Strategies for Visualization and Processing of Diffusion Tensor MRI Volumes

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Rough Outline

Glyphs for data inspection

Invariants for describing shape

Invariant gradients

- WRT position: Shading in rendering
- WRT tensor components: DOF of shape

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- WRT position: Shading in rendering
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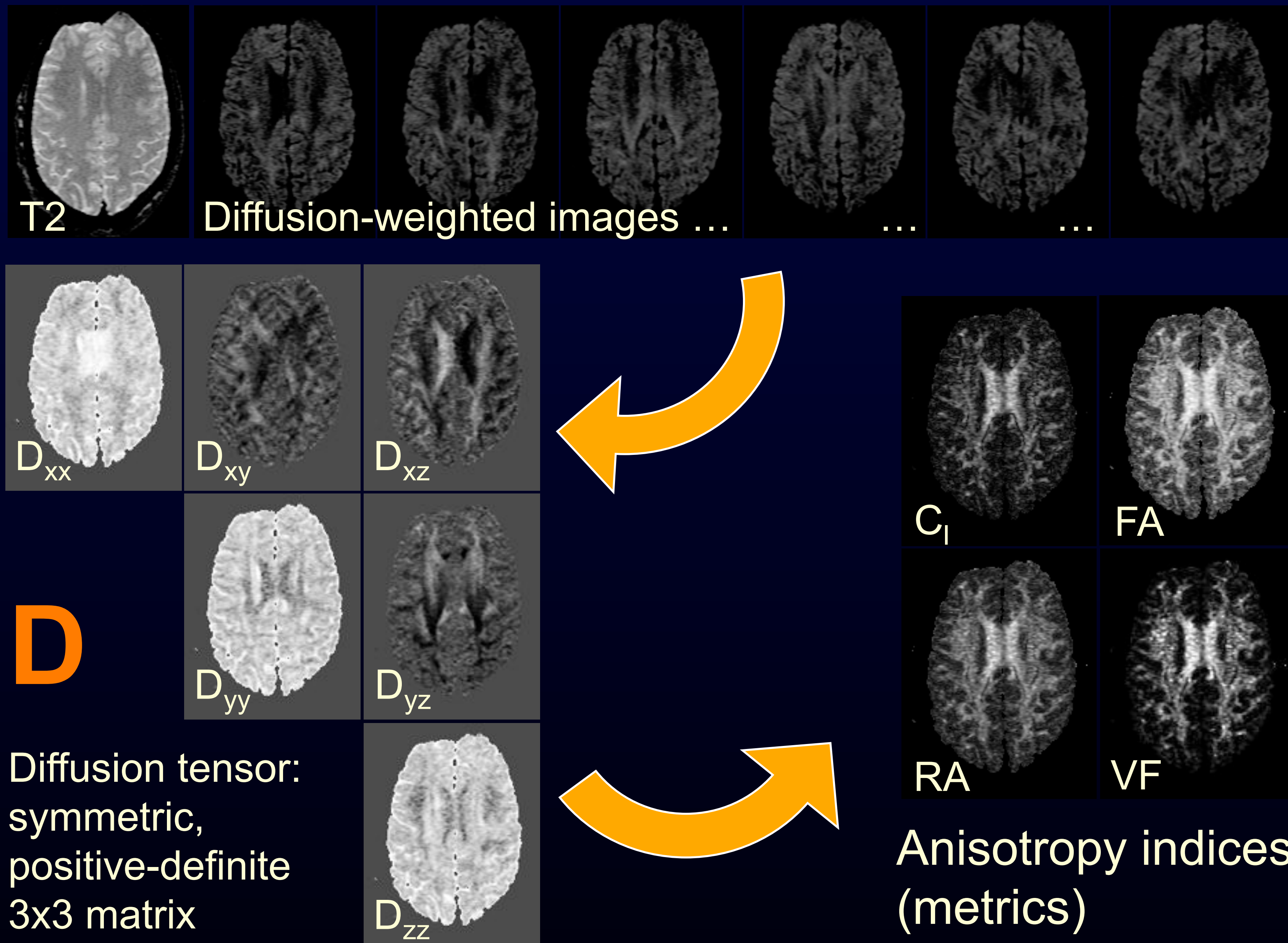
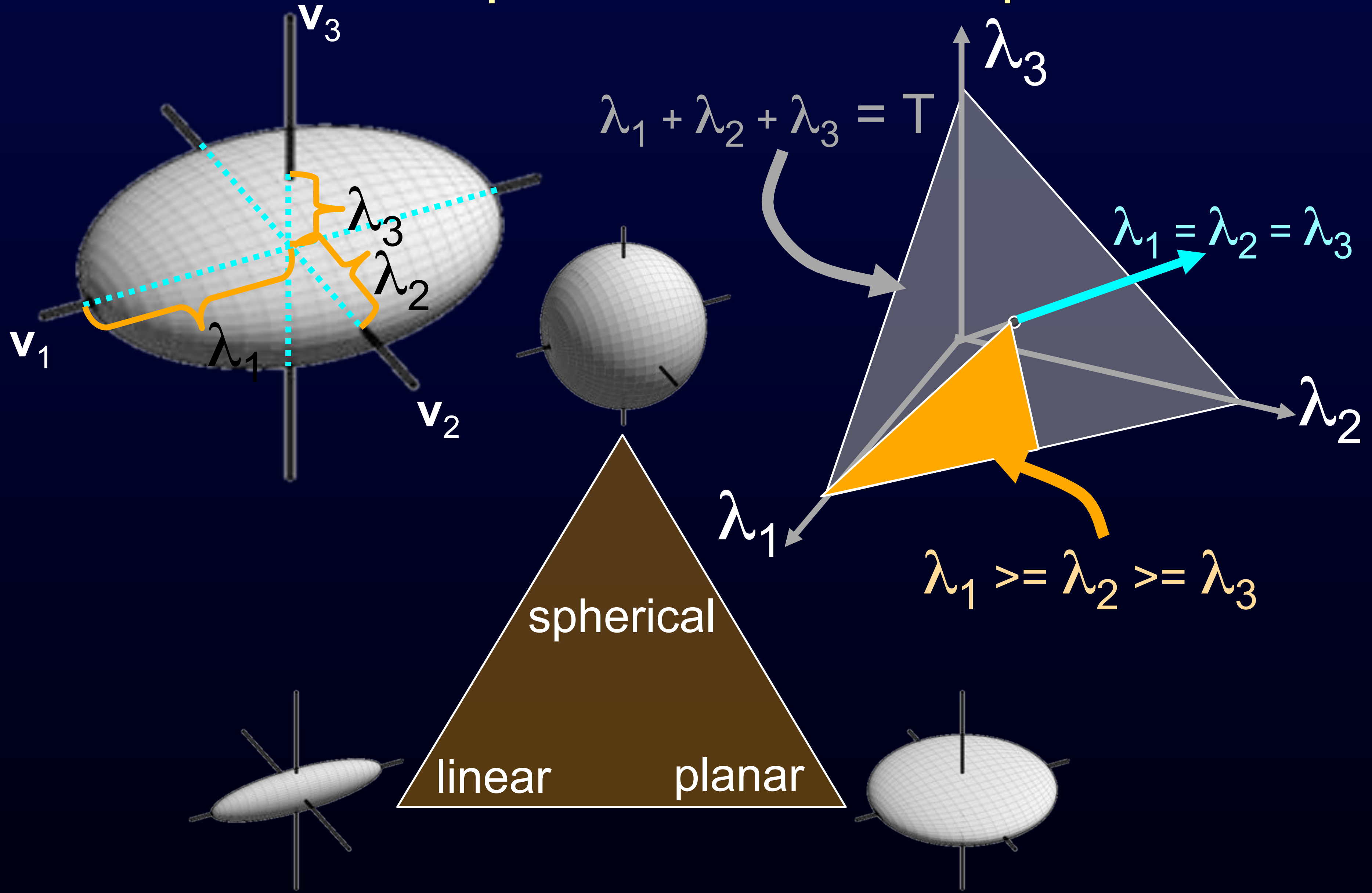
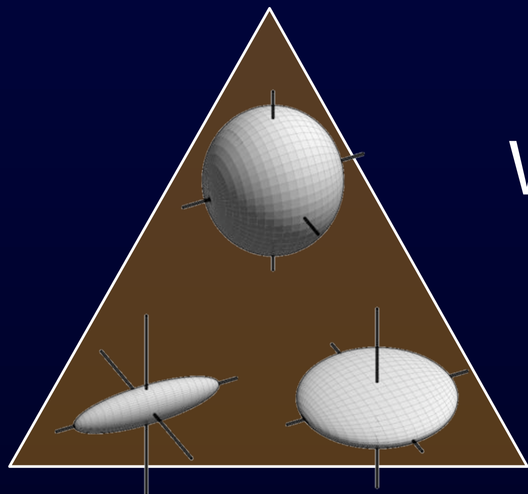
Diffusion-Tensor MRI



