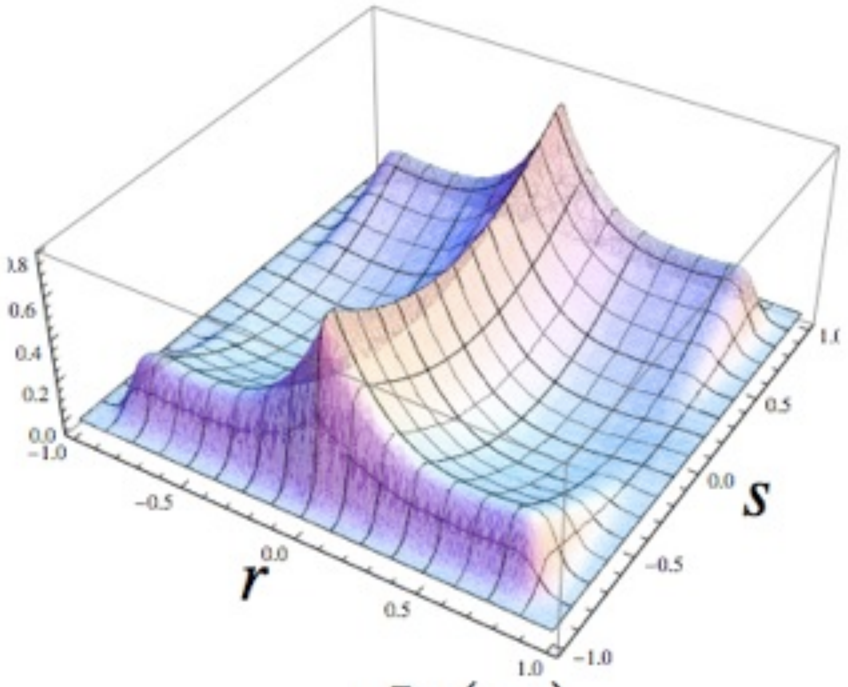
$b(\omega)$

$$\Phi_2(r,s) \quad = \quad (1-\beta)\phi_c(r)b(s) + \beta b(r)b(s)s^2$$

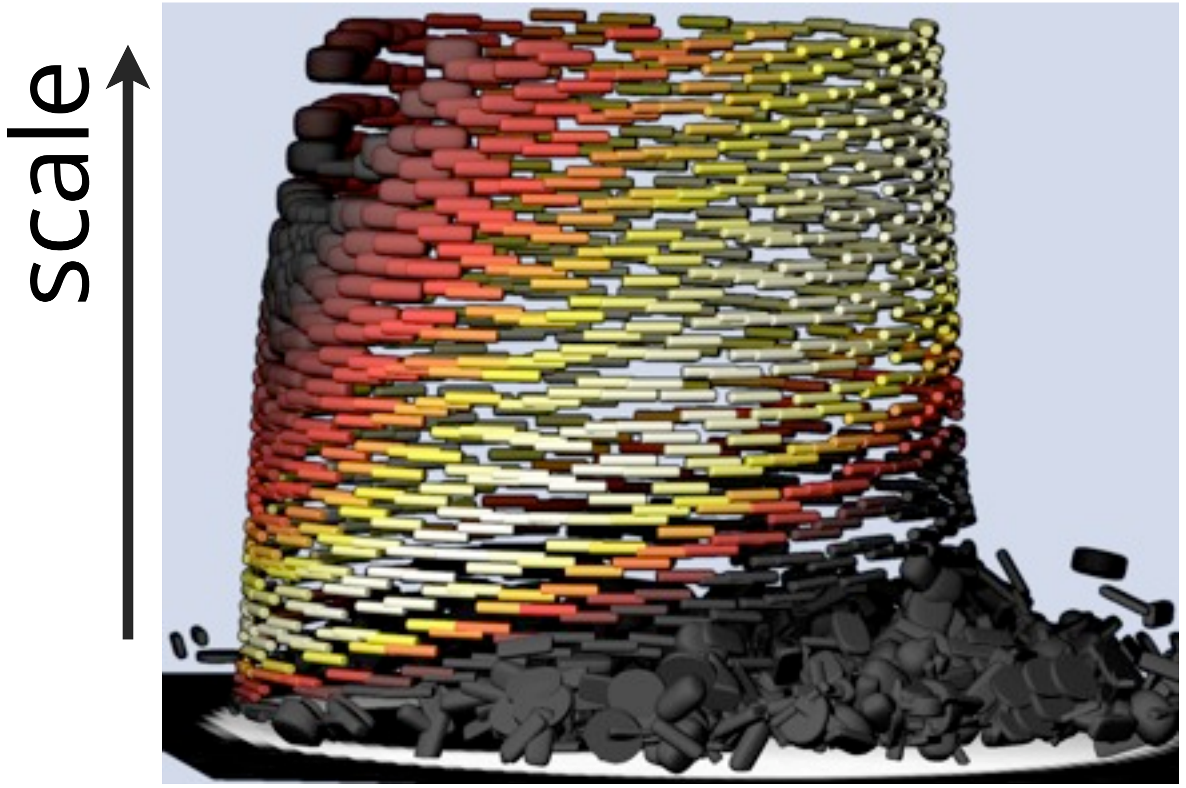
Inter-particle Energy E_{ij}

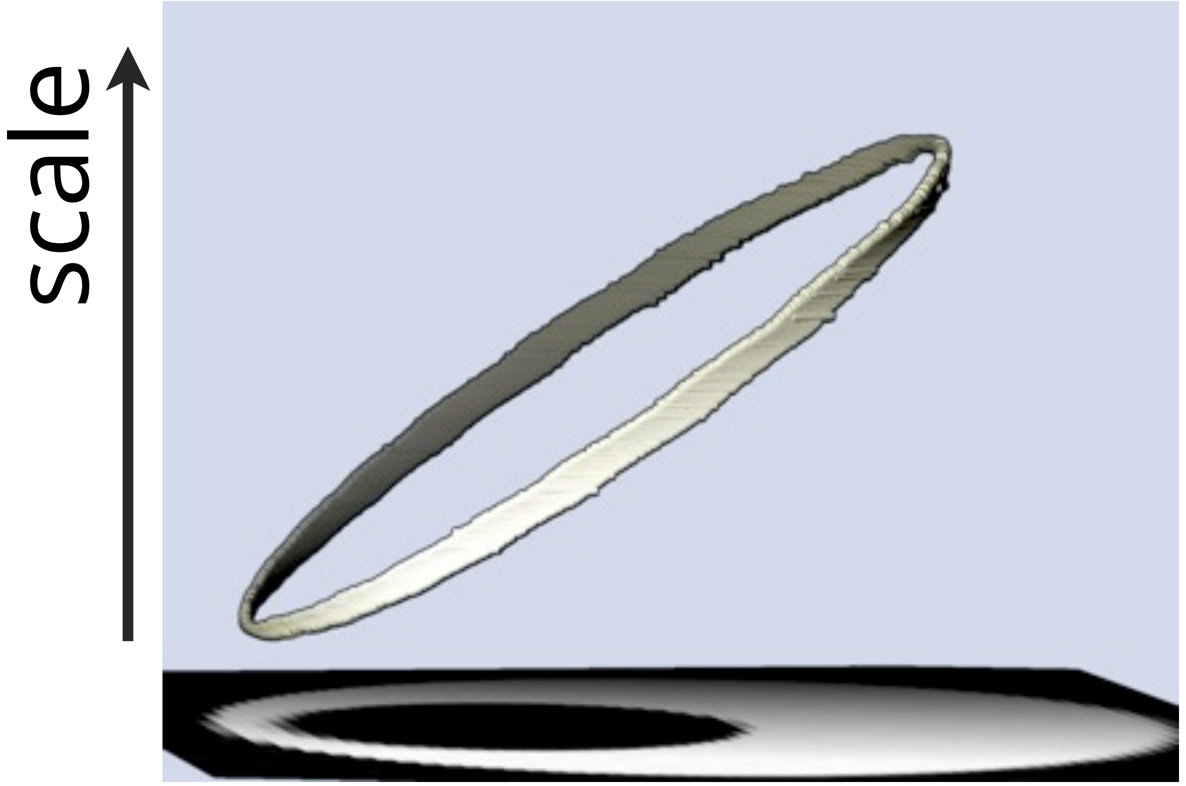


$\Phi_1(r,s)$



$\Phi_2(r,s)$





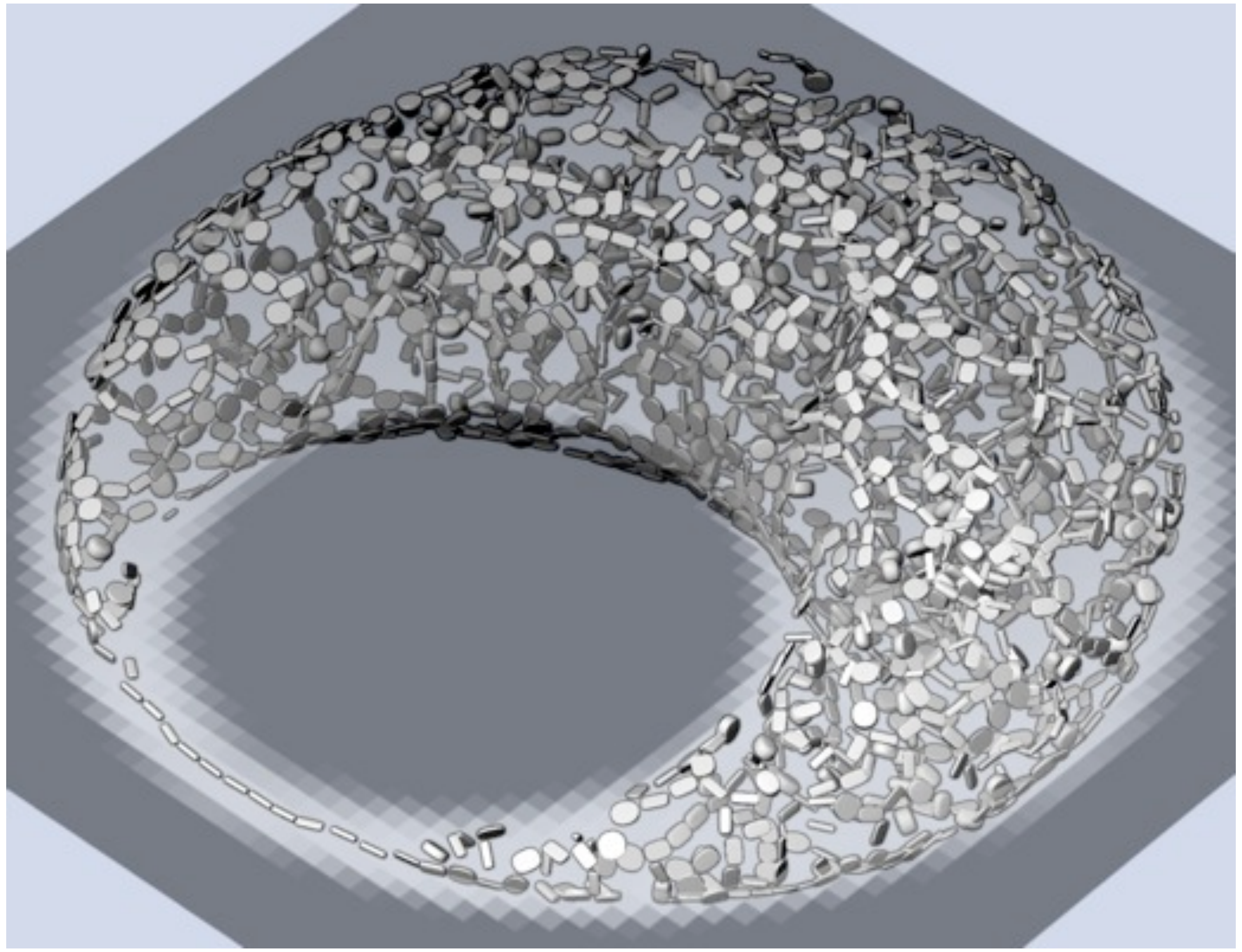
System Visualization

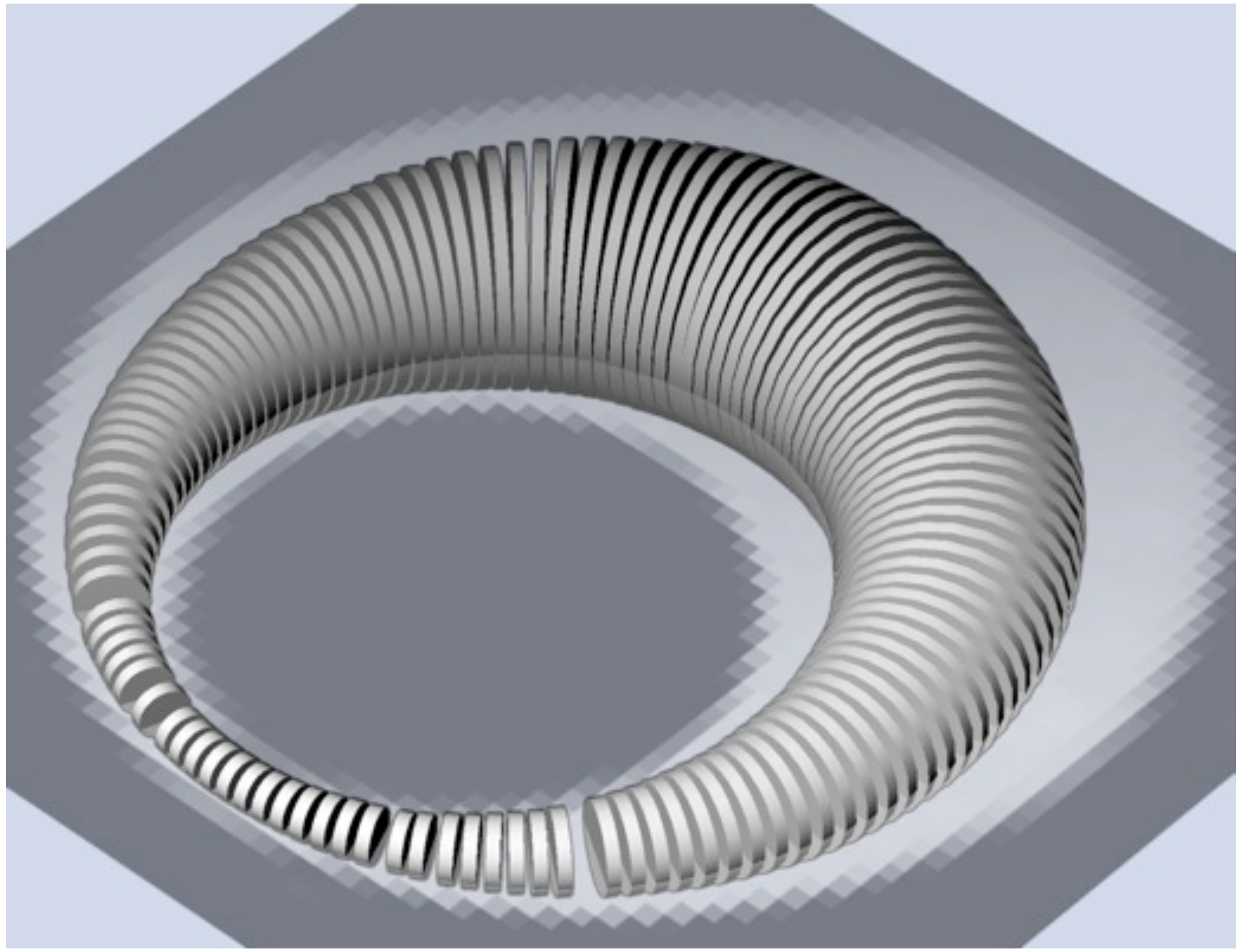
Glyphs at particle locations, possibly colormapped

Glyphs show tensors related to local Hessian

- crease surfaces → discs, lines → rods

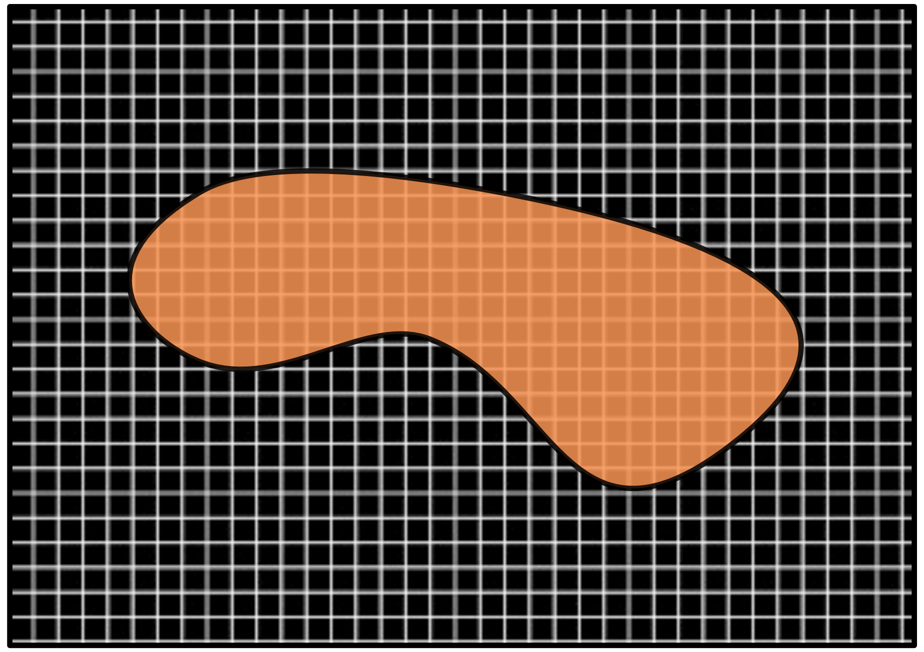