Space of tensor shape

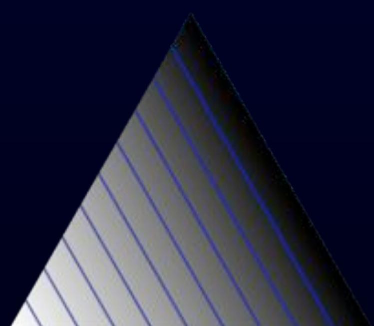


Scalar shape metrics

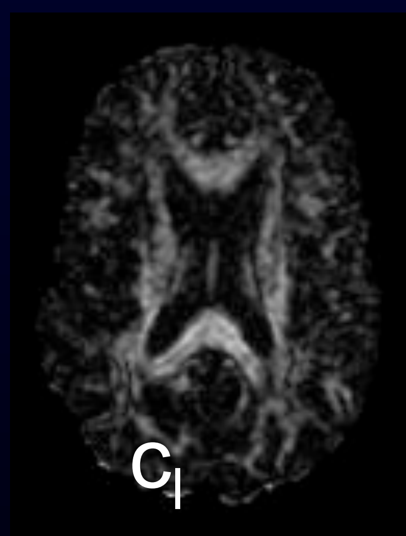


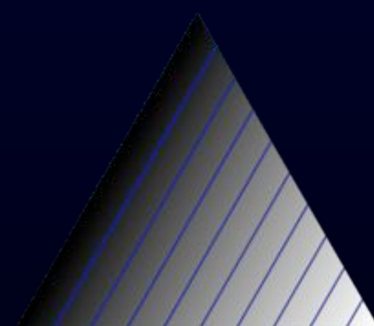
Westin, 1997

c_s

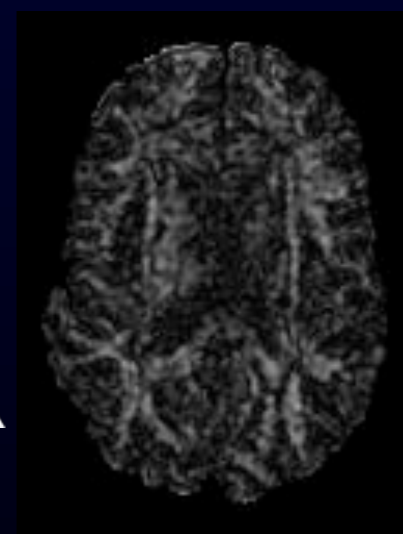


c_l

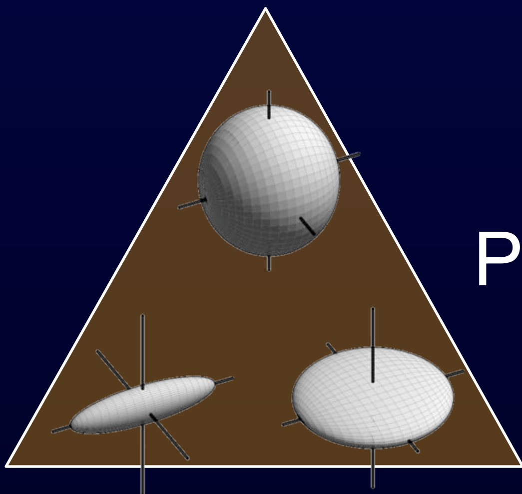




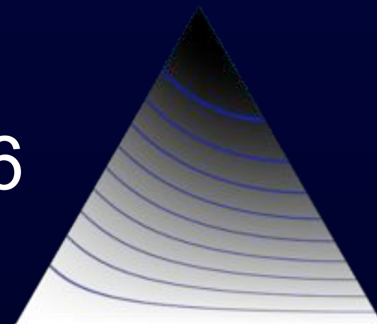
c_p

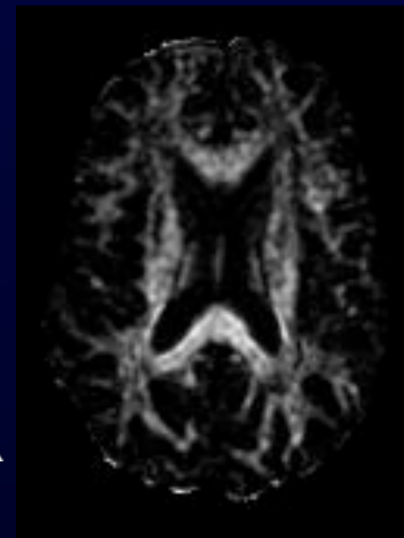


Scalar shape metrics



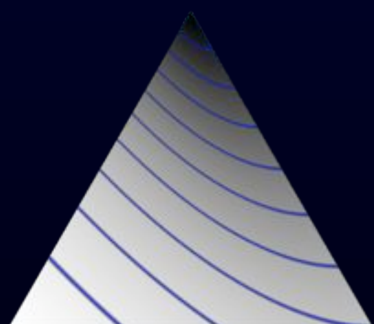
Basser + Pierpaoli, 1996

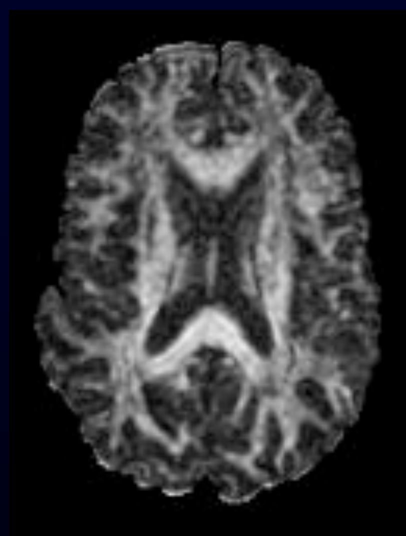




1 - VR

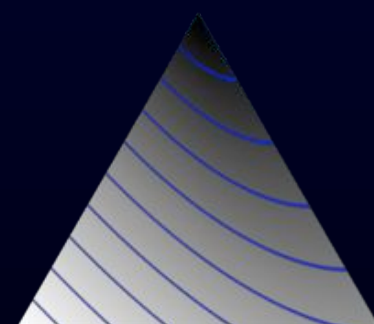
1 - volume ratio

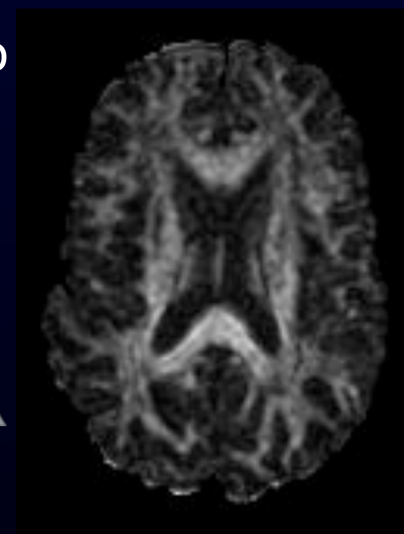




FA

fractional anisotropy

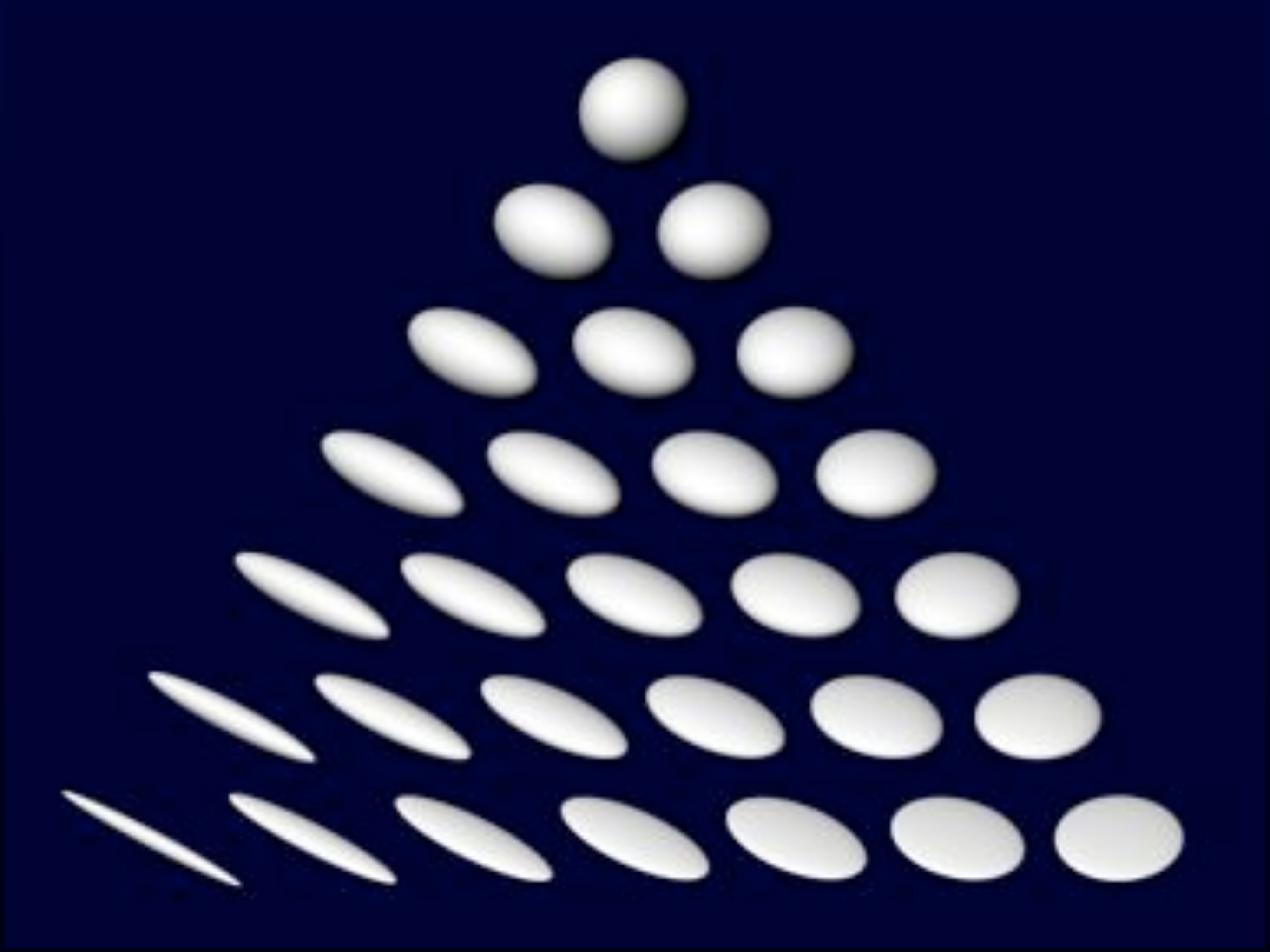




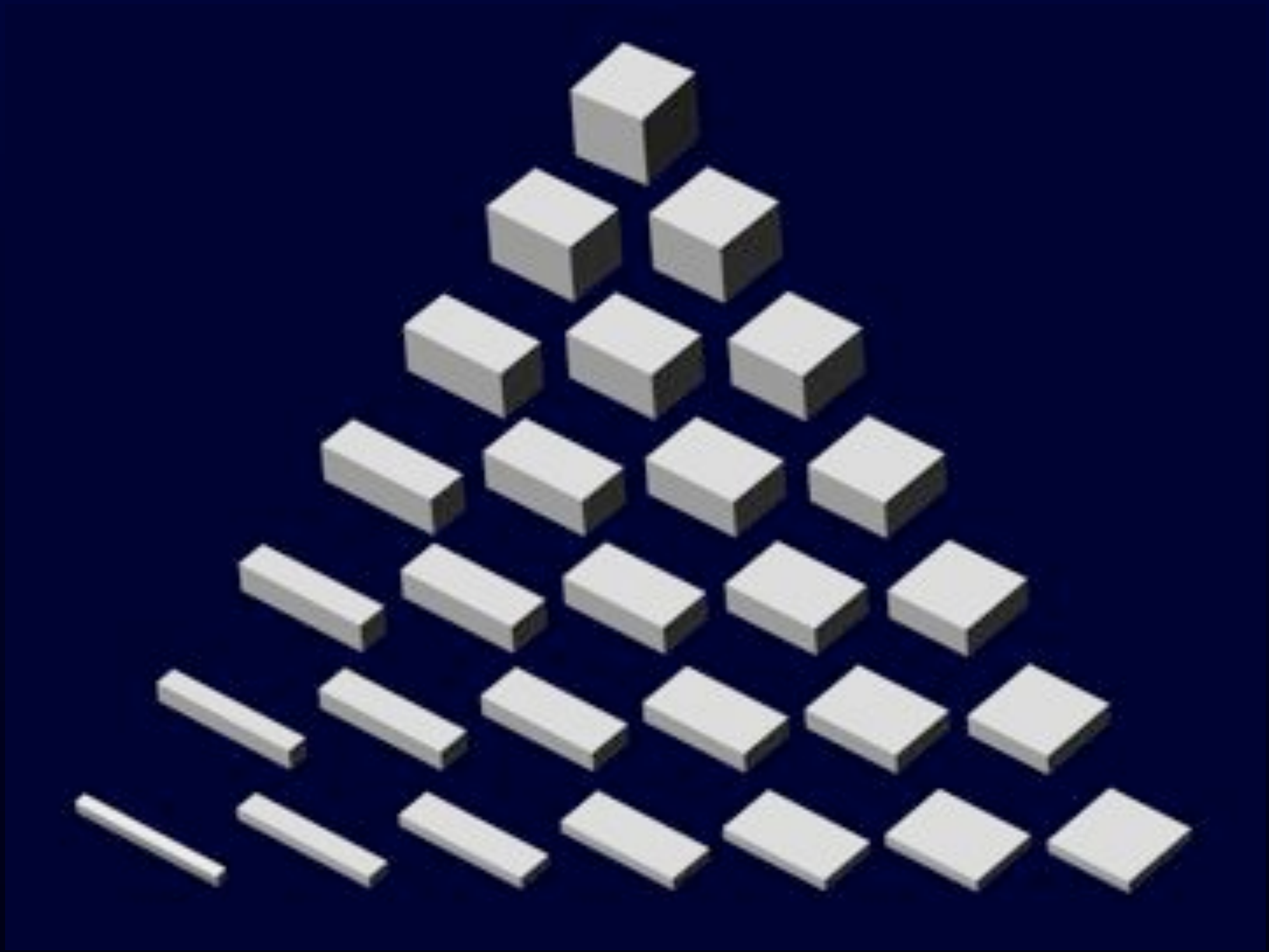
RA

relative anisotropy

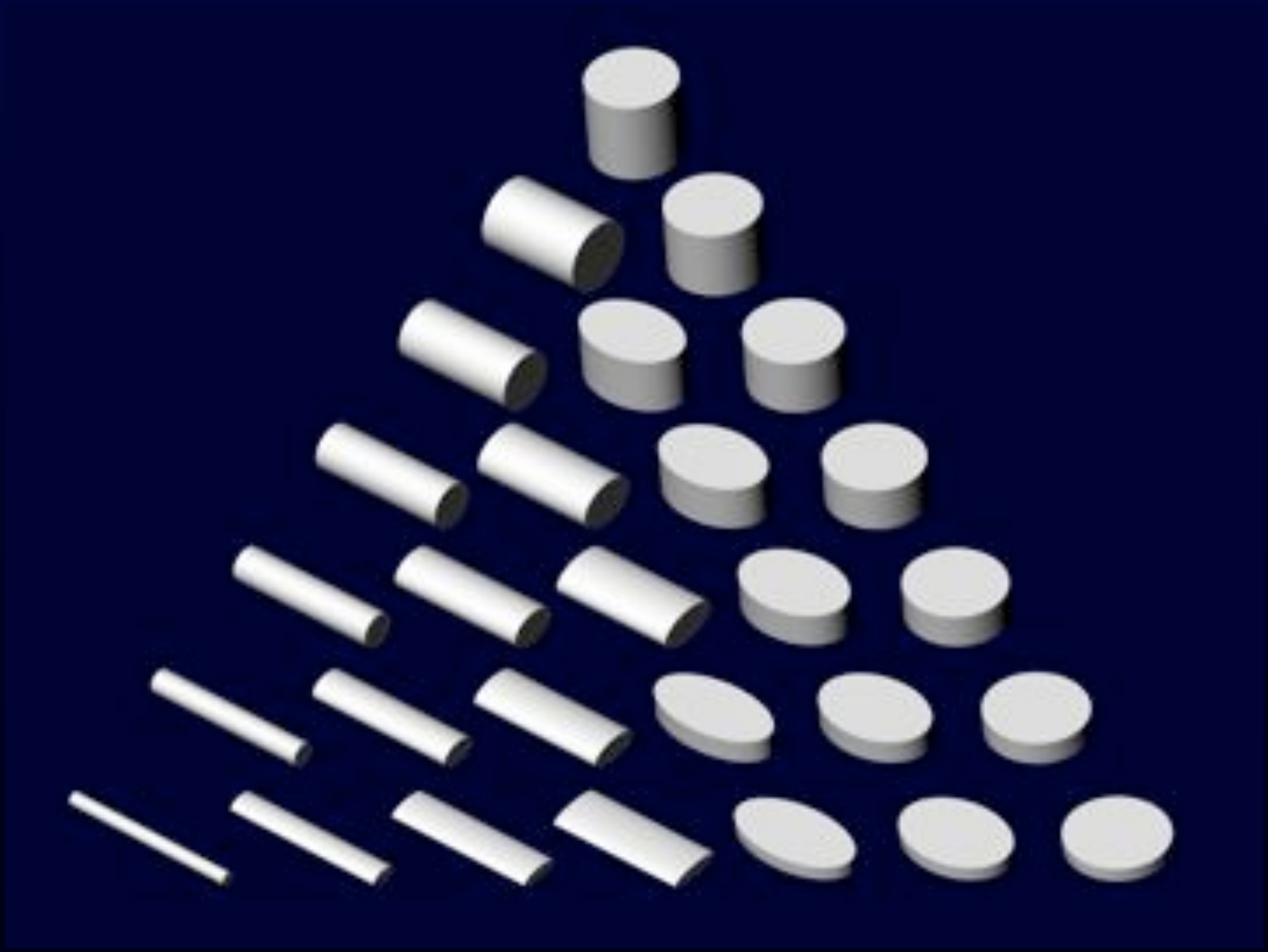
Glyph shapes



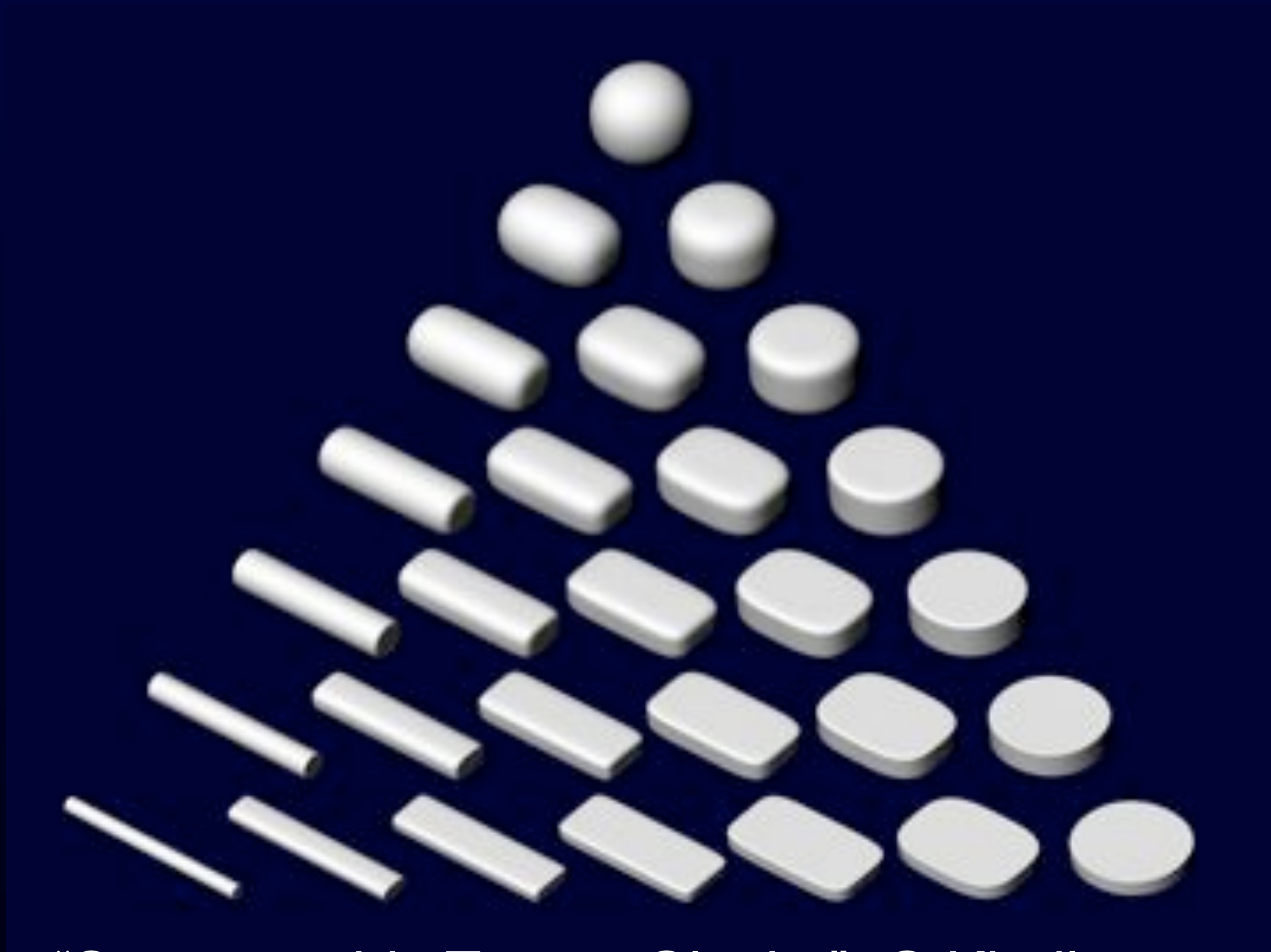
Glyph shapes



Glyph shapes

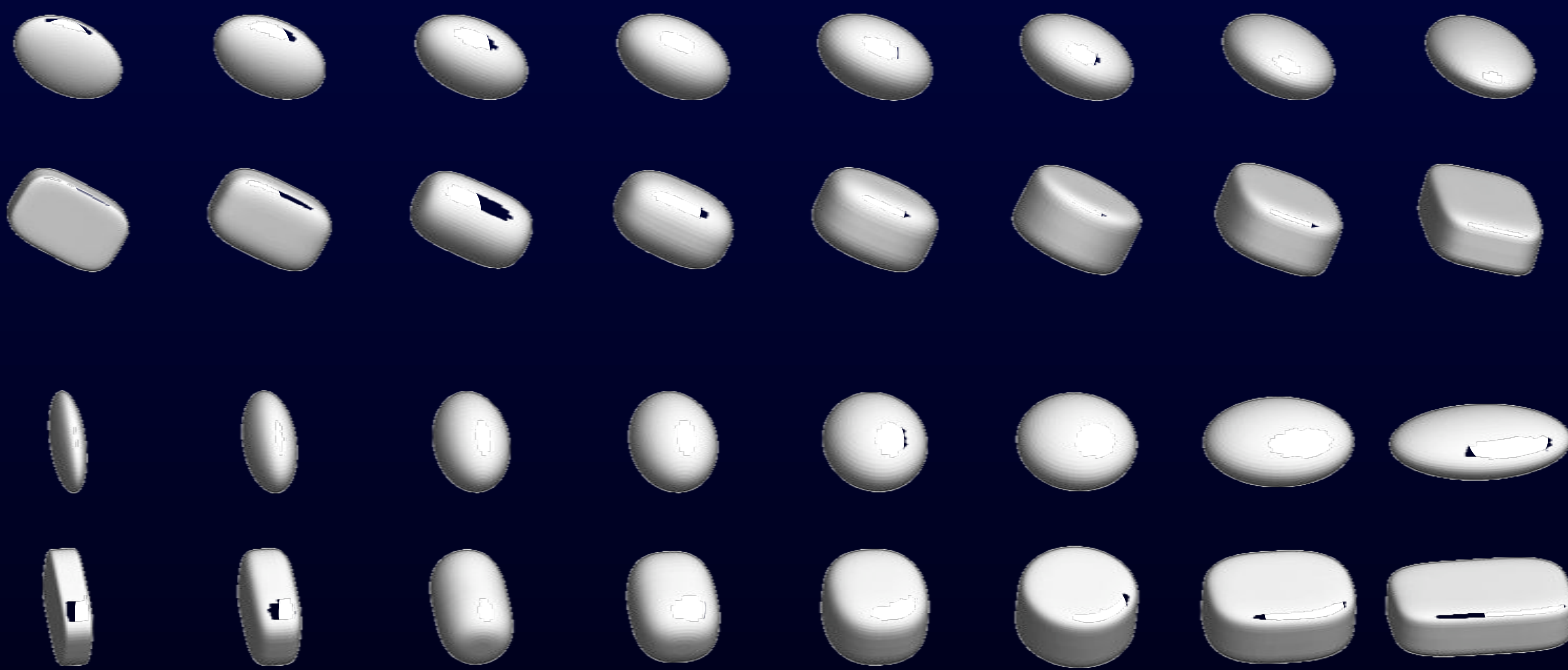


Glyph shapes



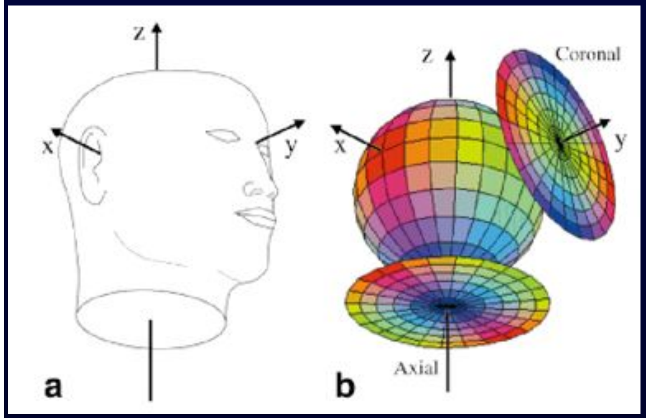
“Superquadric Tensor Glyphs”, G Kindlmann, Proceedings IEEE/TVCG VisSym 2004
http://www.cs.utah.edu/~gk/papers/vissym04

Worst case scenario

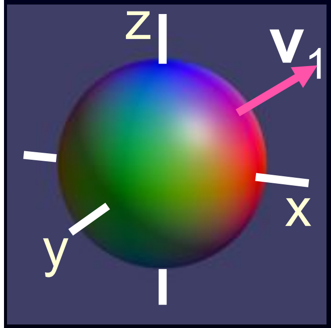


"Superquadric Tensor Glyphs", G Kindlmann, Proceedings IEEE/TVCG VisSym 2004
http://www.cs.utah.edu/~gk/papers/vissym04