Or, encode scale instead of Hessian eigenvalues

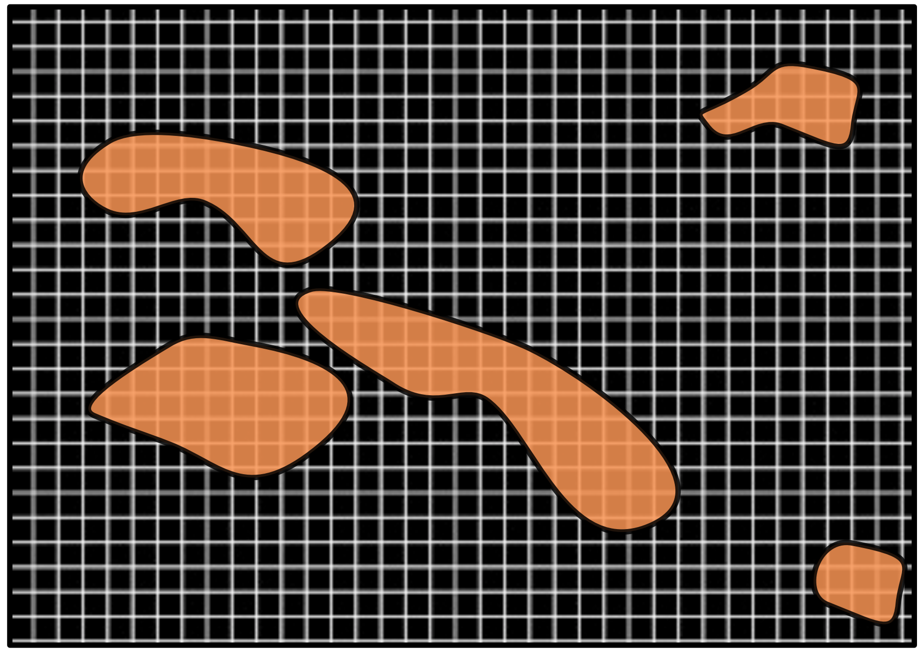




System Computation

- Initialize with particle at every Nth voxel
  - “CPM: A Deformable Model for Shape Recovery and Segmentation Based on Charged Particles” [Jalba-PAMI-2004]
  - Sampling one vs. detecting all
  - Currently bottleneck
- Every iteration decreases energy
  - Move particles, with spatial constraint
  - Periodically try adding or nixing particles
- “Connected components” (CC)
  - Connected if non-zero inter-particle energy





(demo)

(doesn't show scale-space)