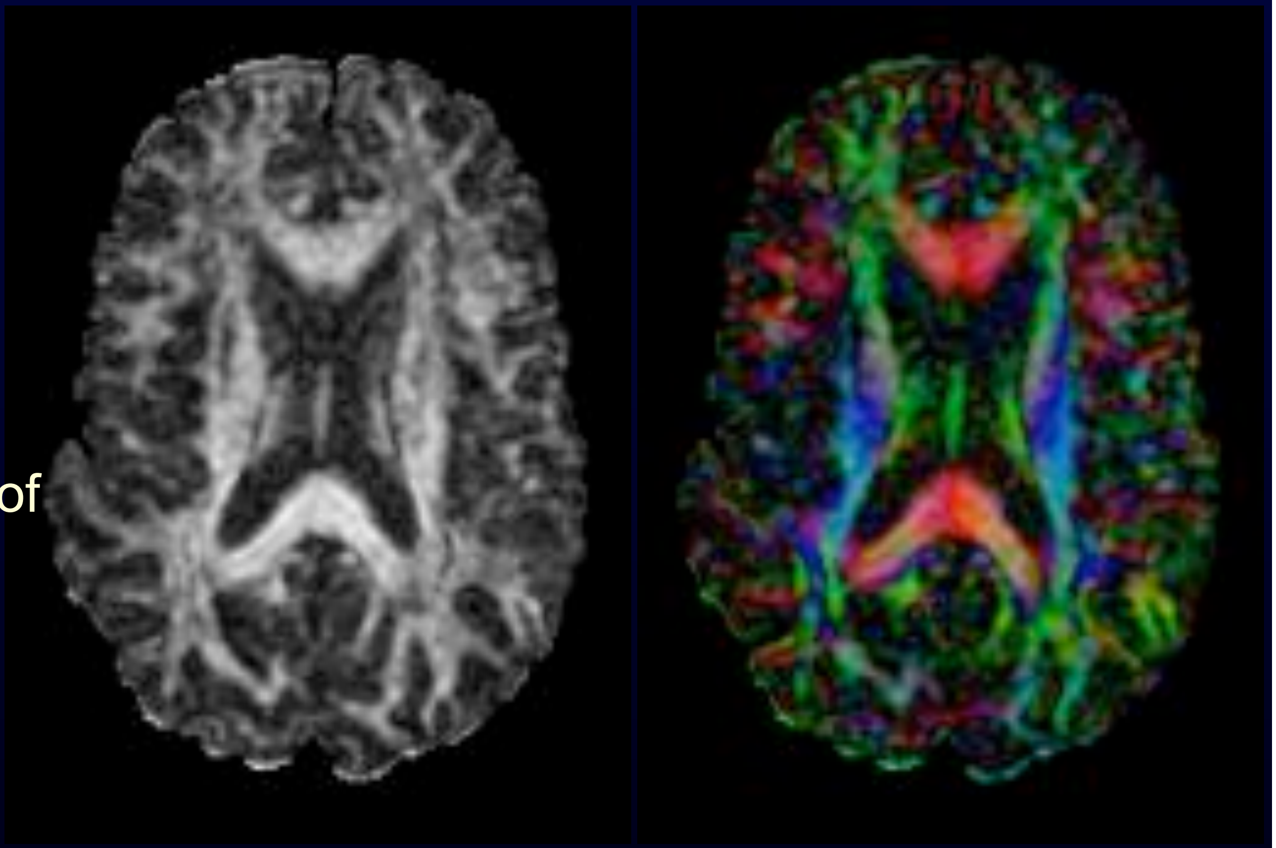
Colored by dominant diffusion direction



"Color Schemes to Represent the Orientation of Anisotropic Tissues …" Pajevic and Pierpaoli, MRM 42:526-540 1999

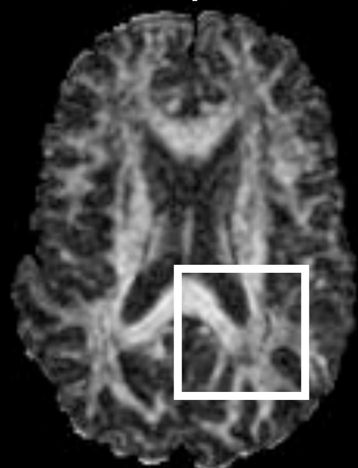


R=| $\mathbf{v}_1$.x |
G=| $\mathbf{v}_1$.y |
B=| $\mathbf{v}_1$.z |

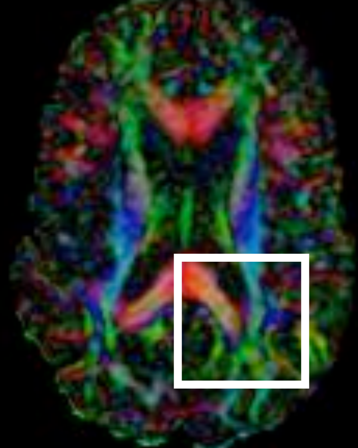


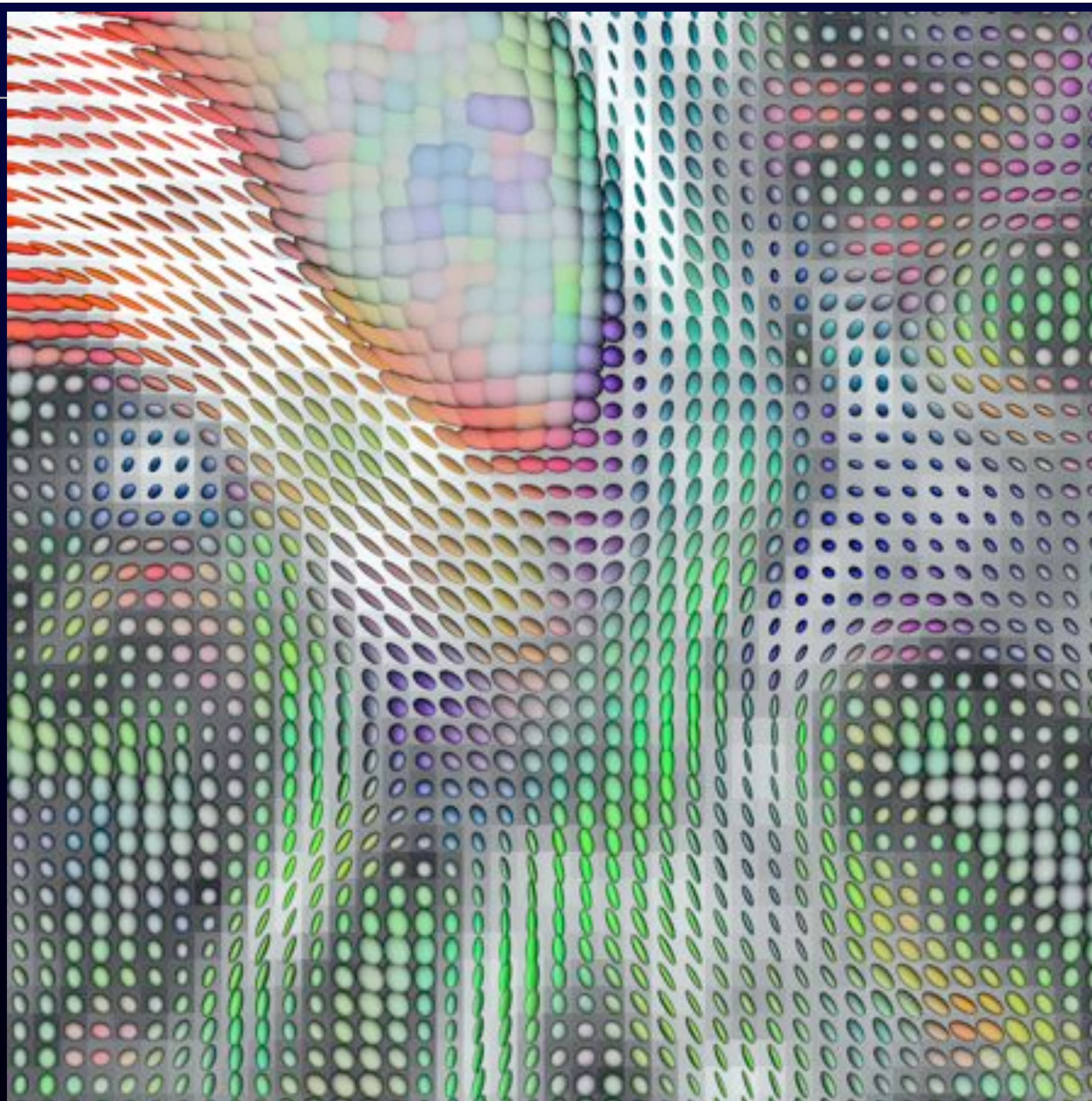
Fractional anisotropy (FA)   RGB(1[st] eigenvector $\mathbf{v}_1$)

Backdrop: FA

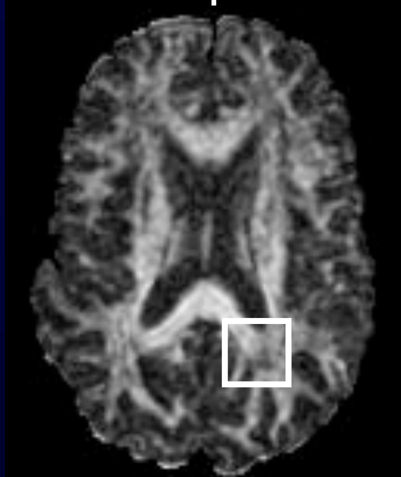


Color: RGB($\mathbf{v}_1$)

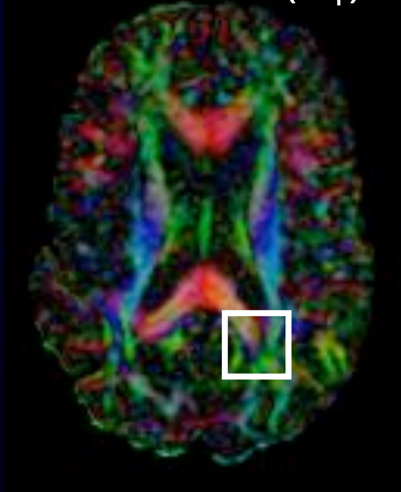


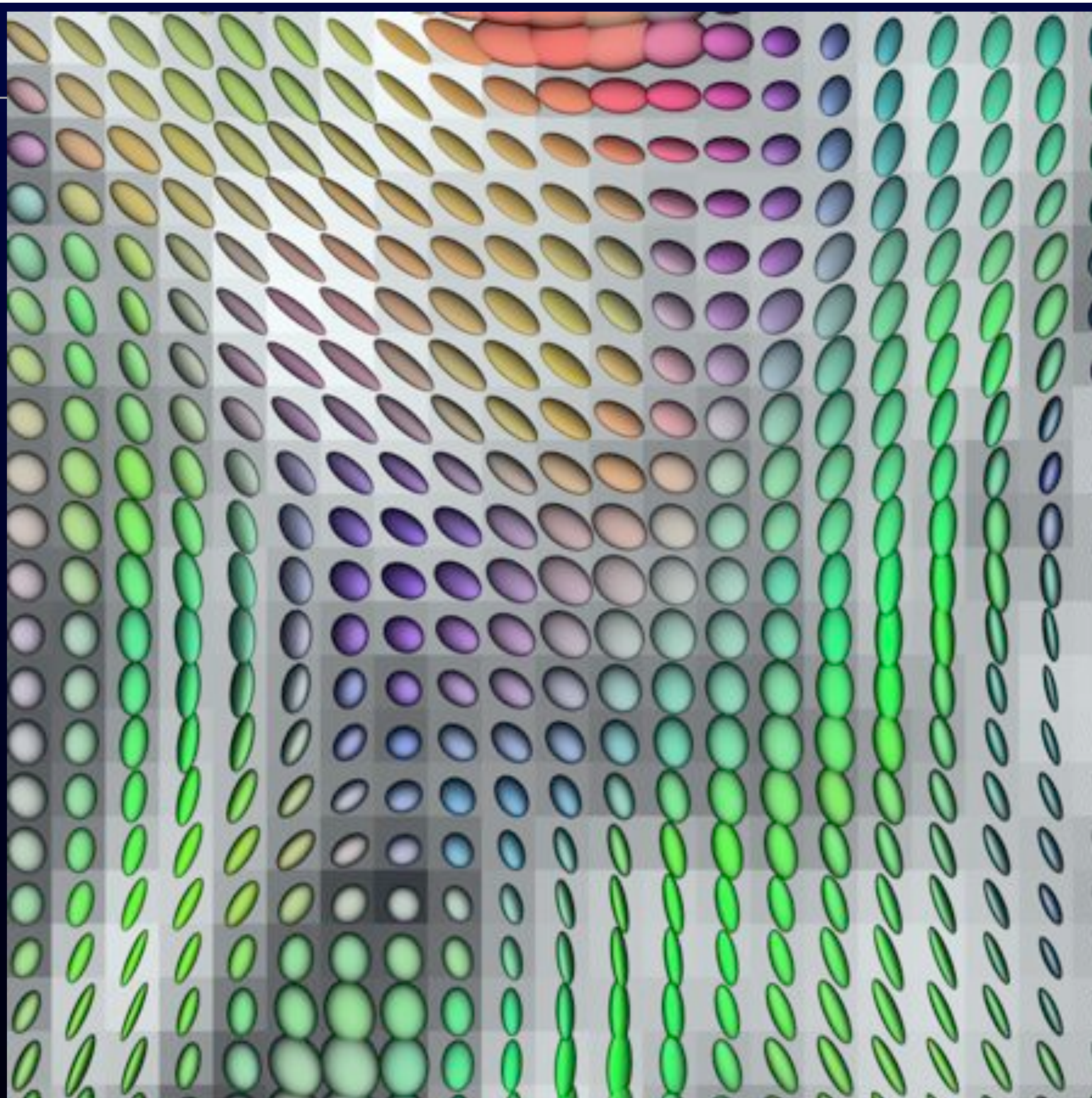


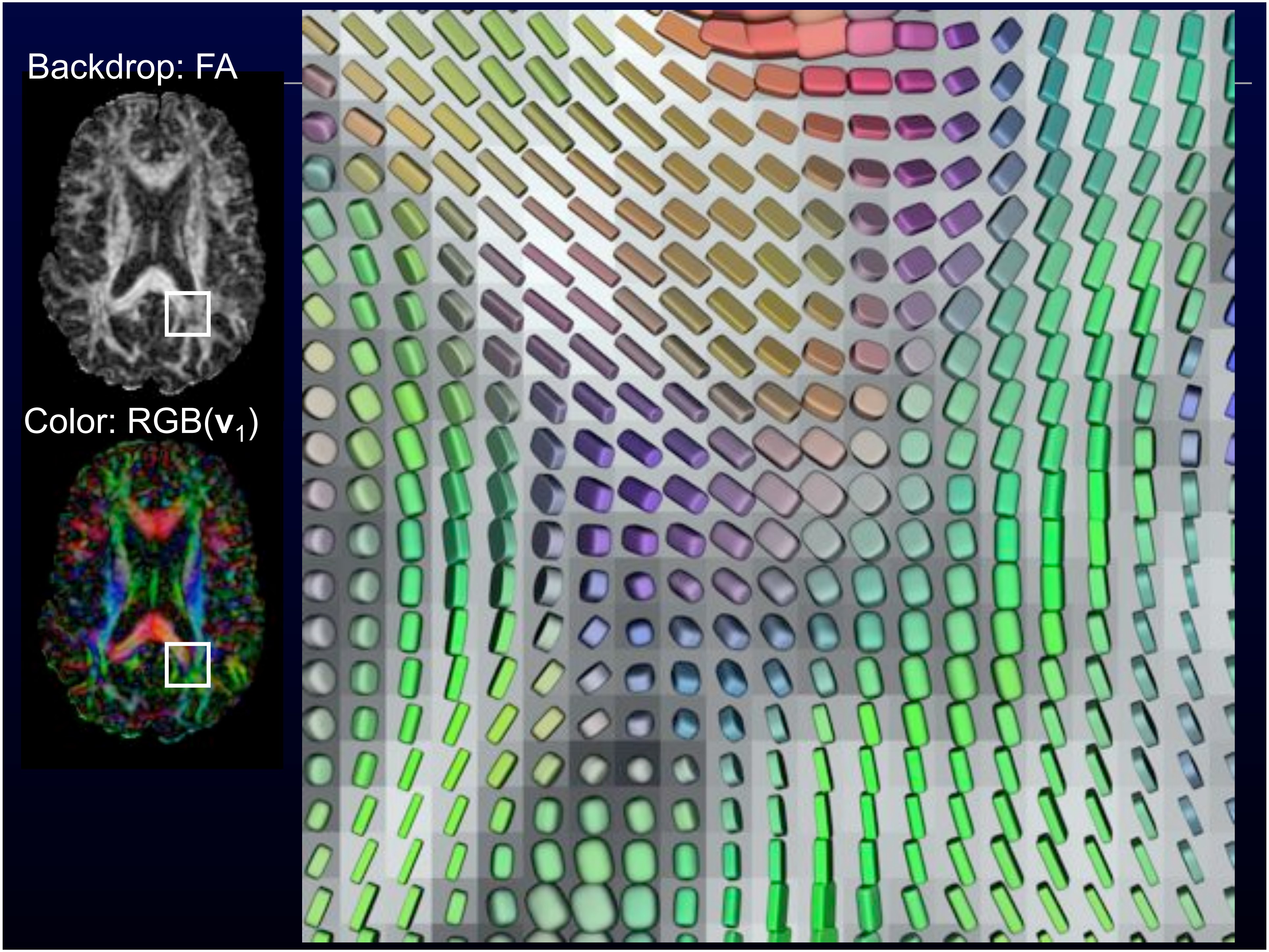
Backdrop: FA



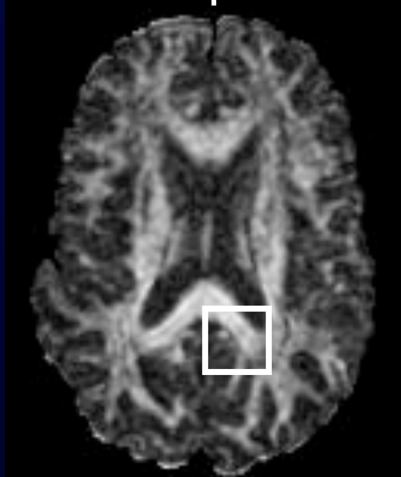
Color: RGB($\mathbf{v}_1$)