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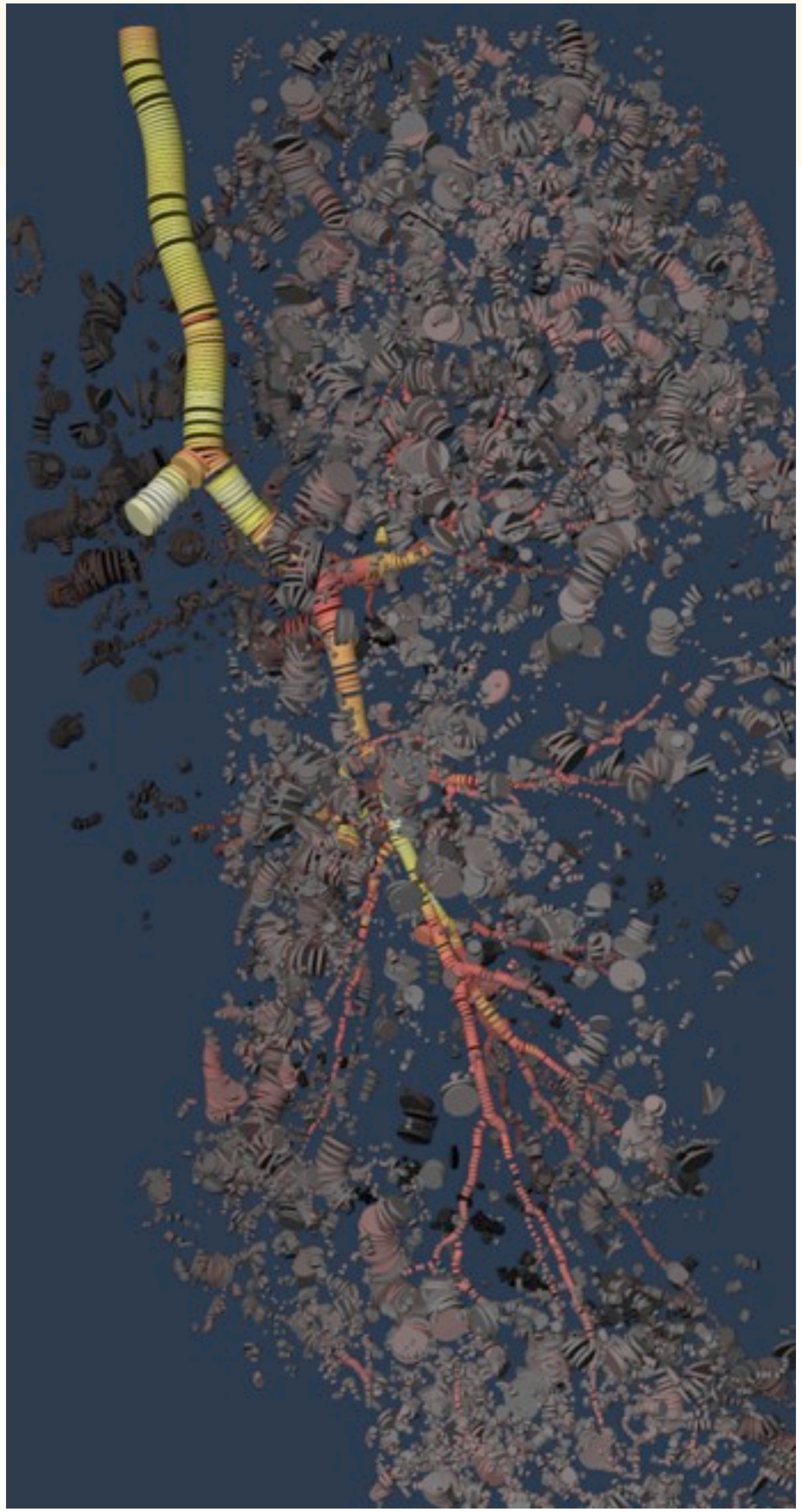
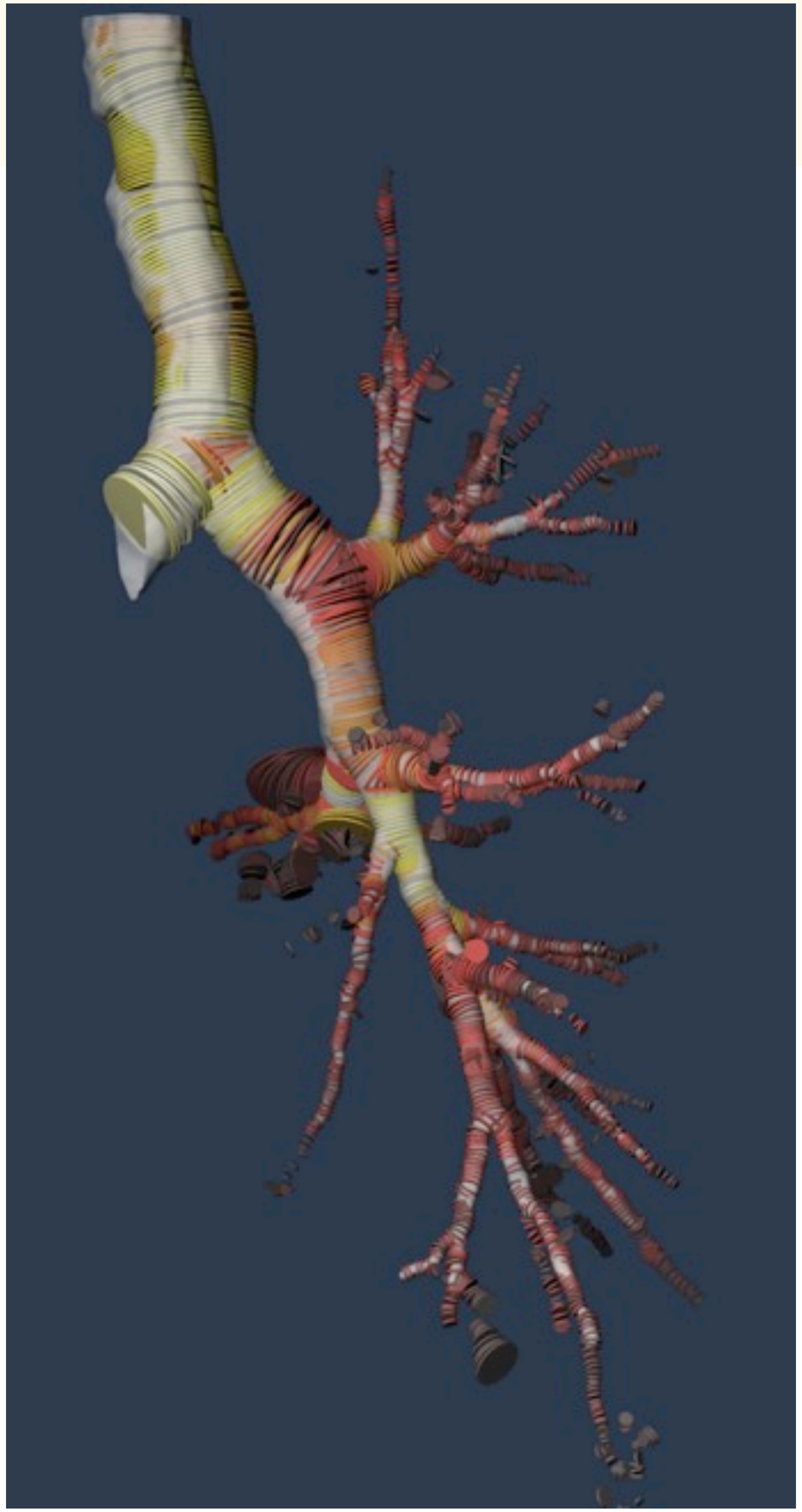
Lung CT Results

- Lung airway segmentation still not quite solved
- Particles captured 4-5 levels of branching, as well as size



All CCs



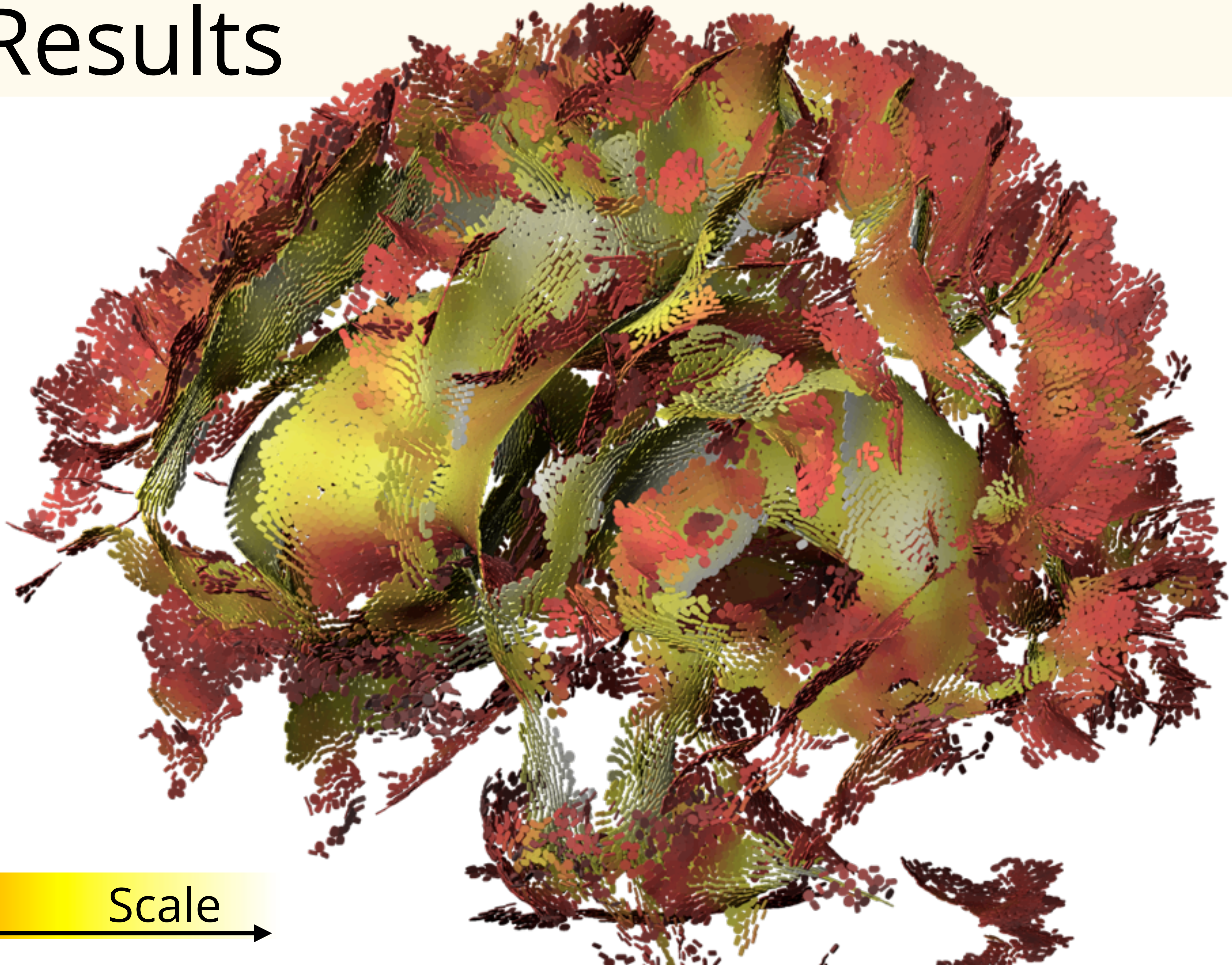
Biggest CC

Lung CT Results

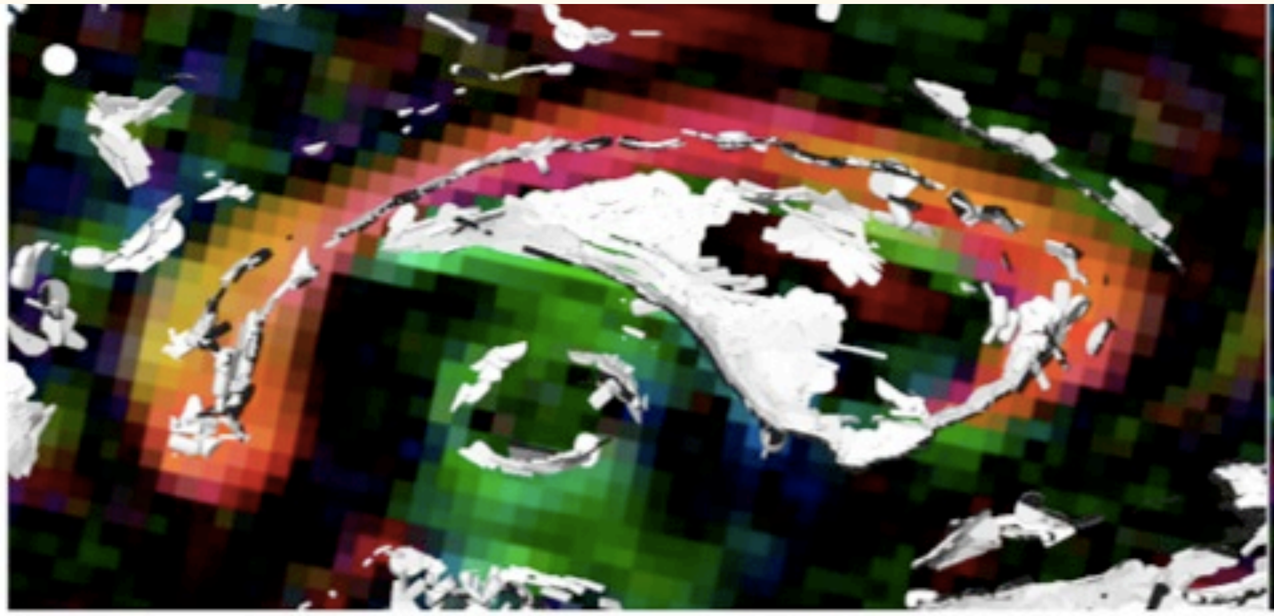
Smallest airways not much larger than voxels
(benefit of working on continuous domain)

Brain DTI Results

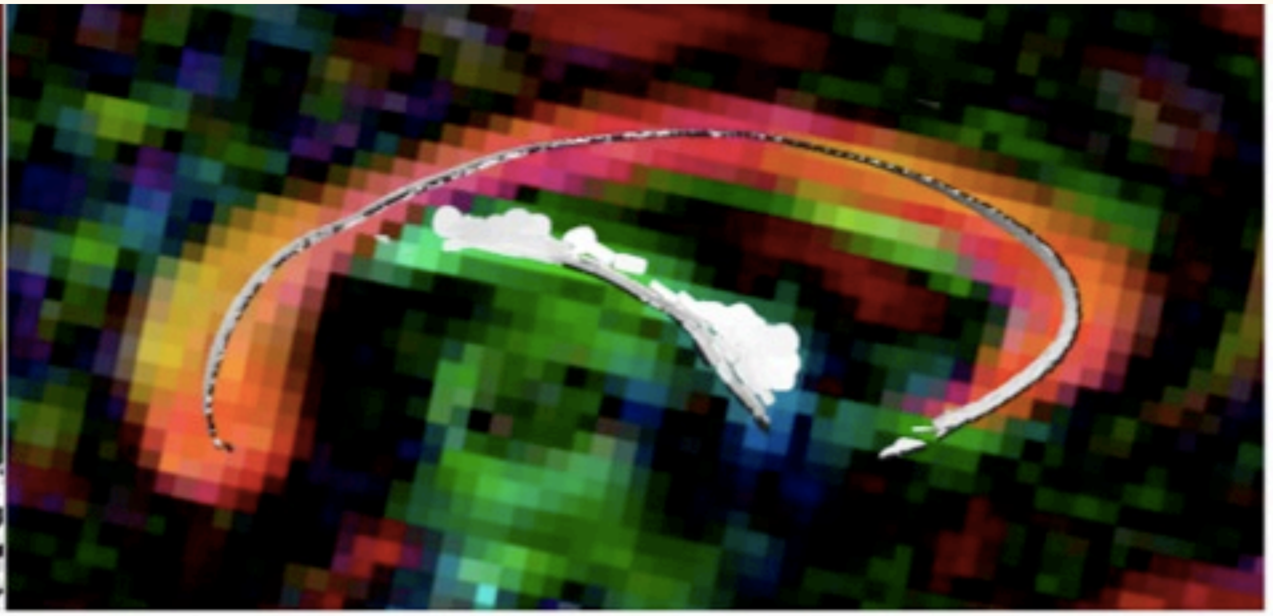
Fractional Anisotropy (FA) ridge surfaces