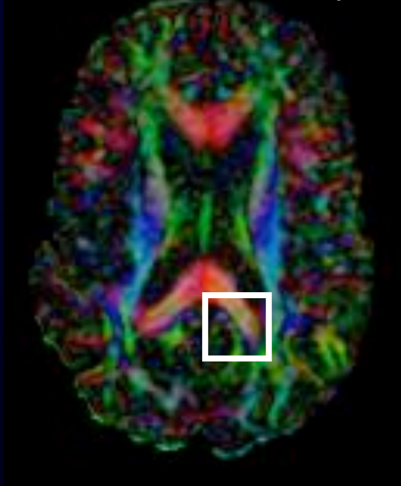


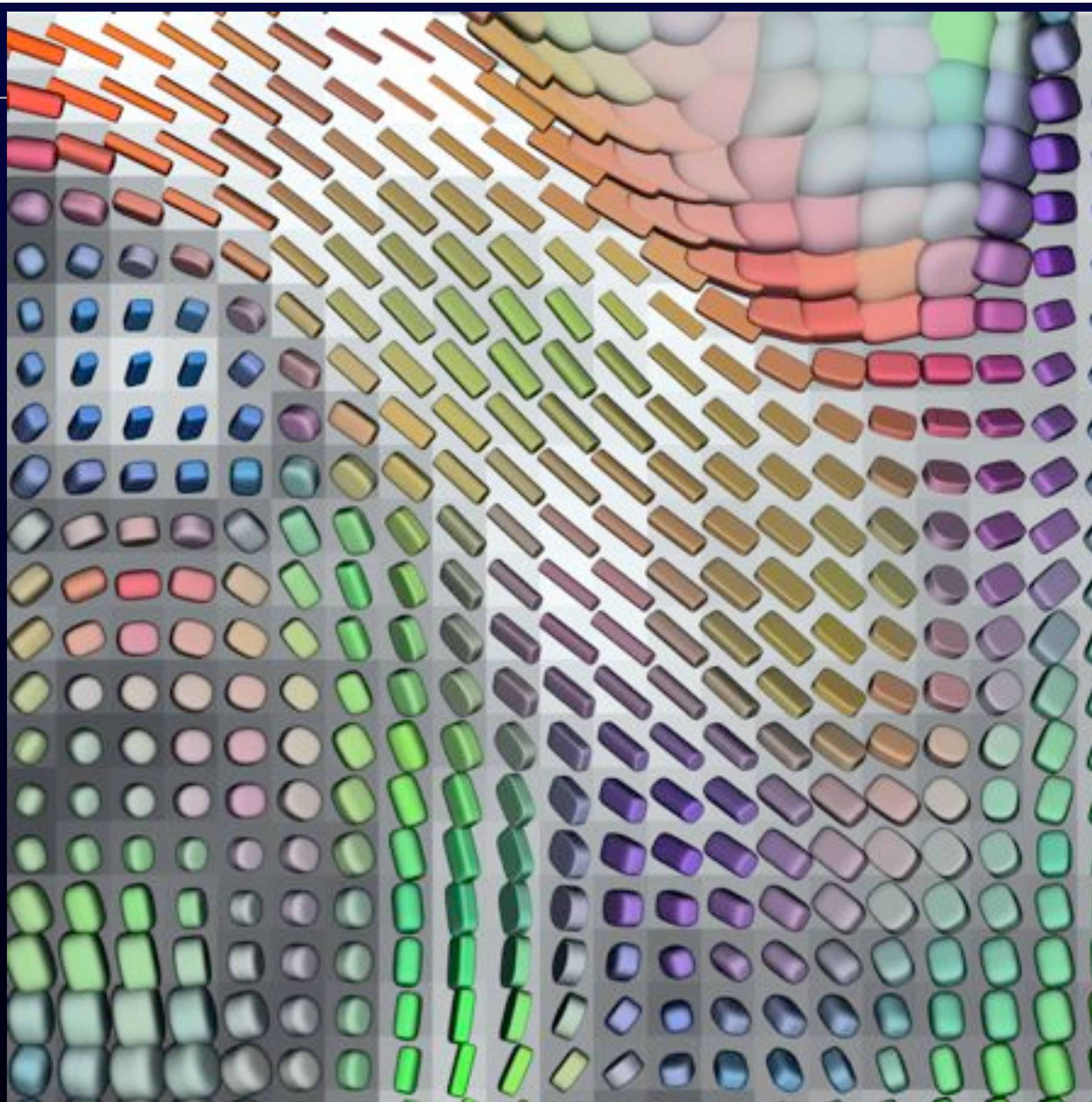

Backdrop: FA
Color: RGB($\mathbf{v}_1$)

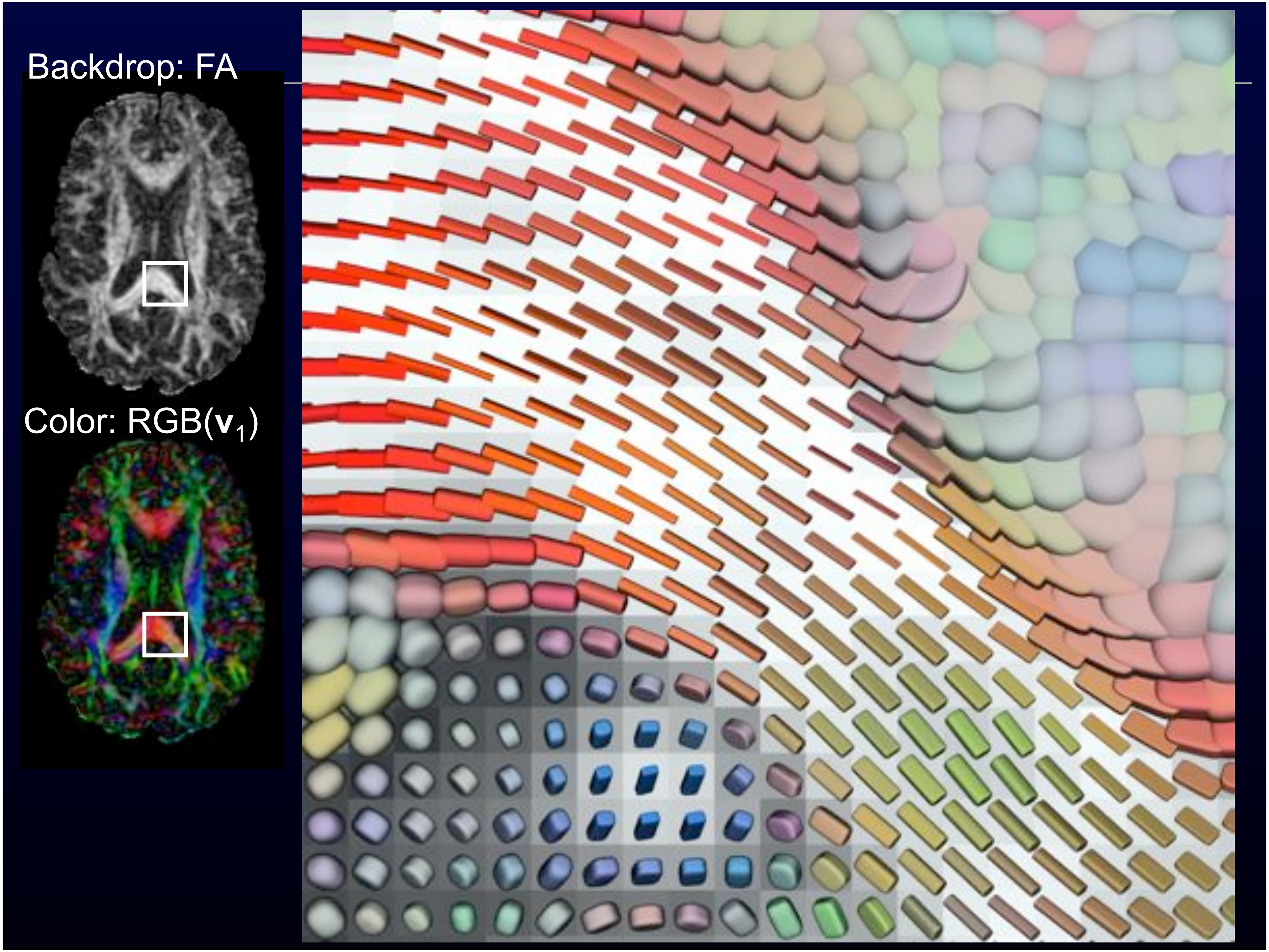
Backdrop: FA



Color: RGB($\mathbf{v}_1$)

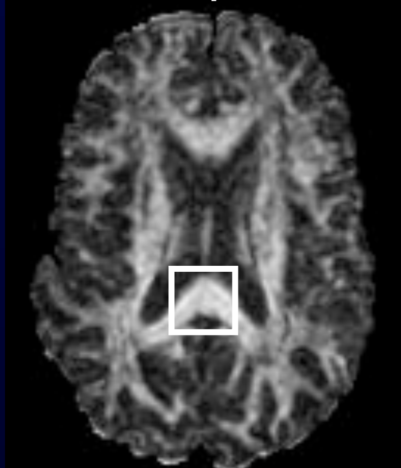




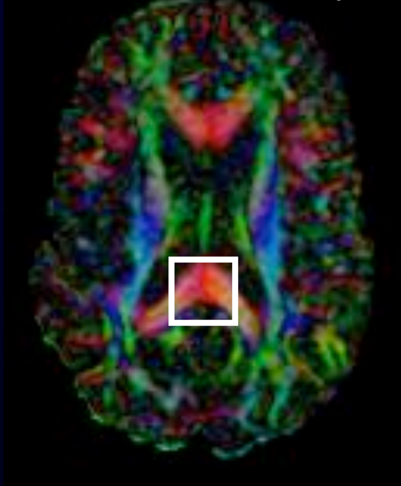


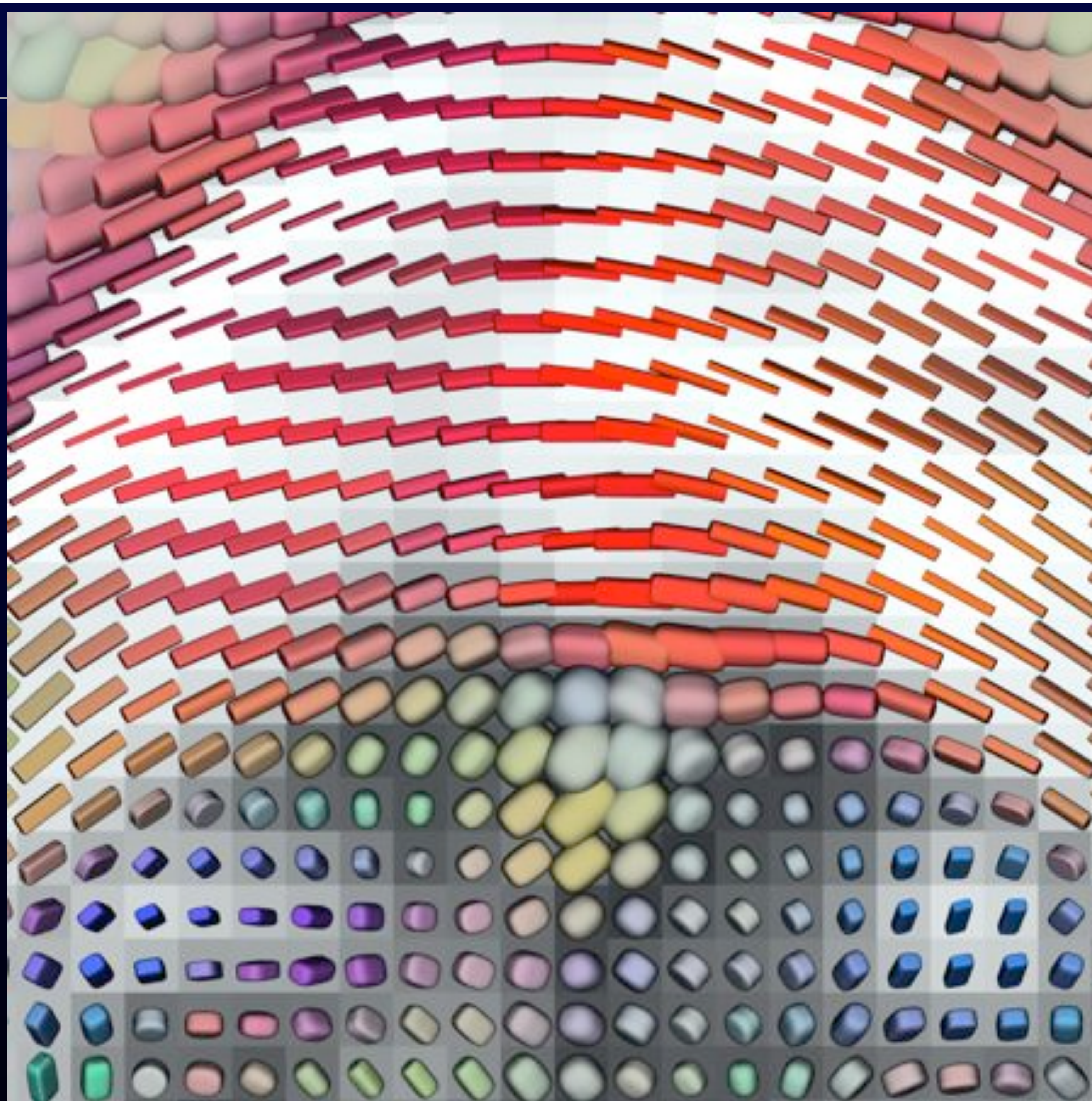
Backdrop: FA
Color: RGB($\mathbf{v}_1$)

Backdrop: FA



Color: RGB($\mathbf{v}_1$)





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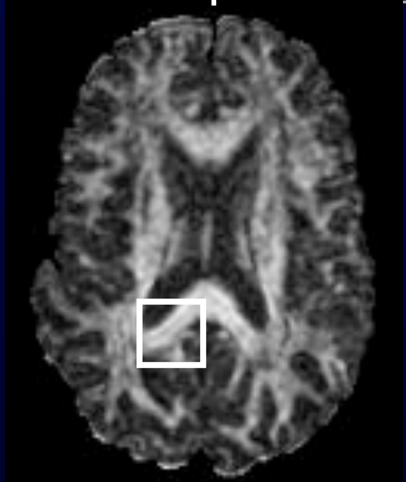
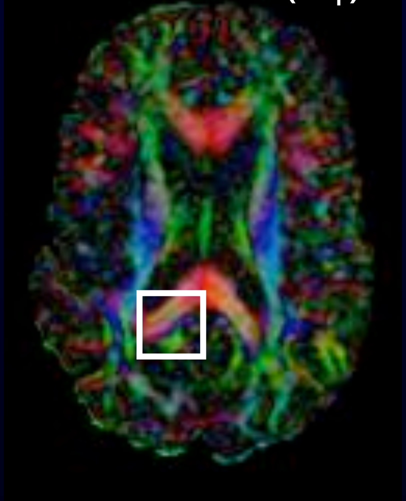
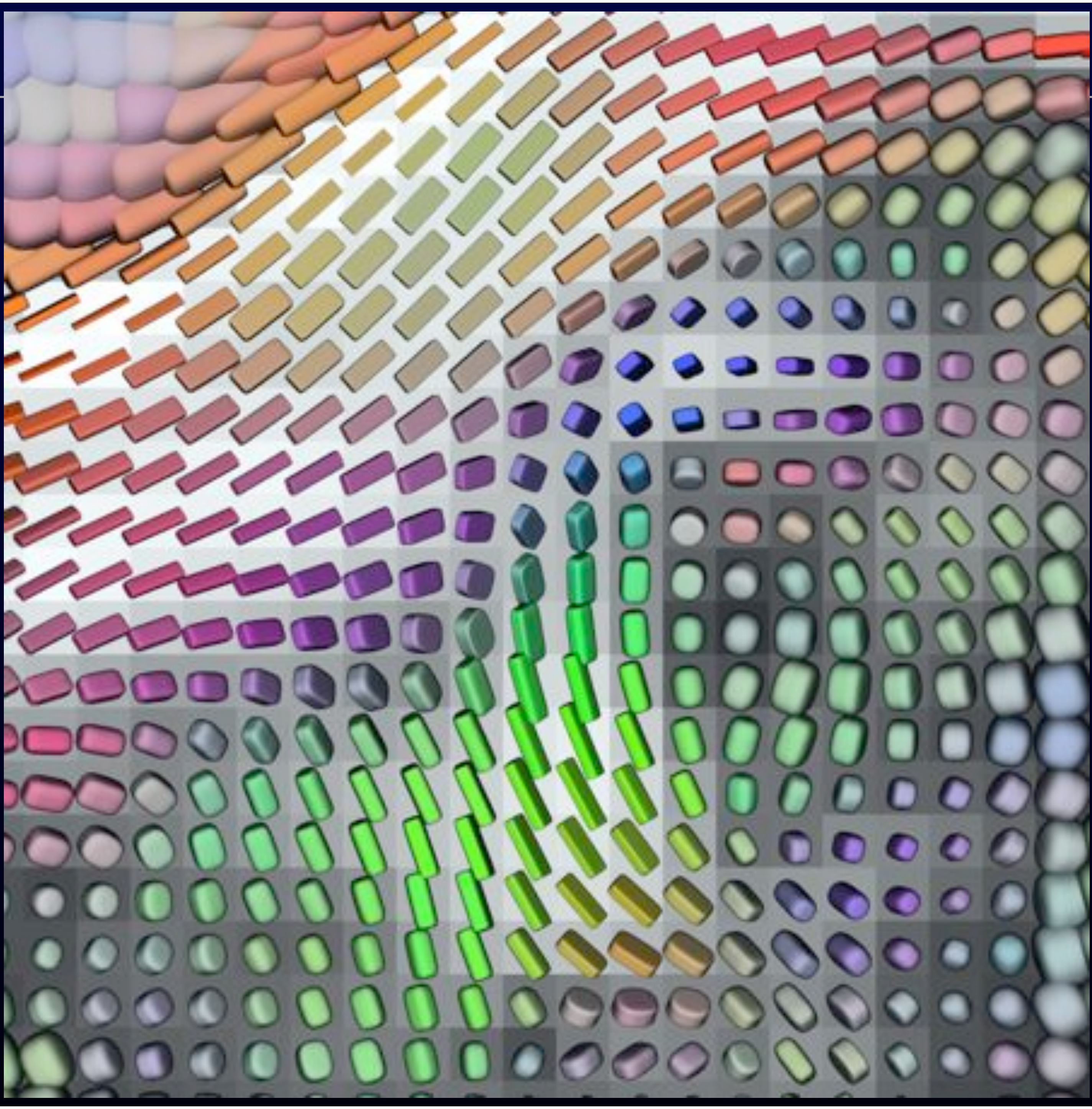
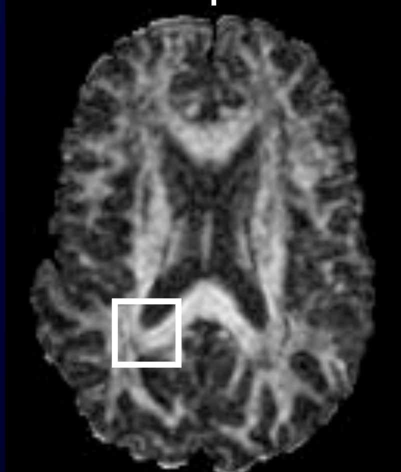