Brain DTI Results

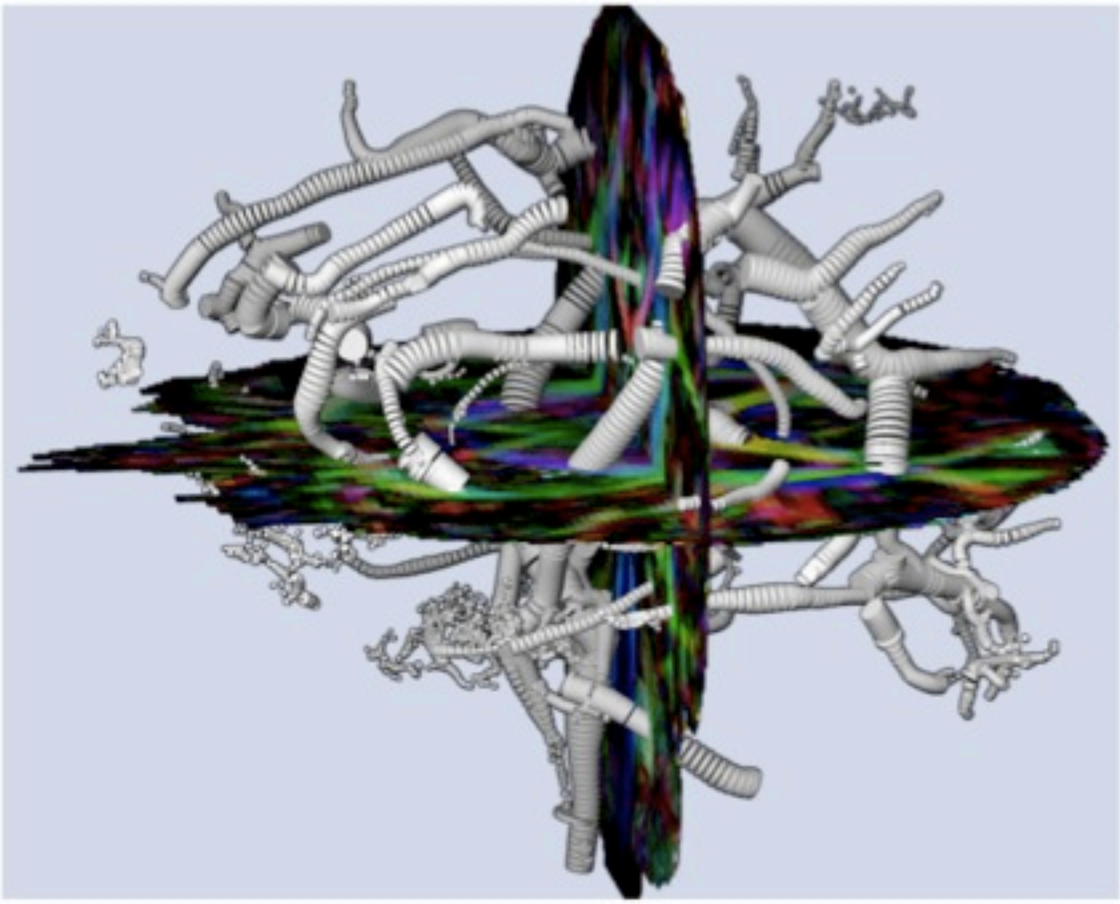


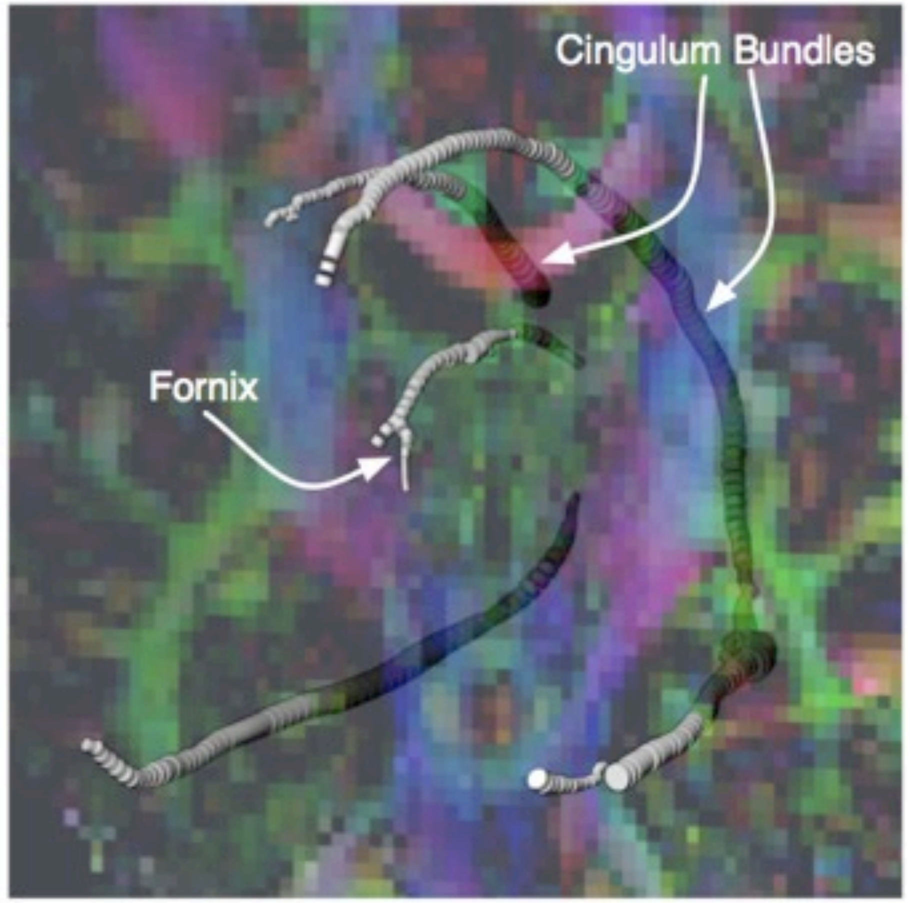
Without Scale-Space



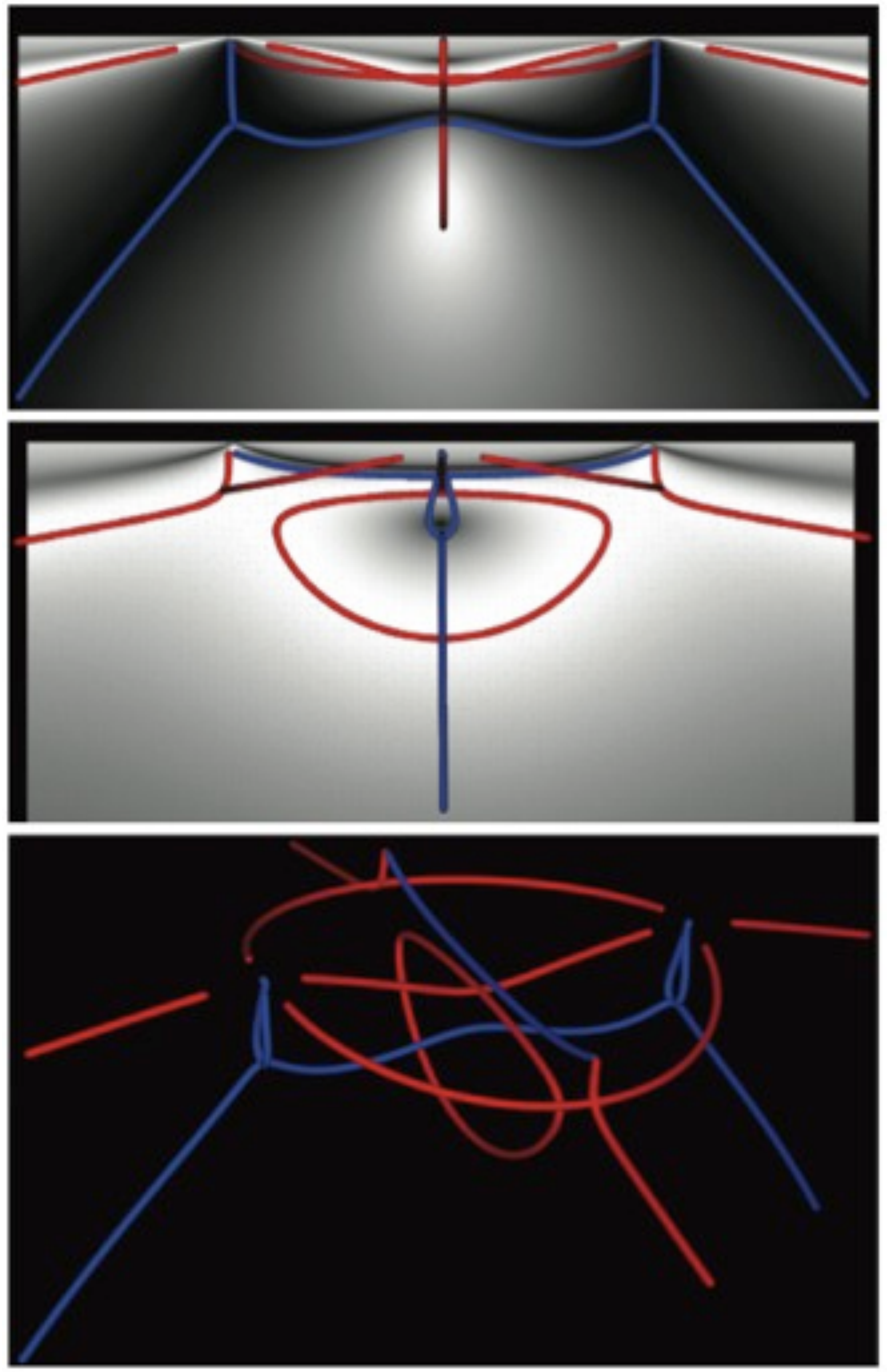
With Scale-Space

FA ridge lines

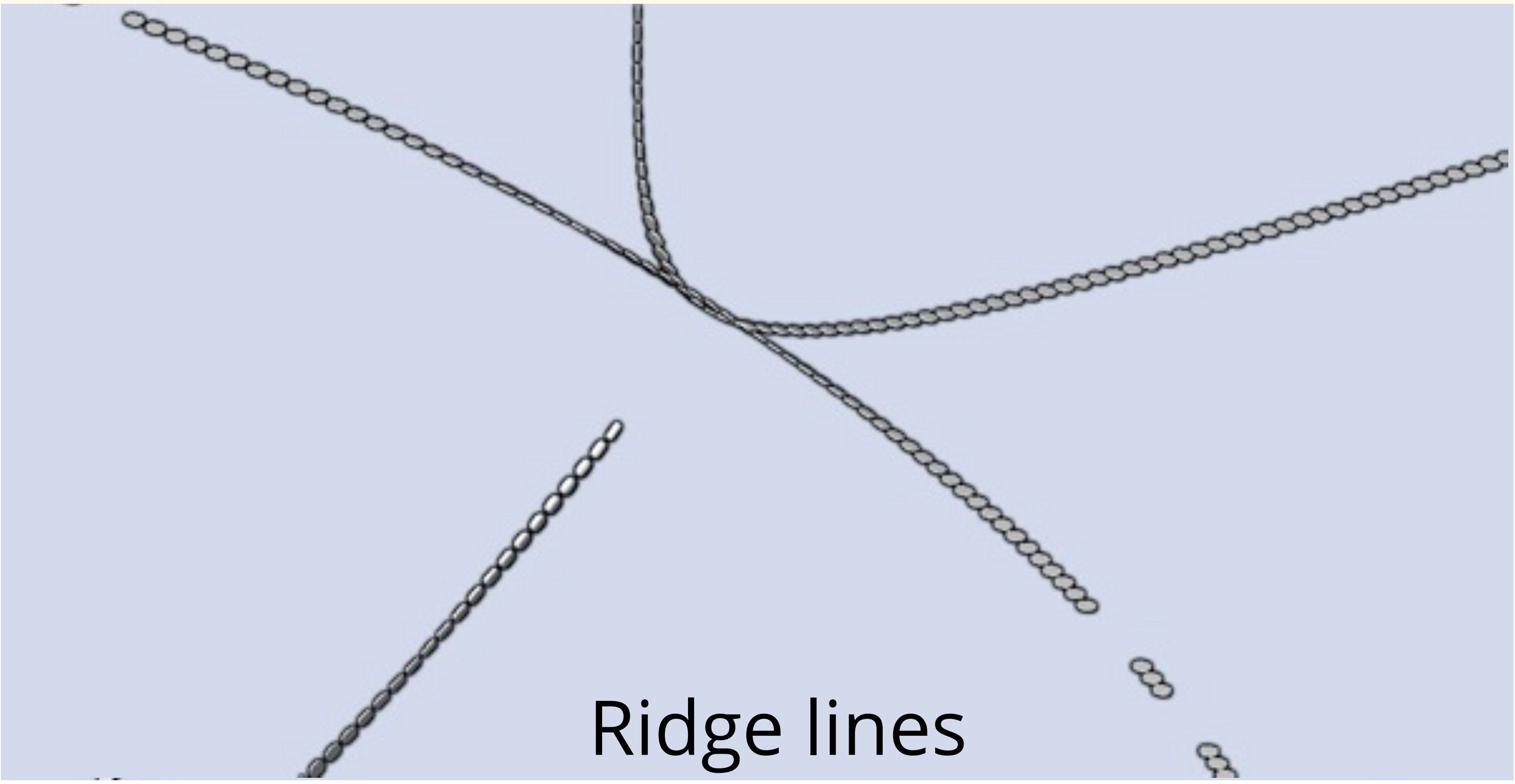


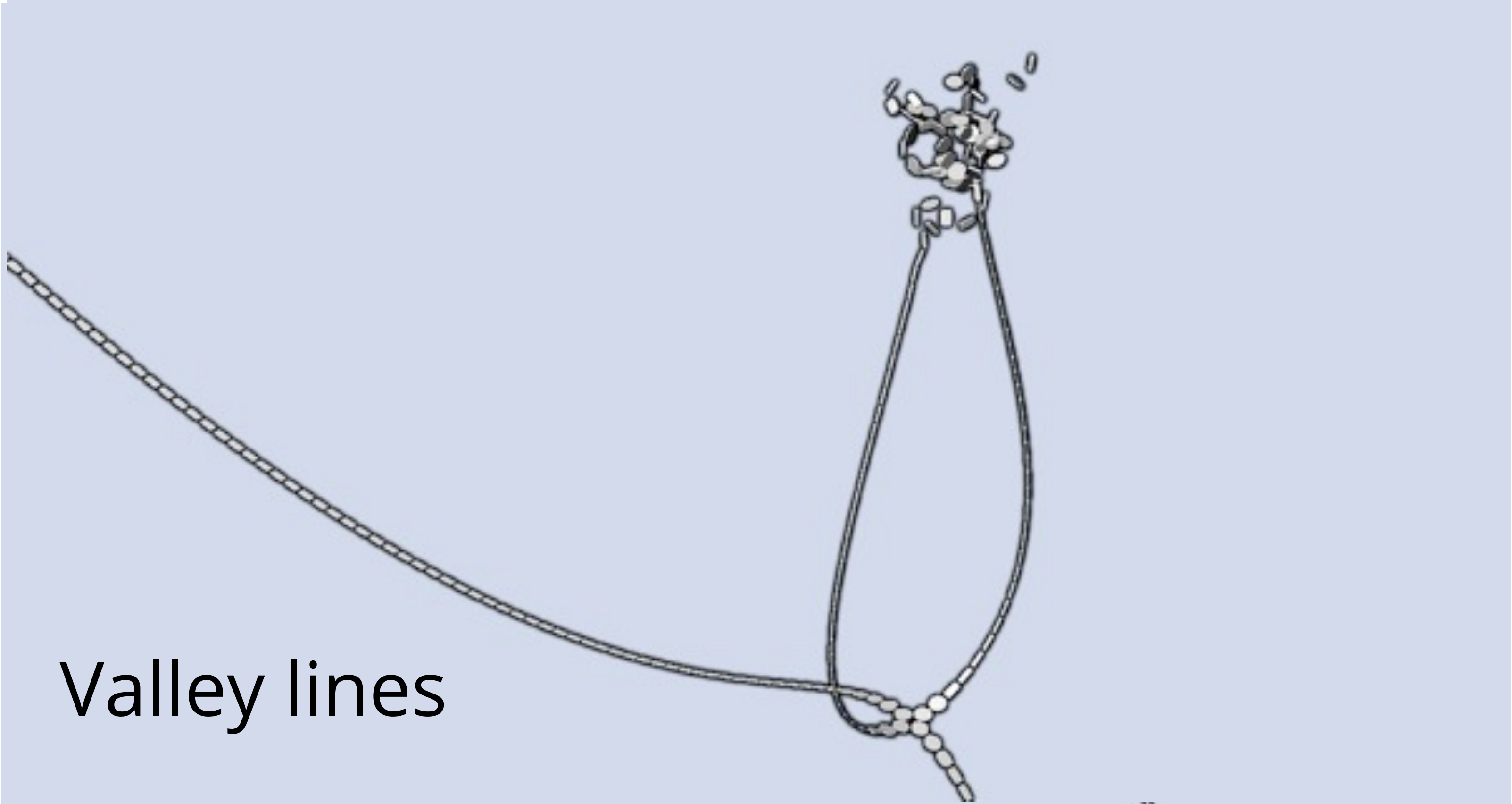


Double Point Load Stress Tensor Field

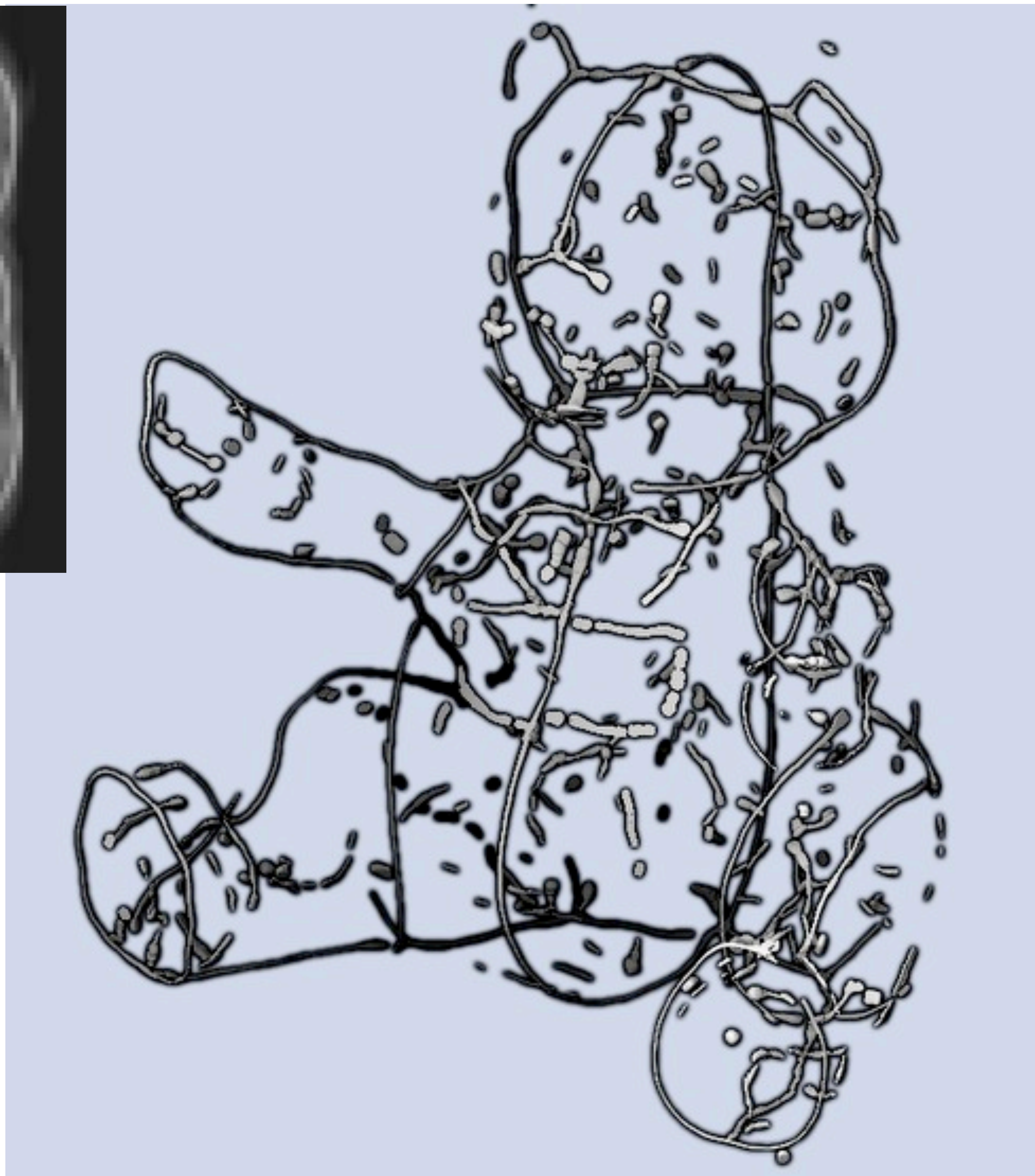


“Invariant Crease Lines for Topological and Structural Analysis of Tensor Fields” [Tricoche-VIS-2008]





Teddy Bear



Outline

Introduction: motivation, example, contributions

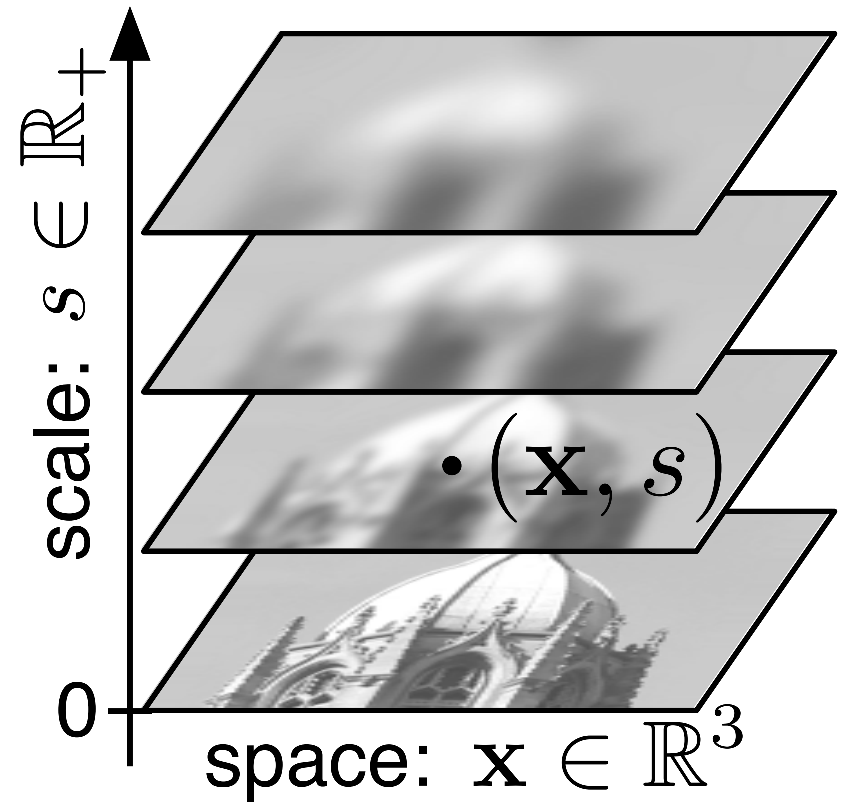
Method: interpolation, features, energy, visualization, computation

Results: lung CT, brain DTI, more

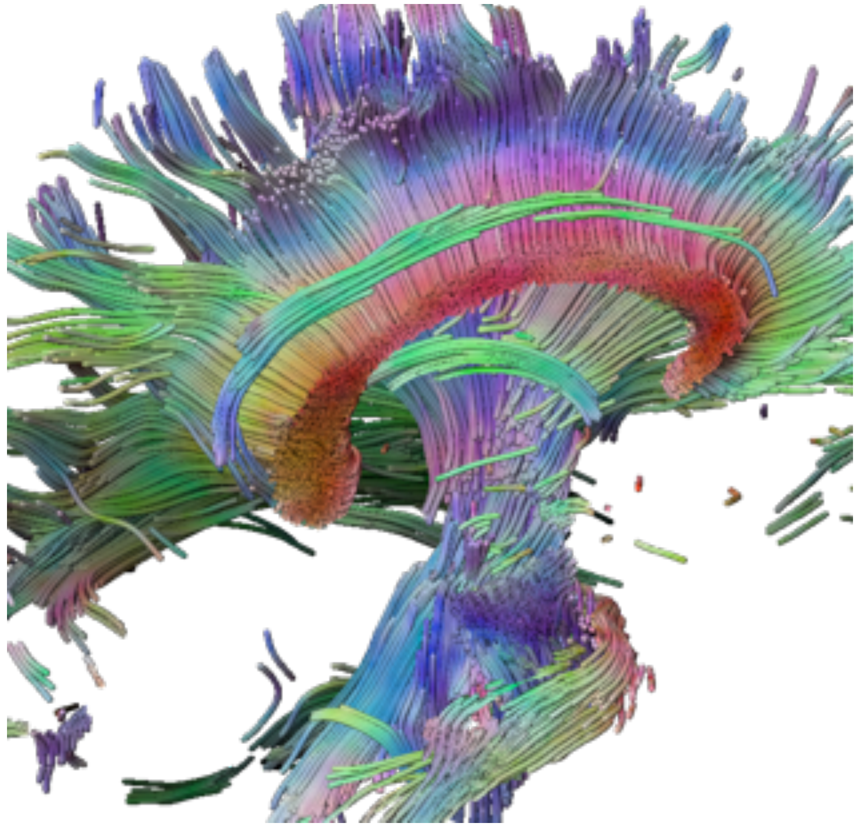
Discussion: scale, particles, analysis, future

Why not more Scale Space in Vis?

What are relative strengths, weaknesses of scale-space (with strength measures) versus topological simplification (with persistence measure)



Can always ask: but at what scale, and how stable with respect to scale?