Color: RGB($\mathbf{v}_1$)

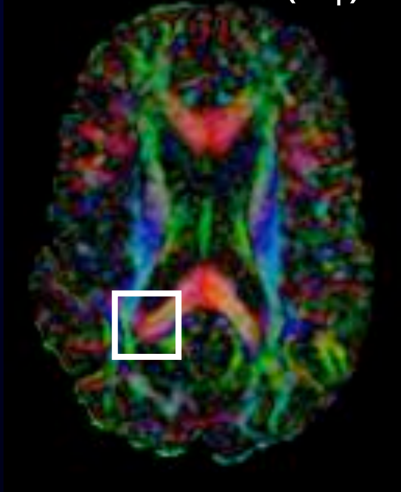


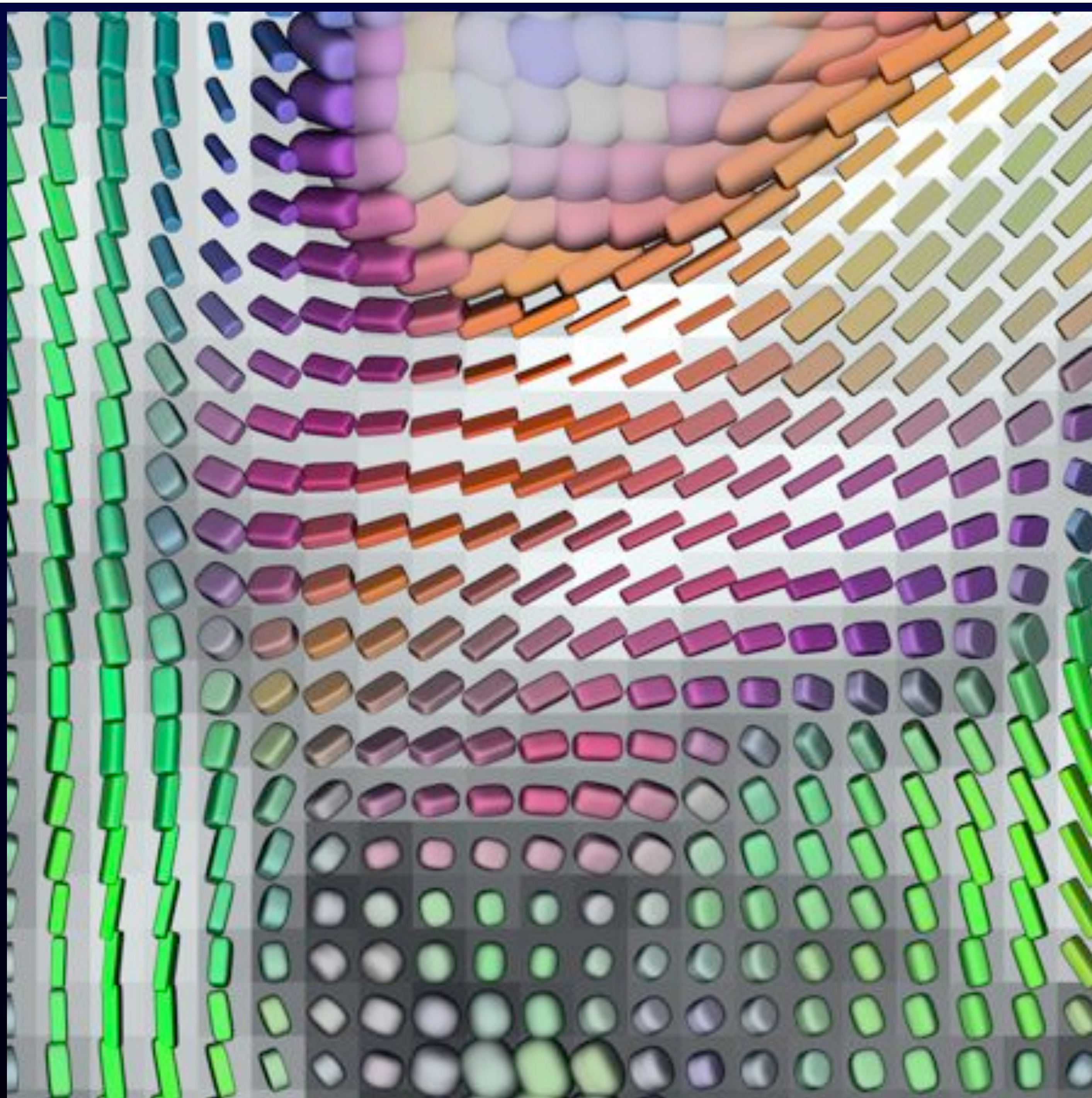


Backdrop: FA



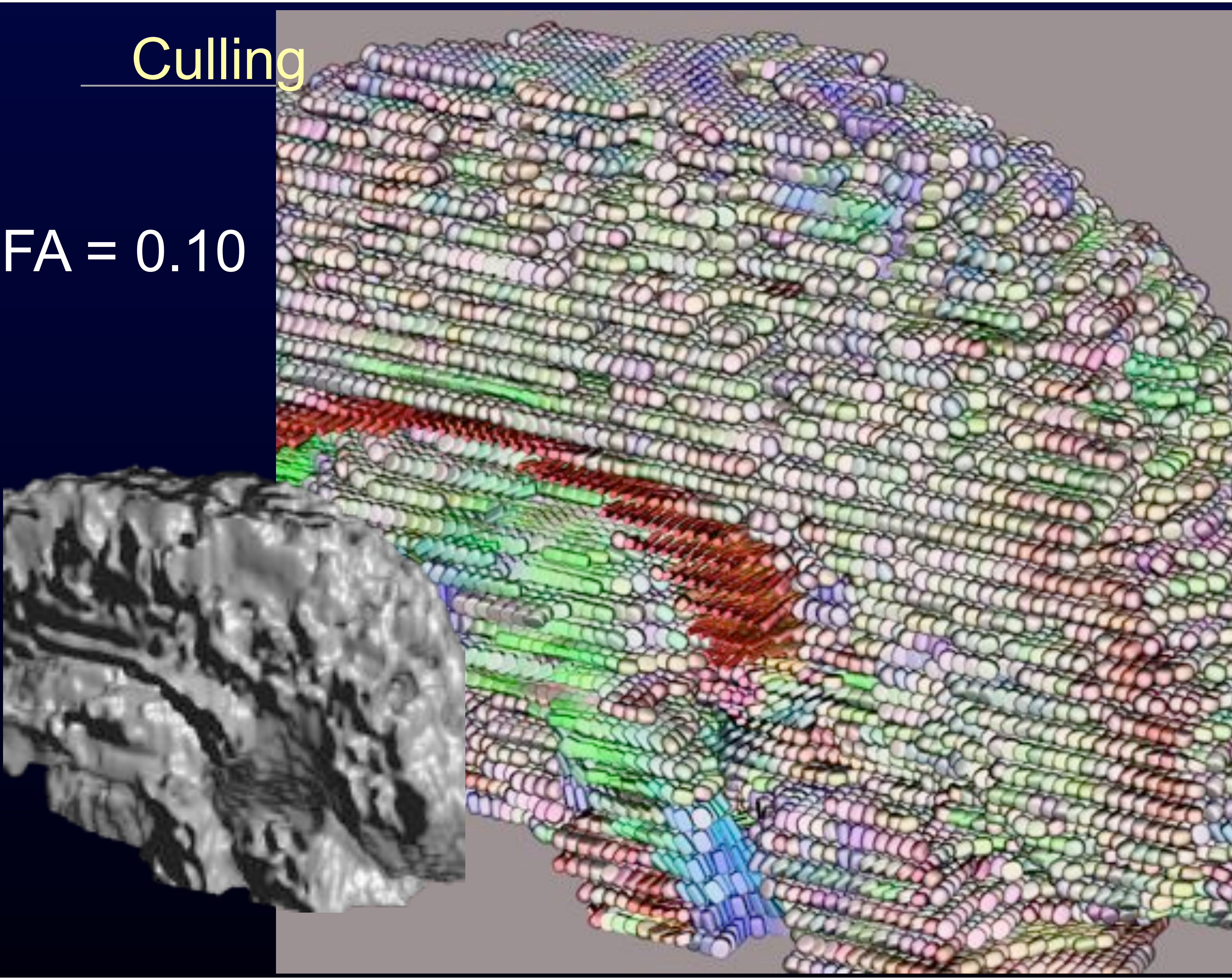
Color: RGB($\mathbf{v}_1$)