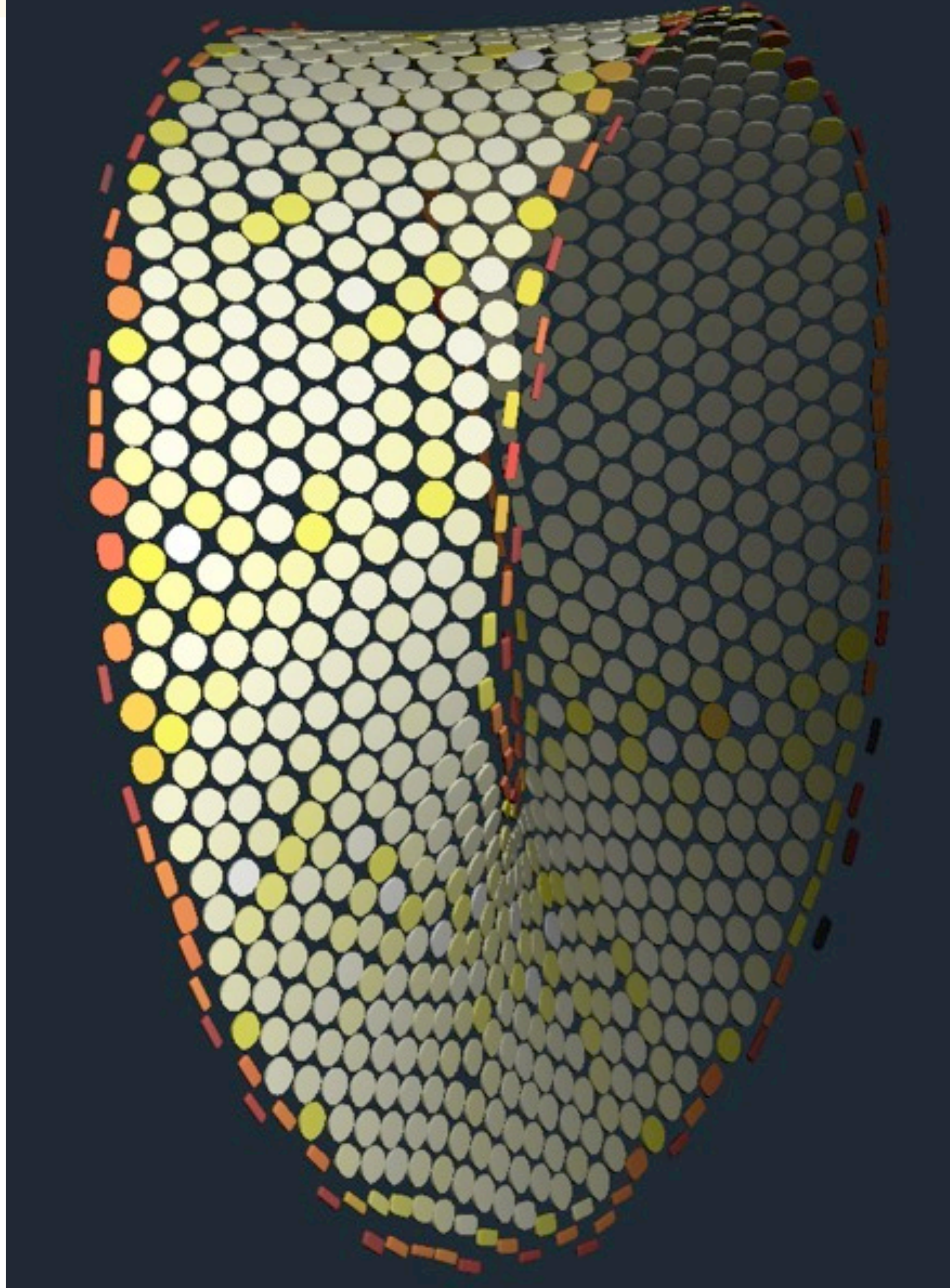


Joy of Particles

Map from crease definition to particle motion

Implement at level of individual particle behavior, watch group evolve

Same system for curves or surfaces, with or without scale space



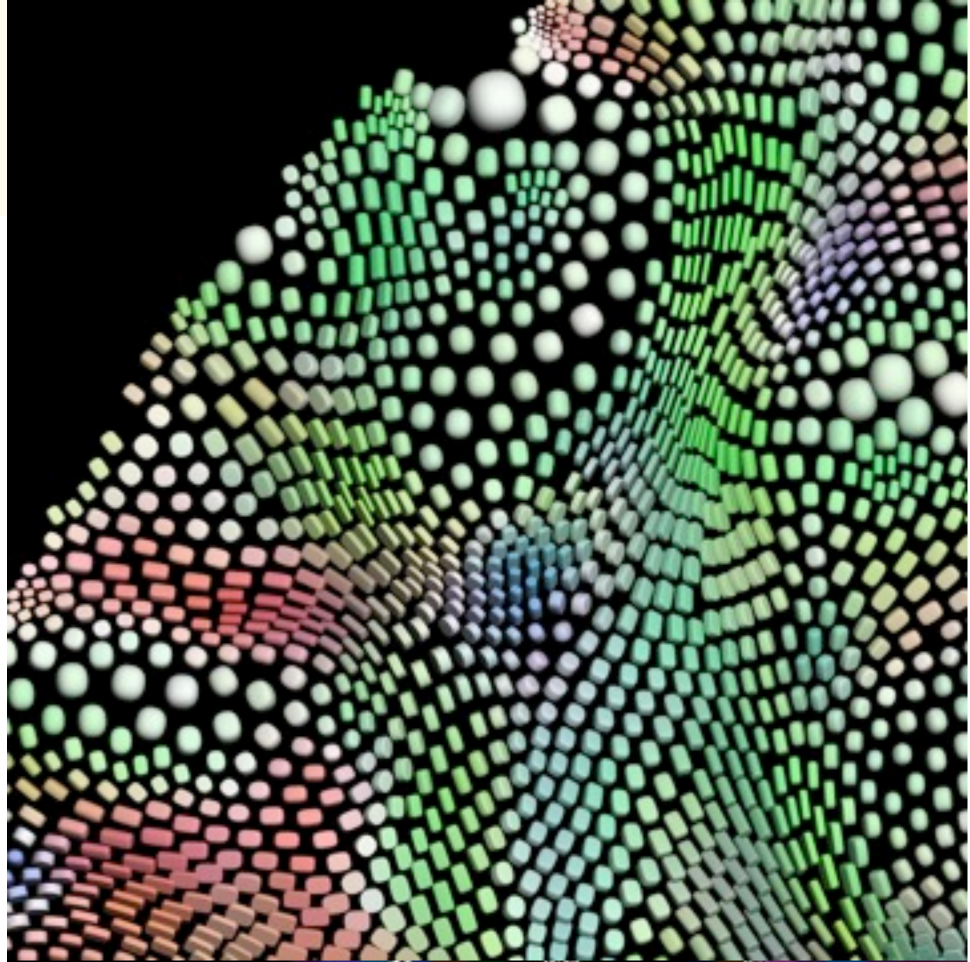
Visualization or Analysis?

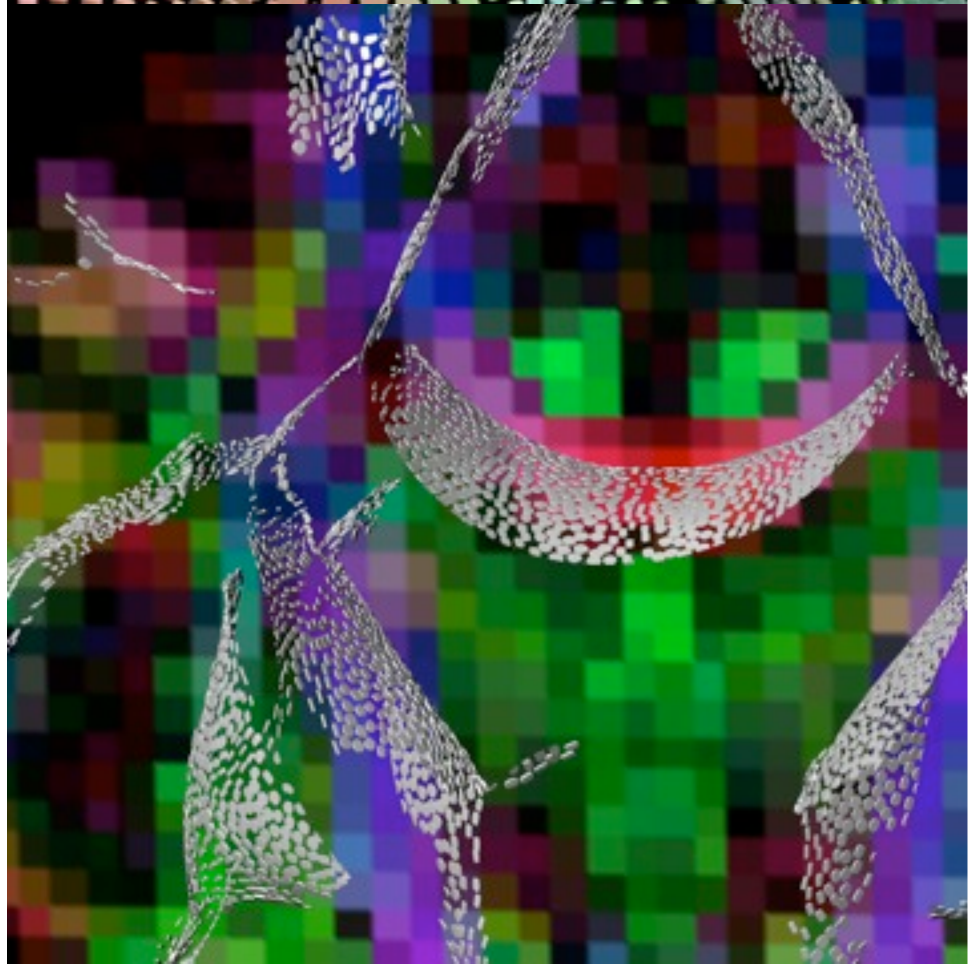
Intersection of both

Vis, by **methods** (particles, glyphs) and **modality** (CT, DT-MRI)

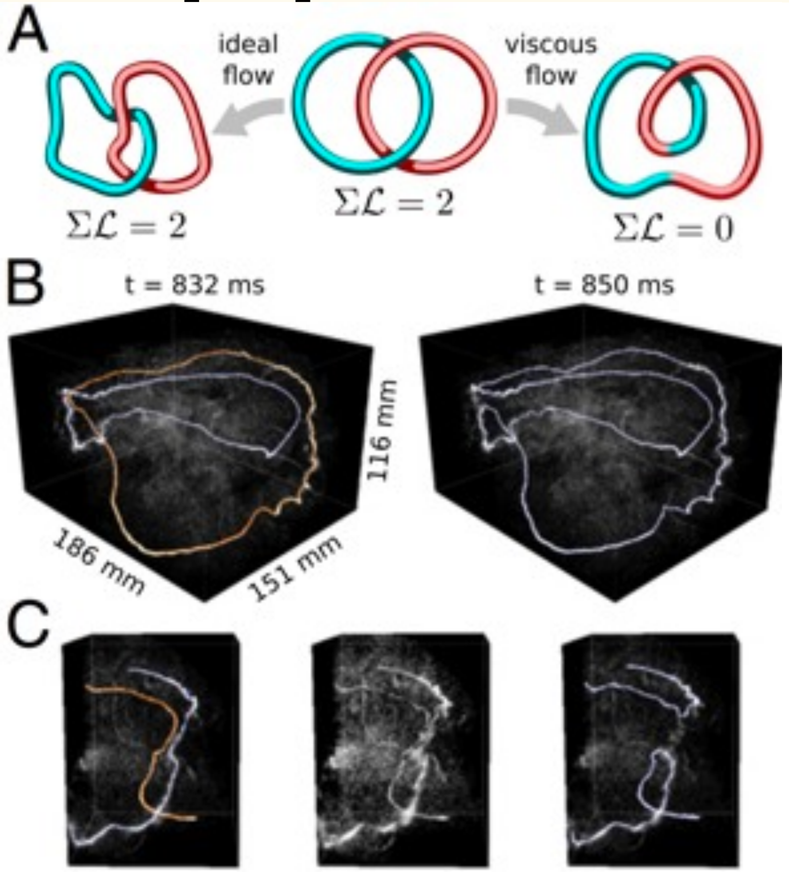
Vis & Analysis, by **strategy**: inspecting computation, feature detection, optimizing sampling

Analysis, by **goals**: quantitative studies of local properties, shape variation, registration



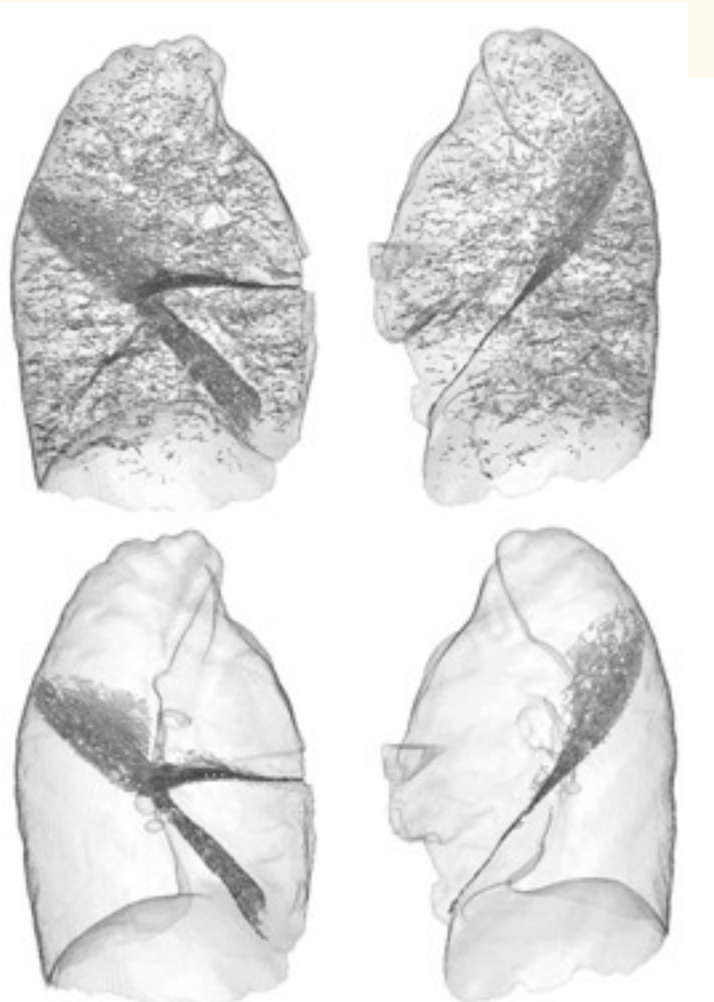


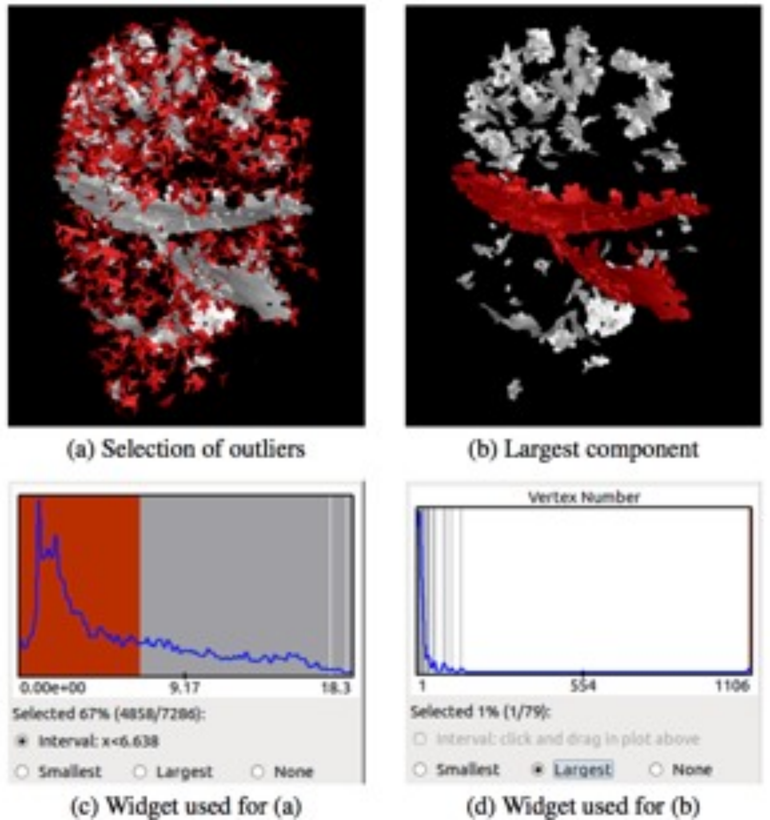
Applications since 2009



Helicity conservation by flow across scales in reconnecting vortex links and knots. MW Scheeler, D Kleckner, D Proment, GL Kindlmann, and WTM Irvine. *Proceedings of the National Academy of Sciences*, 111(43):15350–15355, October 2014.

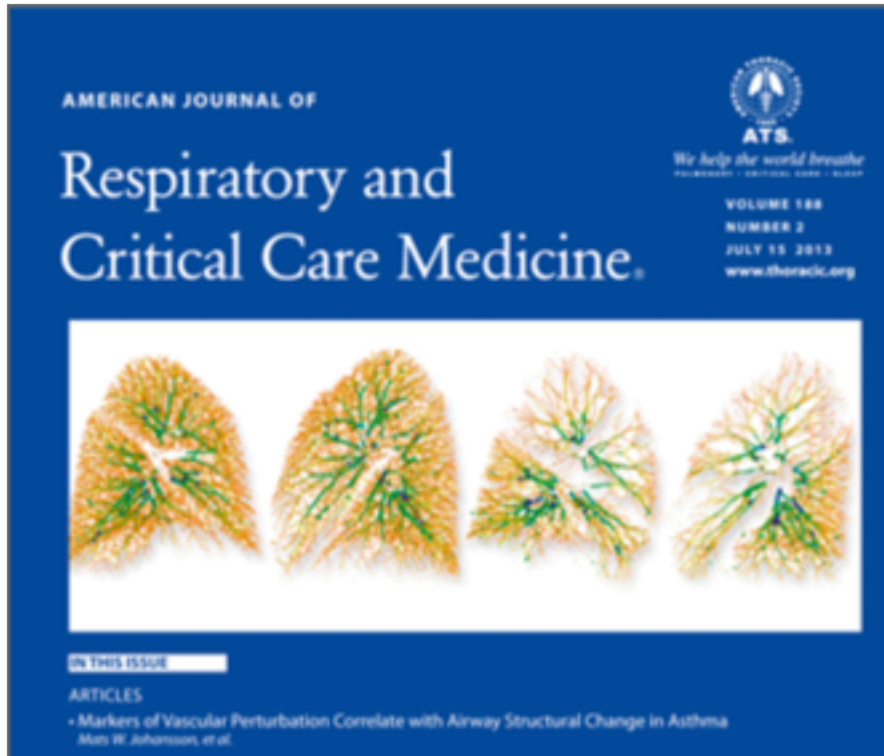
Pulmonary lobe segmentation based on ridge surface sampling and shape model fitting. JC Ross, GL Kindlmann, Y Okajima, H Hatabu, AA Díaz, EK Silverman, GR Washko, J Dy, and R San José Estépar. *Medical Physics*, 40(12):121903, December 2013





Open-Box Spectral Clustering: Applications to Medical Image Analysis. T Schultz and G Kindlmann. *IEEE Transactions on Visualization and Computer Graphics (Proceedings of VIS 2013)*, 19(12):2100–2108, December 2013.

Computed Tomographic Measures of Pulmonary Vascular Morphology in Smokers and Their Clinical Implications. R San José Estépar, GL Kinney, JL Black-Shinn, RP Bowler, GL Kindlmann, JC Ross, R Kikinis, MK Han, CE Come, AA Díaz, MH Cho, CP Hersh, JD Schroeder, JJ Reilly, DA Lynch, JD Crapo, JM Wells, MT Dransfield, JE Hokanson, GR Washko, and COPDGene Study. *American Journal of Respiratory and Critical Care Medicine*, 188(2):231—239, July 2013.



Future work

Other kinds of features: Canny edges

(should be possible with future Diderot!)

Varying sampling density (curvature, local size)

More parameter automation (did scale)

(maybe with Tuner?)

Meshing, where needed

GPU-based computation (also with future Diderot)

References

[Eberly-JMIV-1994] **Ridges for Image Analysis.** D. Eberly and R. Gardner and B. Morse and S. Pizer. *Journal of Mathematical Imaging and Vision* 4 353--373 (1994)

[Eberly-1996] **Ridges in Image and Data Analysis.** D. Eberly. Kluwer Academic Publishers. 1996

[Jalba-PAMI-2004] **CPM: A Deformable Model for Shape Recovery and Segmentation Based on Charged Particles.** A. C. Jalba and M. H. F. Wilkinson and J. B. T. M. Roerdink. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 26 1320--1335 (2004)

[Kindlmann-VisSym-2004] **Superquadric Tensor Glyphs**. G Kindlmann. *In Proceedings of the Joint Eurographics - IEEE TCVG Symposium on Visualization ("VisSym")*, pages 147–154, May 2004.

[Kindlmann-VIS-2006] **Diffusion Tensor Visualization with Glyph Packing**. G Kindlmann and C-F Westin. *IEEE Transactions on Visualization and Computer Graphics (Proceedings IEEE Visualization Conference and IEEE Symposium on Information Visualization),* 12(5):1329–1335, September–October 2006.

[Kindlmann-VIS-2009] **Sampling and Visualizing Creases with Scale-Space Particles**. GL Kindlmann, R San José Estépar, SM Smith, and C-F Westin. *IEEE Transactions on Visualization and Computer Graphics (Proceedings IEEE VisWeek),* 15(6):1415–1424, November–December 2009.

[Lindeberg-PAMI-1990] **Scale-Space for Discrete Signals**. T Lindeberg. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 12 234--254 (1990)

[Lindeberg-IJCV-1998] **Edge detection and ridge detection with automatic scale selection**. T Lindeberg. *International Journal of Computer Vision* 30 117--154 (1998)

[Möller-TVCG-1997] **Evaluation and Design of Filters Using a Taylor Series Expansion**. T. Möller and R. Machiraju and K. Mueller and R. Yagel. **IEEE Trans. on Visualization and Computer Graphics** 3 184--199 (1997)

[Smith-TBS-2006] **Tract-based spatial statistics: Voxelwise analysis of multi-subject diffusion data.** SM Smith, M Jenkinson, H Johansen-Berg, D Rueckert, TE Nichols, CE Mackay, KE Watkins, O Ciccarelli, MZ Cader, PM Matthews, TEJ Behrens. *NeuroImage* 31 1487--1505 (2006)

[Tricoche-VIS-2008] **Invariant Crease Lines for Topological and Structural Analysis of Tensor Fields**. X. Tricoche and G. Kindlmann and C.-F. Westin. *IEEE Trans. on Visualization and Computer Graphics* 14 1627--1634 (2008)

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Lung CT courtesy of Dr. George Washko, Brigham And Women's Hospital, Harvard Medical School

Brain DTI courtesy of Dr. Kyle Pattinson, FMRIB Centre, Oxford University

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Code online: http://people.cs.uchicago.edu/~glk/ssp/ (part of Teem) but unfortunately no GUI

Maybe in Hale: https://github.com/kindlmann/hale

Future grant may fund scale-space for Diderot

Thanks for questions, feedback! **GLK@uchicago.edu**