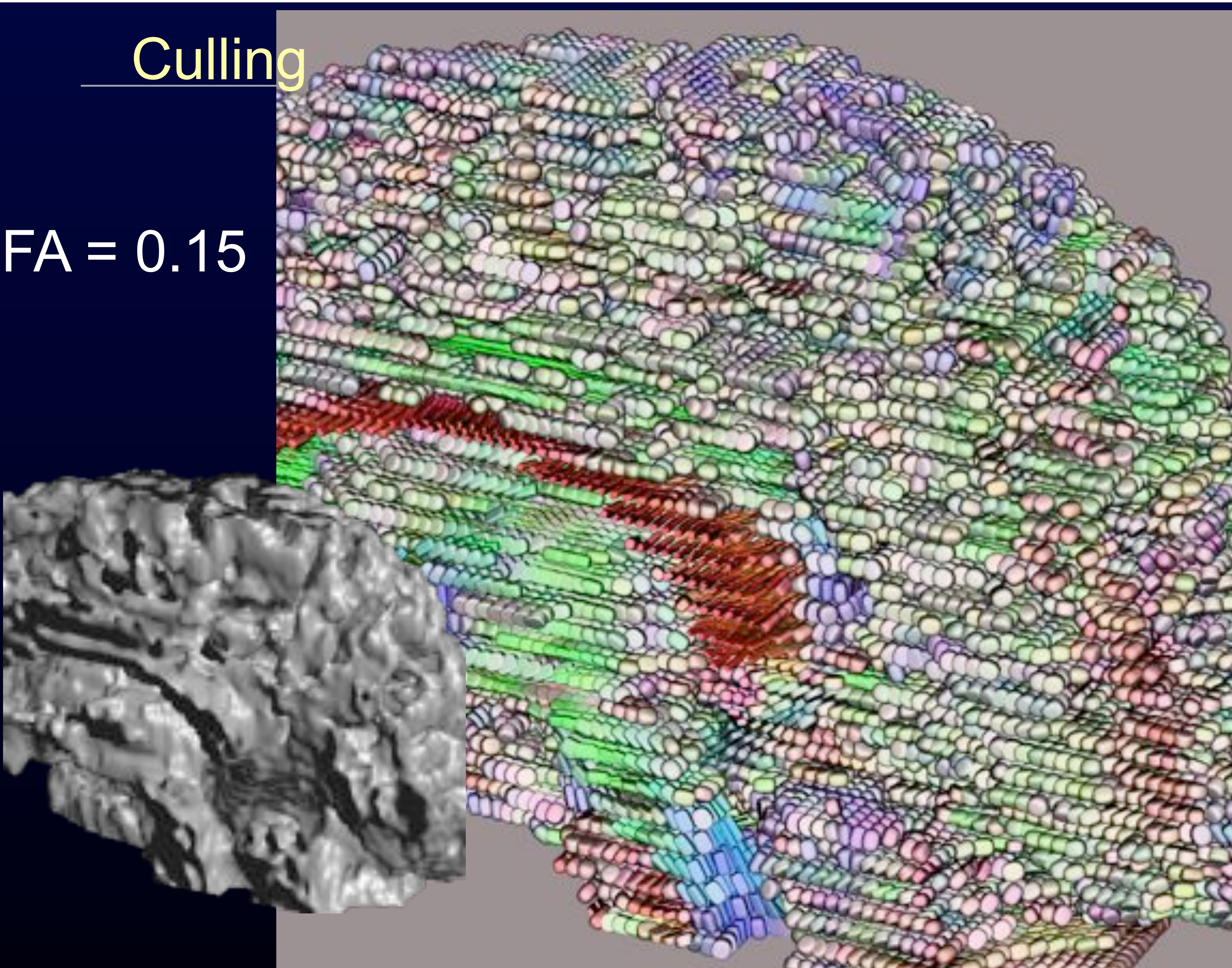
Culling

FA = 0.10



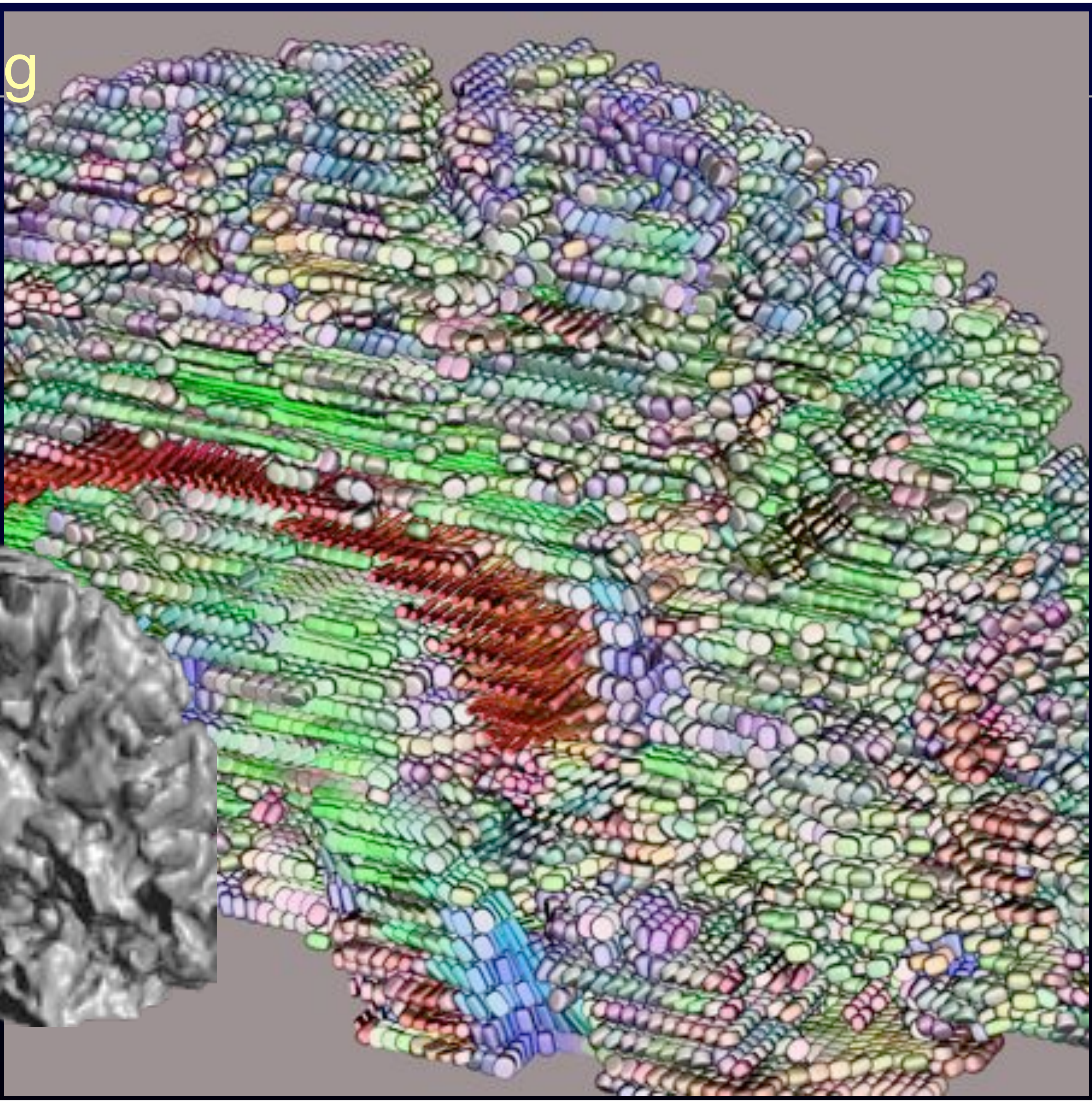
Culling

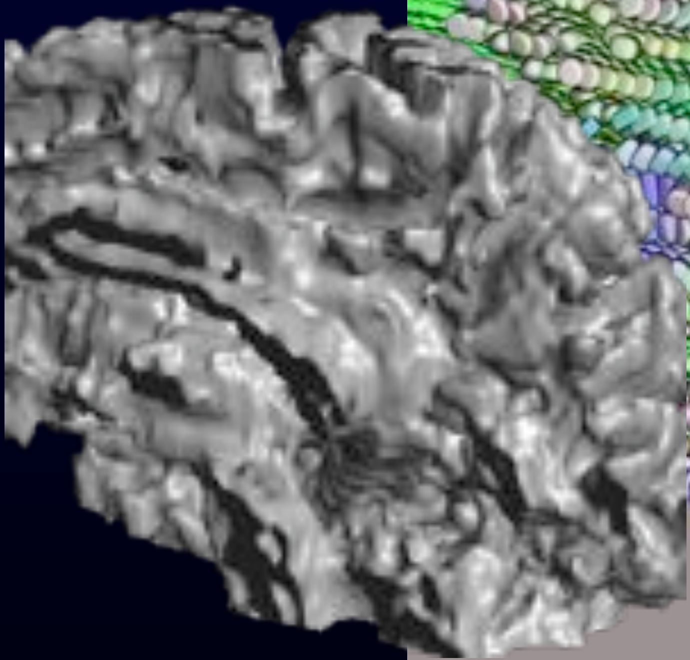
FA = 0.15



Culling

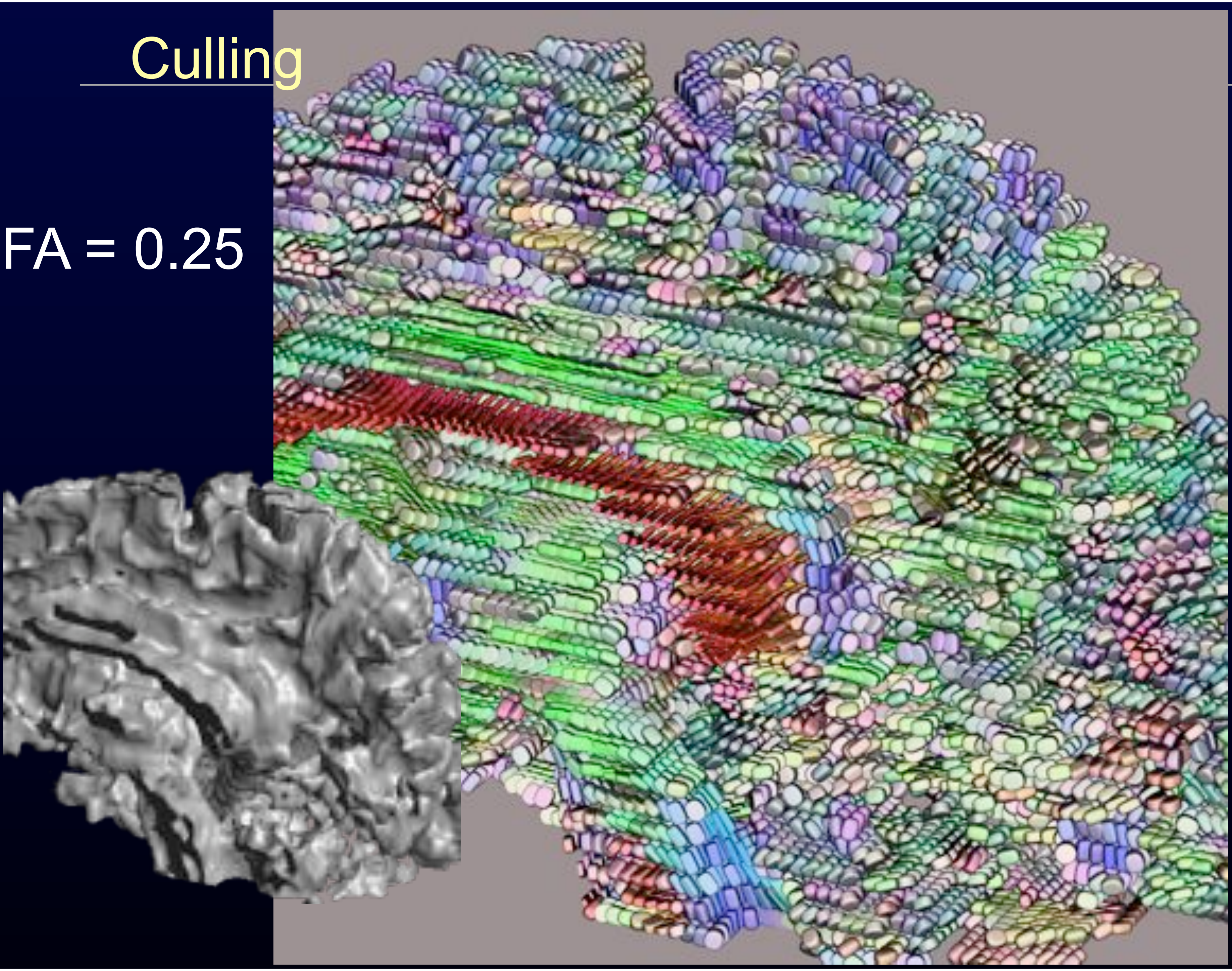
FA = 0.20





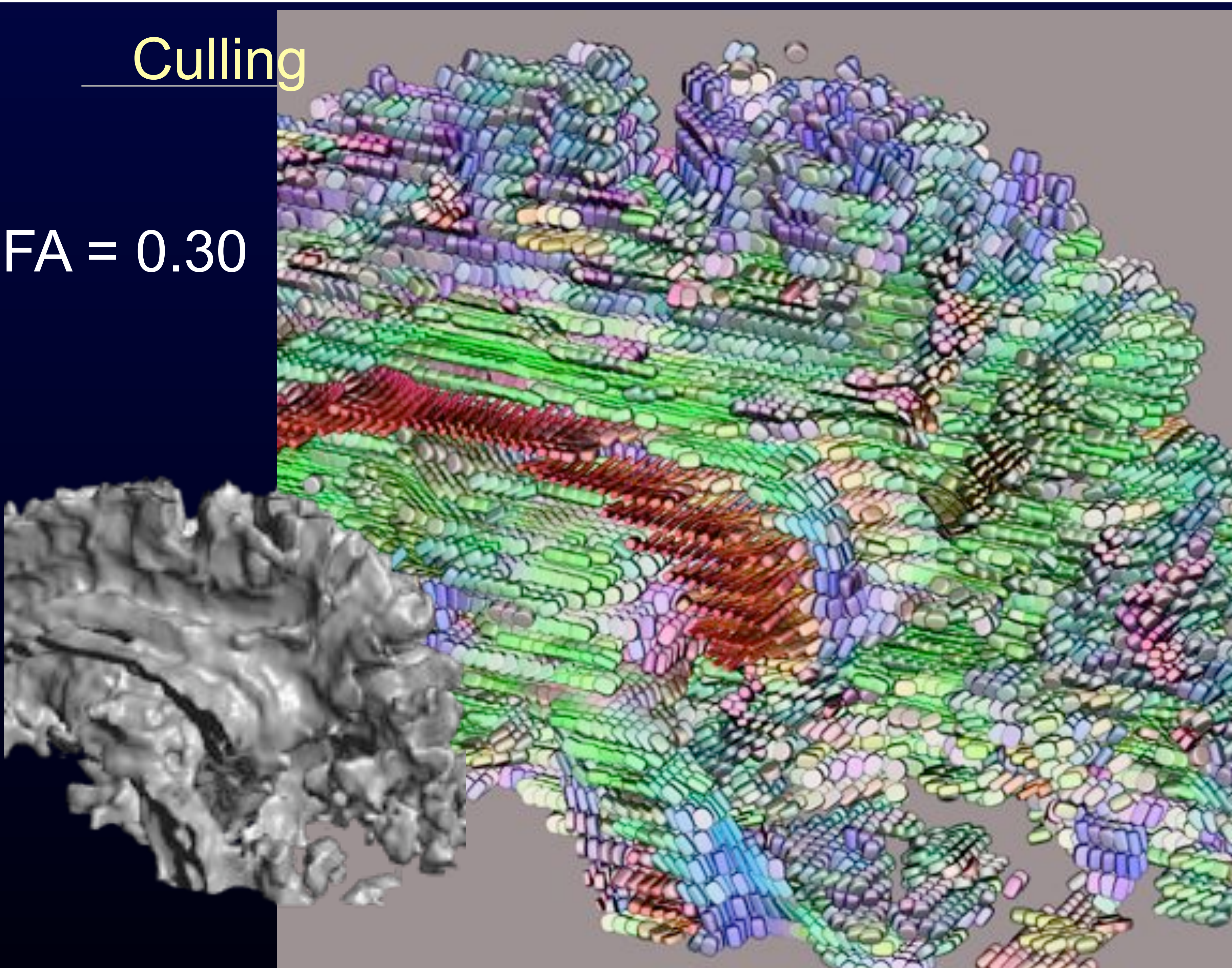
Culling

FA = 0.25



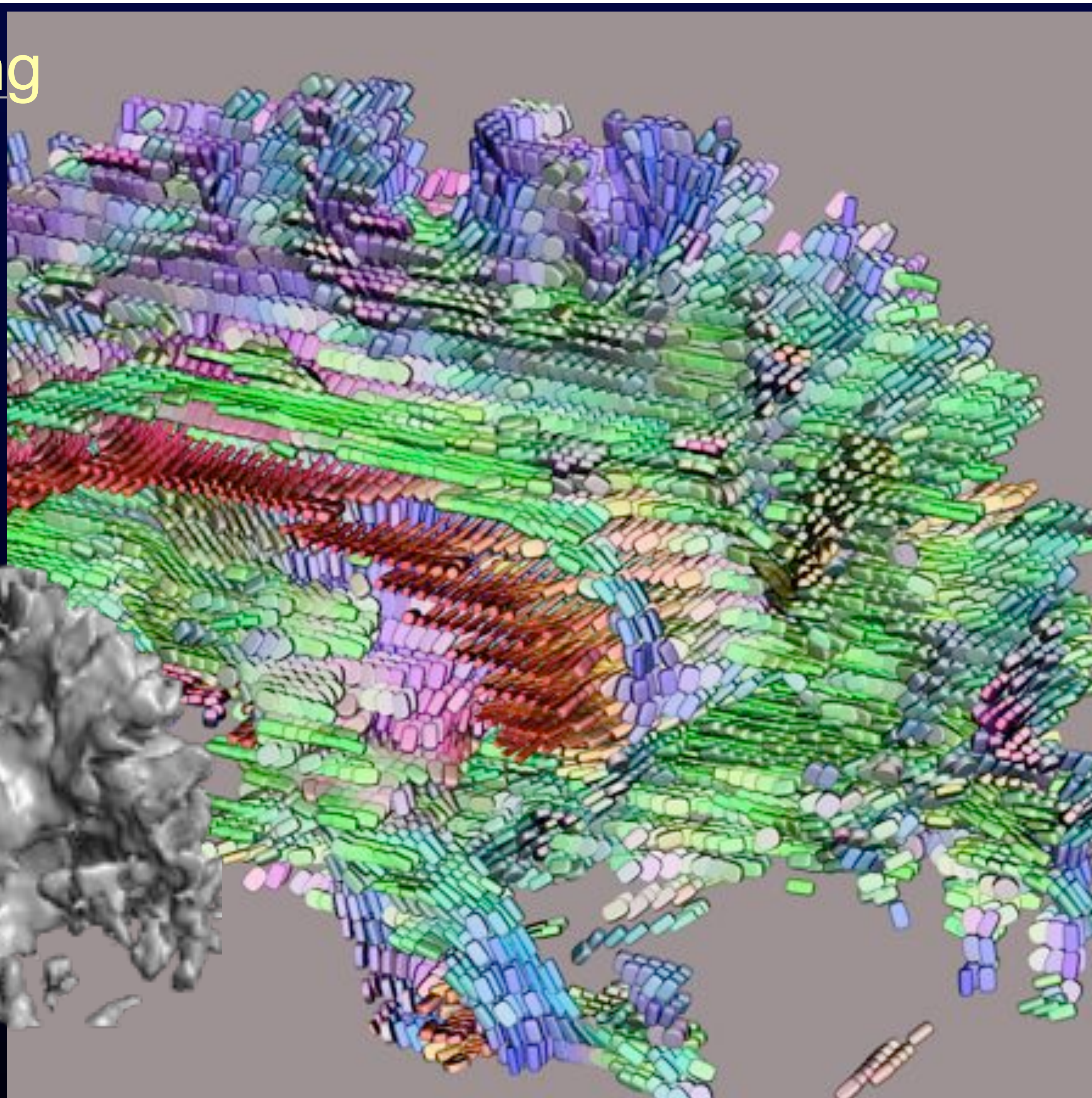
Culling

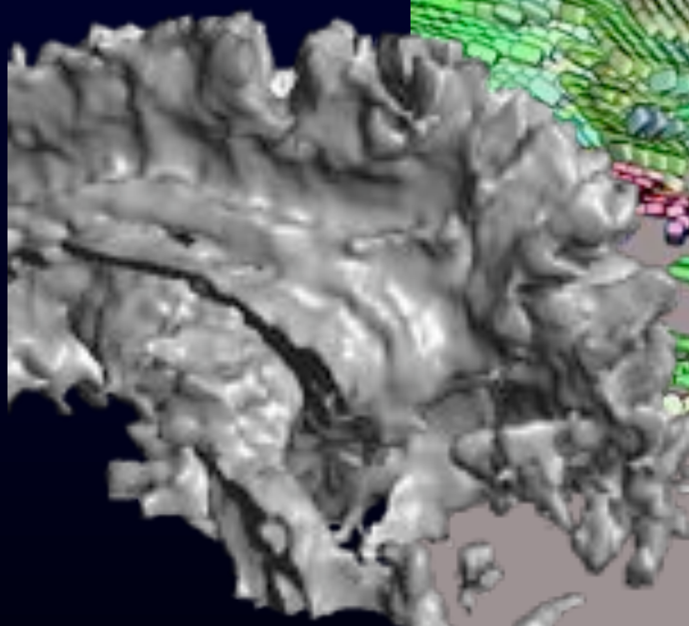
FA = 0.30



Culling

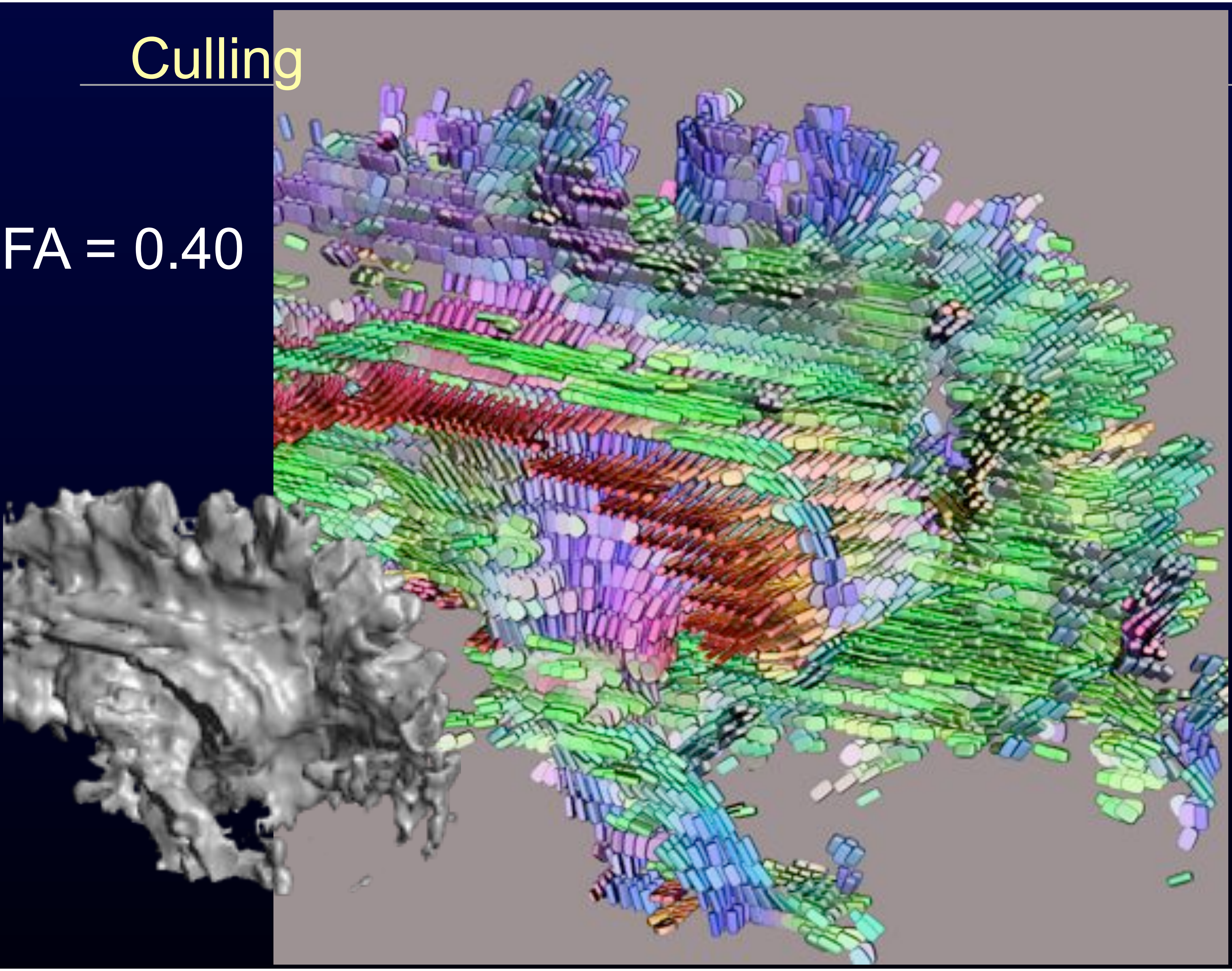
FA = 0.35





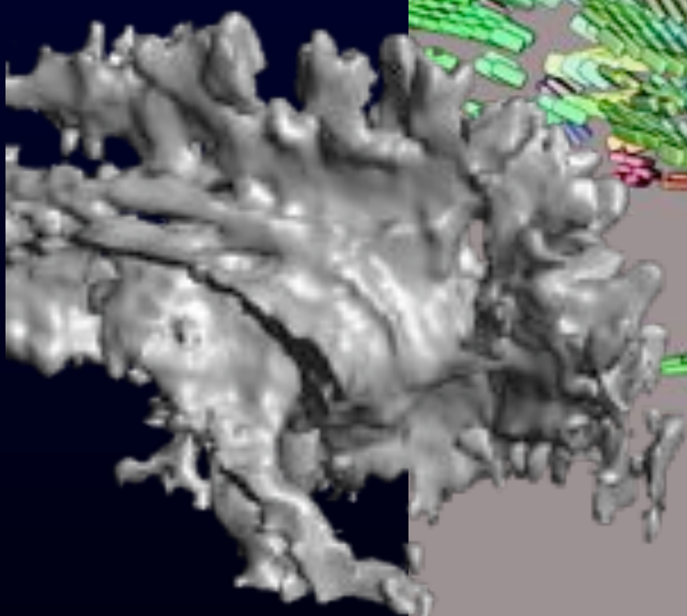
Culling

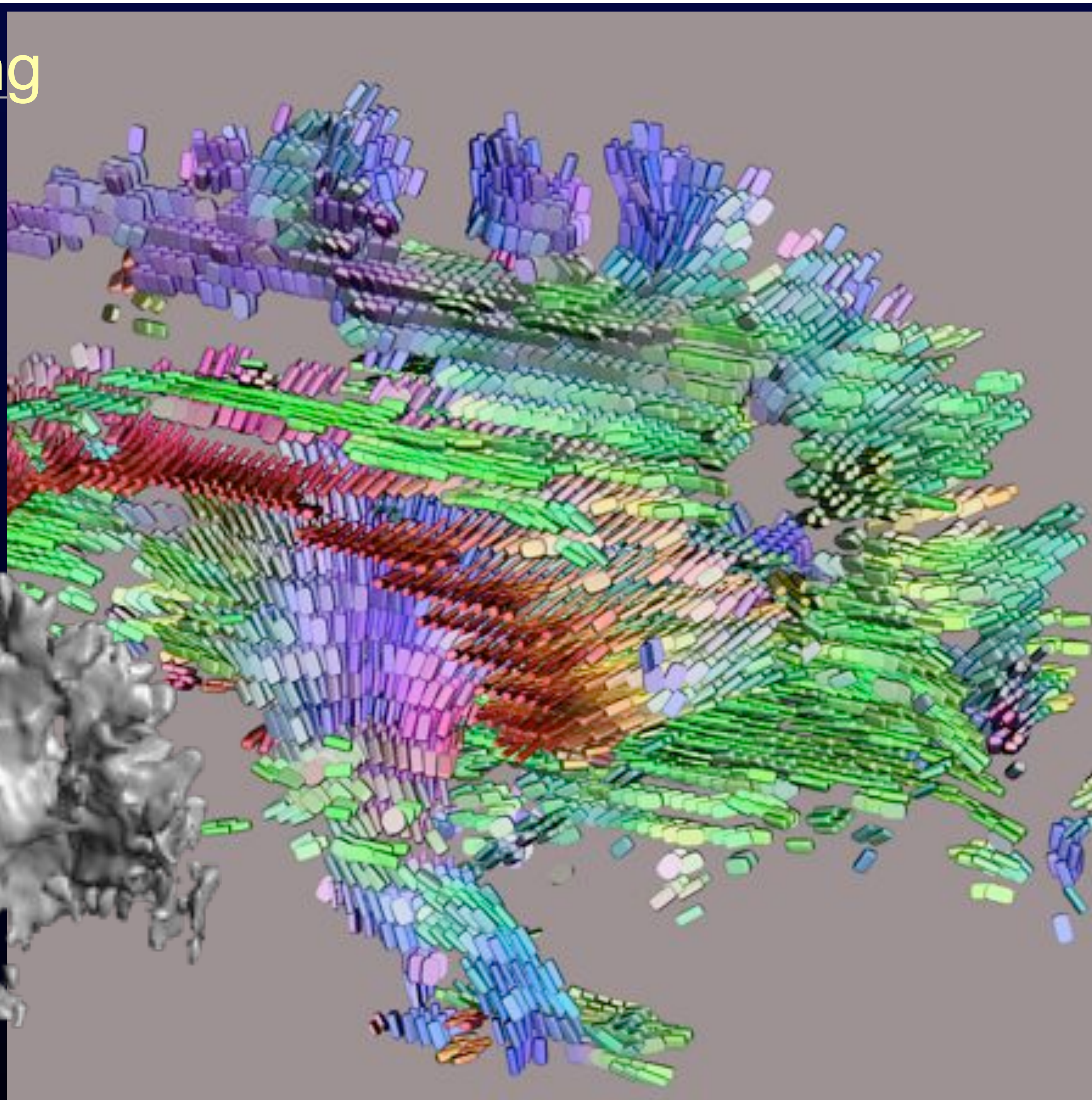
FA = 0.40



Culling

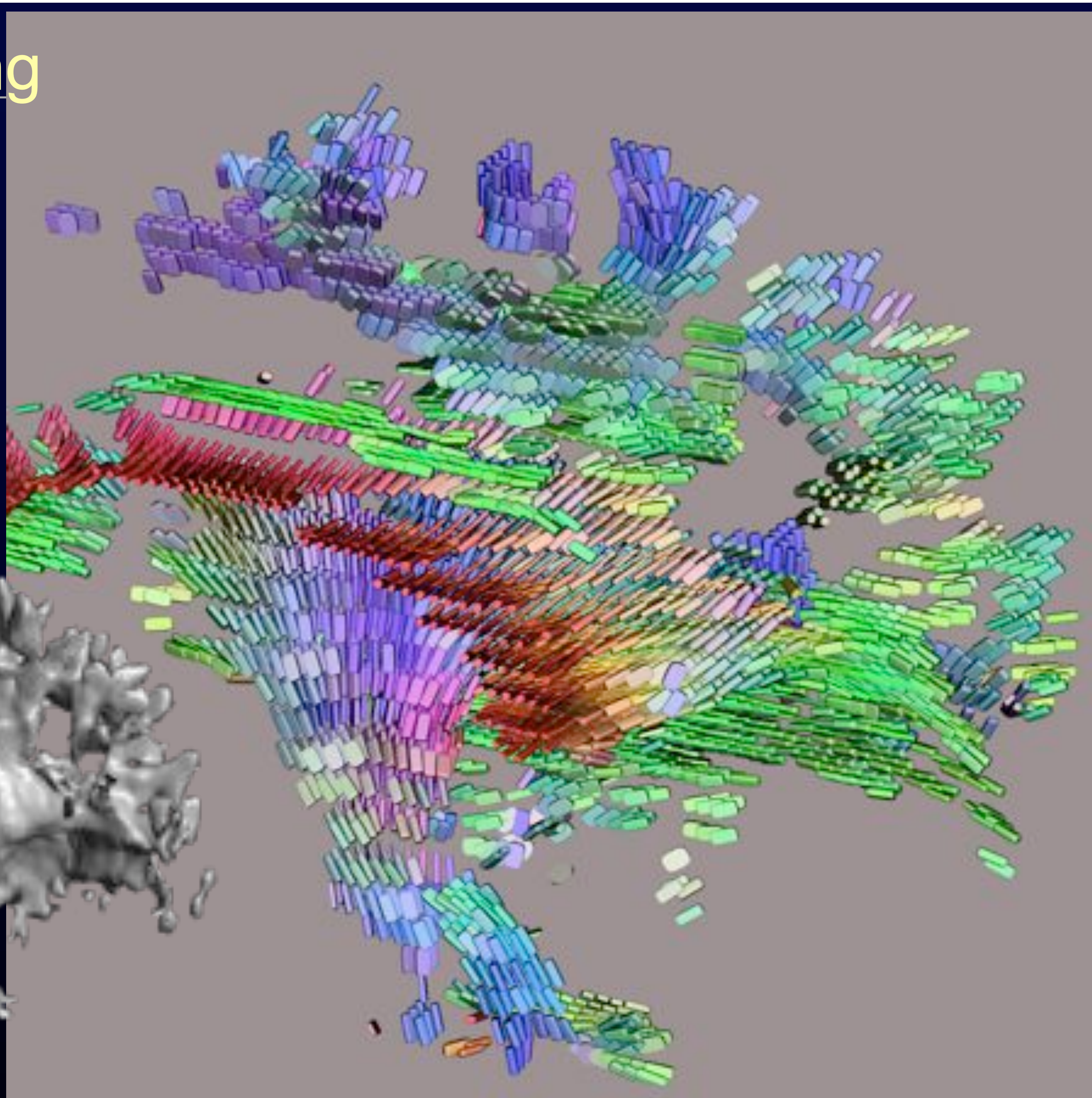
FA = 0.45

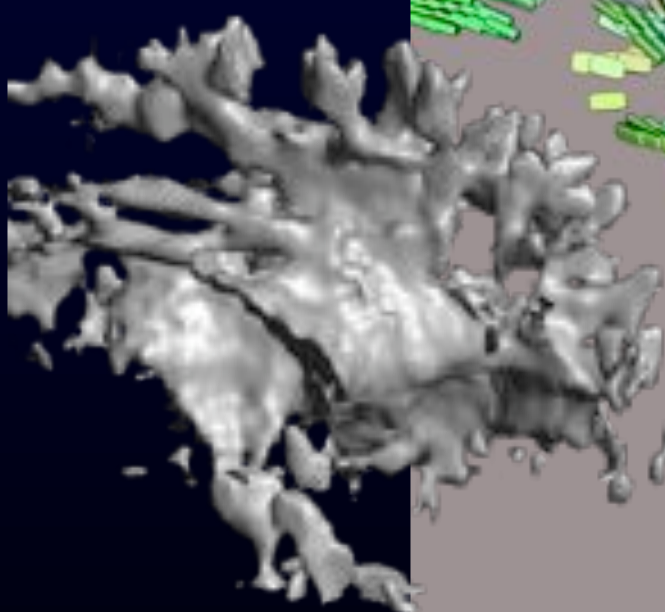




Culling

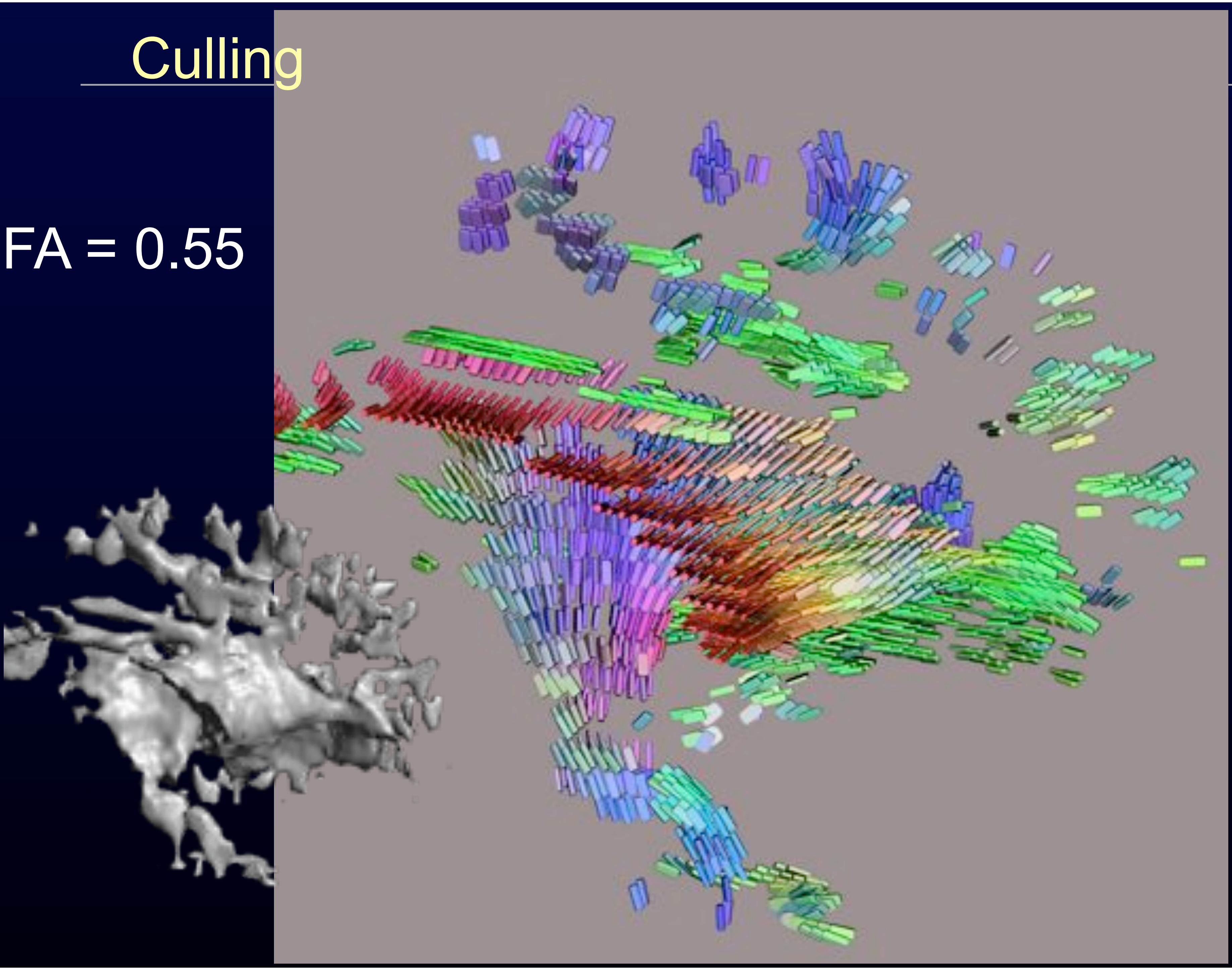
FA = 0.50





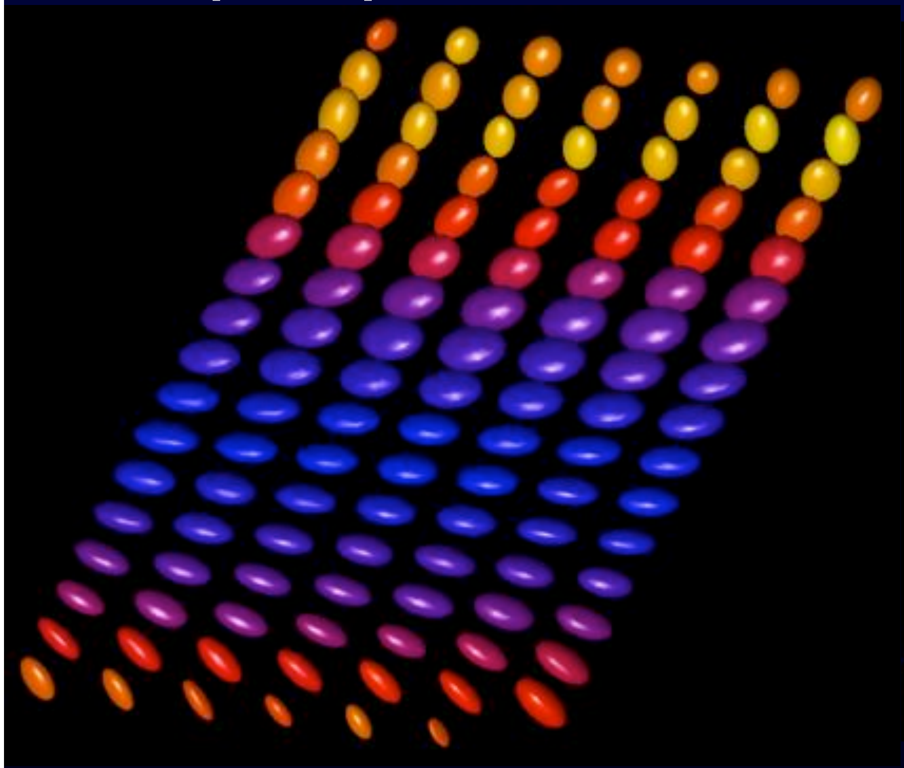
Culling

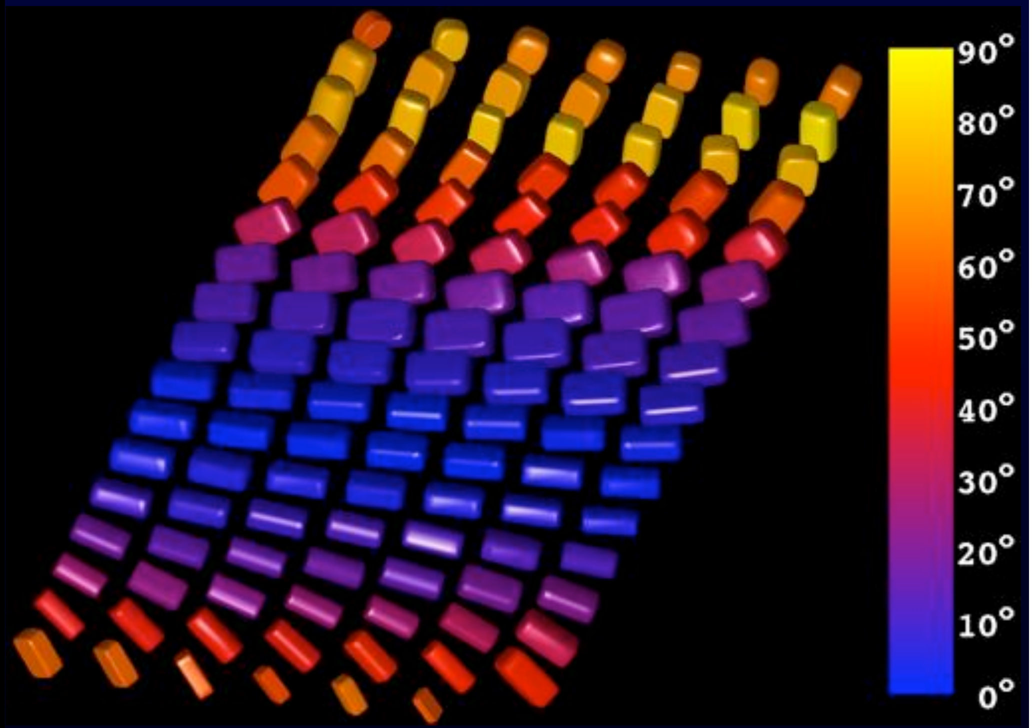
FA = 0.55



Myocardial wall: fiber twist

Superquadrics better highlight orthotropy





Visualization of high resolution myocardial strain and diffusion tensors using superquadric glyphs

D.B. Ennis, G. Kindlmann, P.A. Helm, I. Rodriguez, E.R. McVeigh

ISMRM 2004 e-Poster

Rough Outline

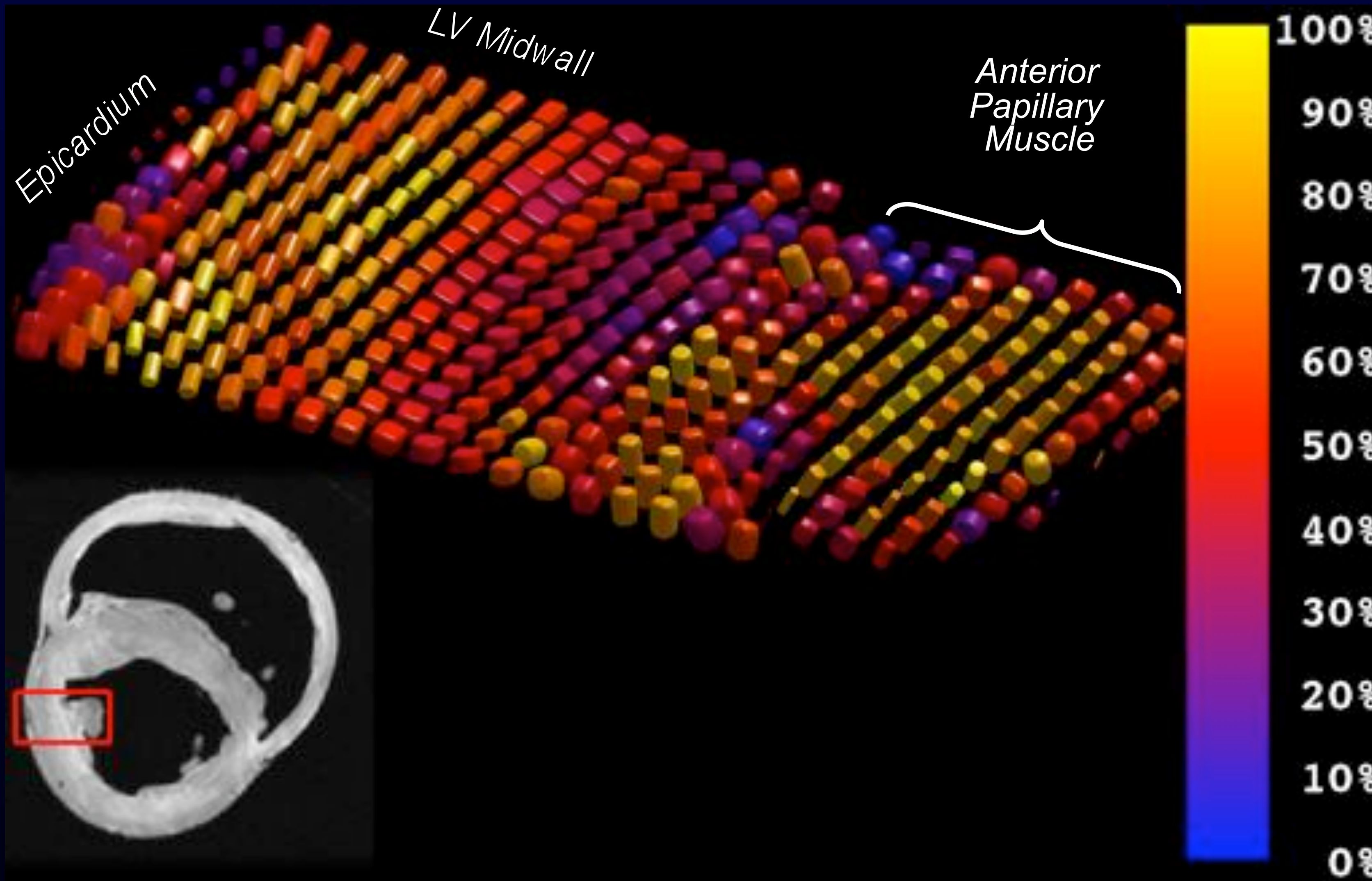


Figure 8. The group of glyphs on the right that are cylindrical have high linear anisotropy (high C_L), high LP-ratio (yellow) and comprise the anterior papillary muscle. The glyphs with greatest orthotropy (~50% LP-ratio, red) appear in the midwall and resemble the expected sheet organization. Most epicardial glyphs appear more cylindrical.

Rough Outline

Glyphs for data inspection

Invariants for describing shape

Invariant gradients

- WRT position: Shading in rendering
- WRT tensor components: DOF of shape

Invariants

$f(\mathbf{D})$ is invariant $\Leftrightarrow f(\mathbf{D}) = f(\mathbf{RDR}^{-1})\ \forall\ \mathbf{R}$

Characteristic equation of **D**: $\det(\mathbf{D} - \lambda\mathbf{I}) = 0 \Rightarrow$

$$\lambda^3 - J_1\lambda^2 + J_2\lambda - J_3 = 0$$

$\det(\mathbf{RDR}^{-1} - \lambda\mathbf{I}) = \det(\mathbf{R})\det(\mathbf{D} - \lambda\mathbf{I})\det(\mathbf{R}^{-1})$

$$= \det(\mathbf{D} - \lambda\mathbf{I}) \Rightarrow$$

J_1, J_2, J_3 are ("principal") invariants:

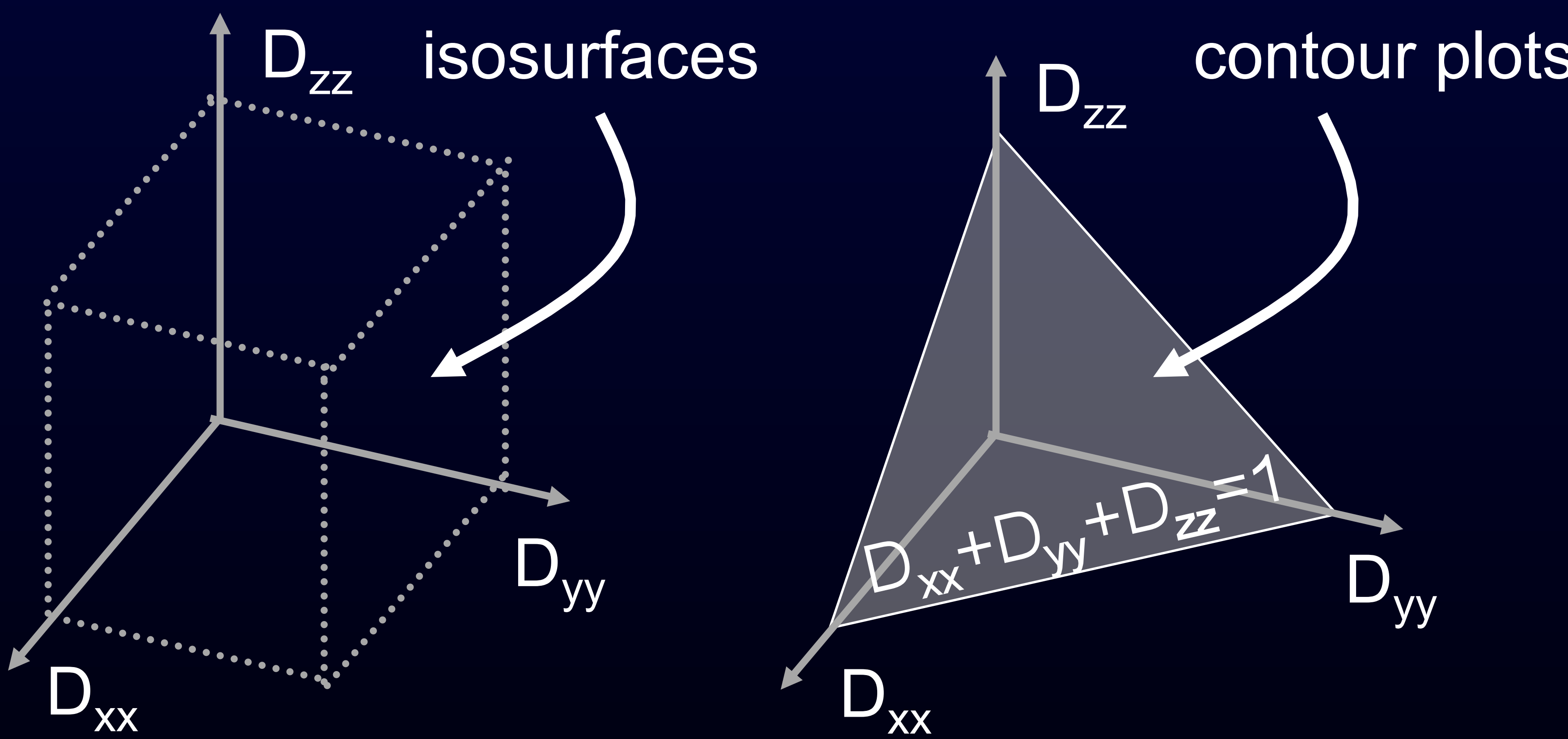
$J_1 = \mathrm{Tr}(\mathbf{D})$

$J_2 = (\mathrm{Tr}(\mathbf{D})^2 - \mathrm{Tr}(\mathbf{D}^2))/2$

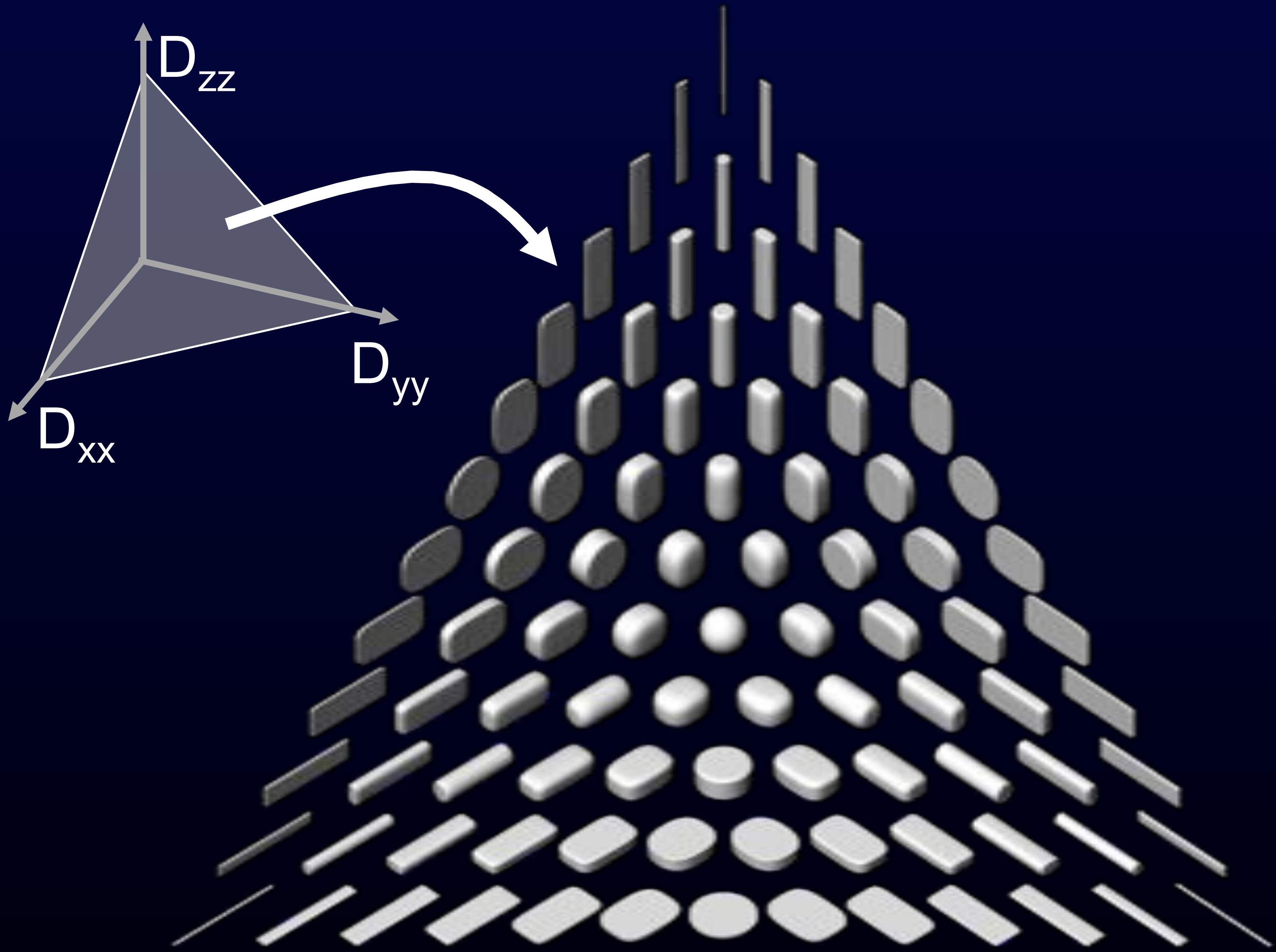
$J_3 = \mathrm{Det}(\mathbf{D})$

But what do invariants **look** like?

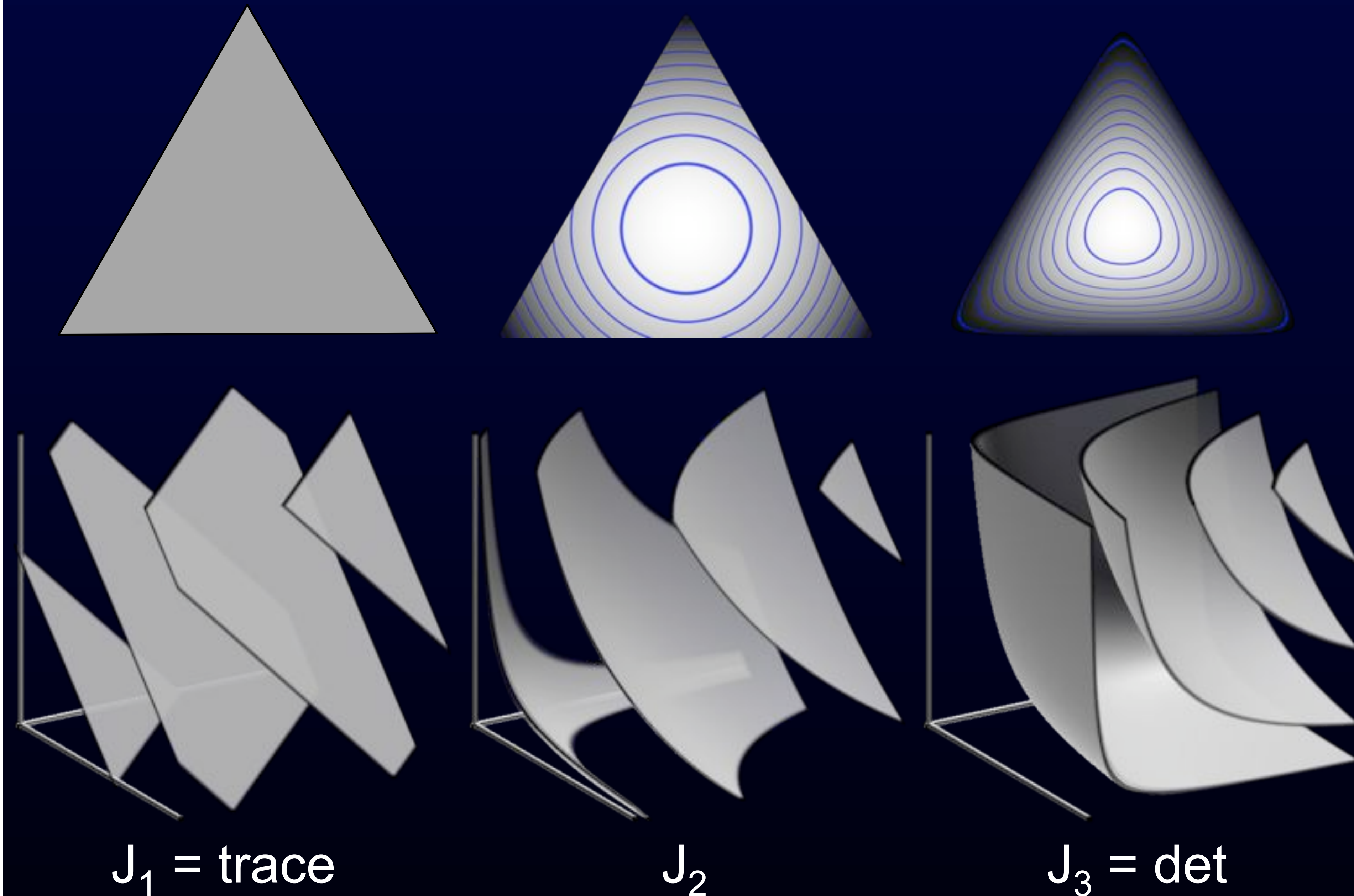
Visualize them in space of diagonal matrices



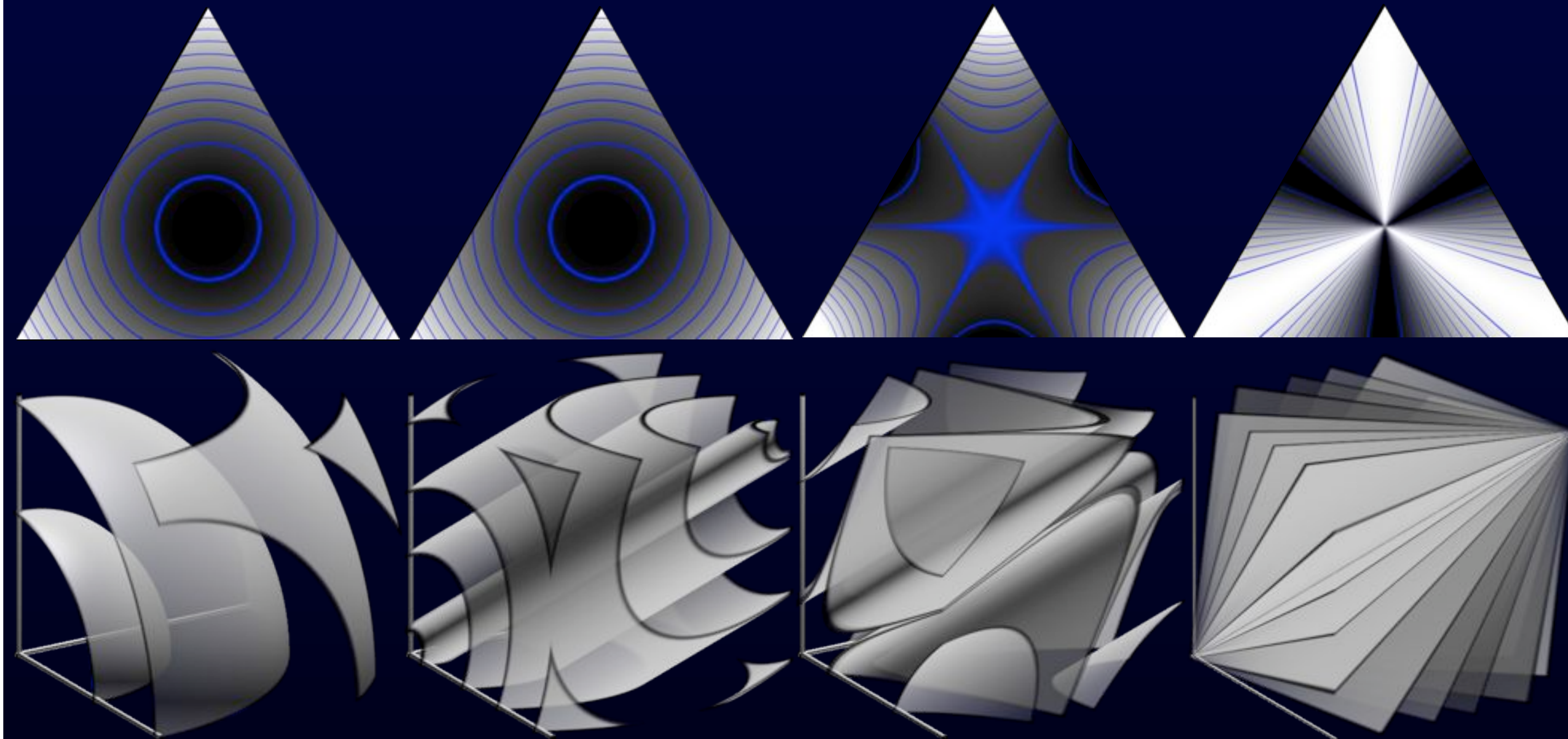
(What do glyphs look like?)



Visualizing principal invariants



More invariants



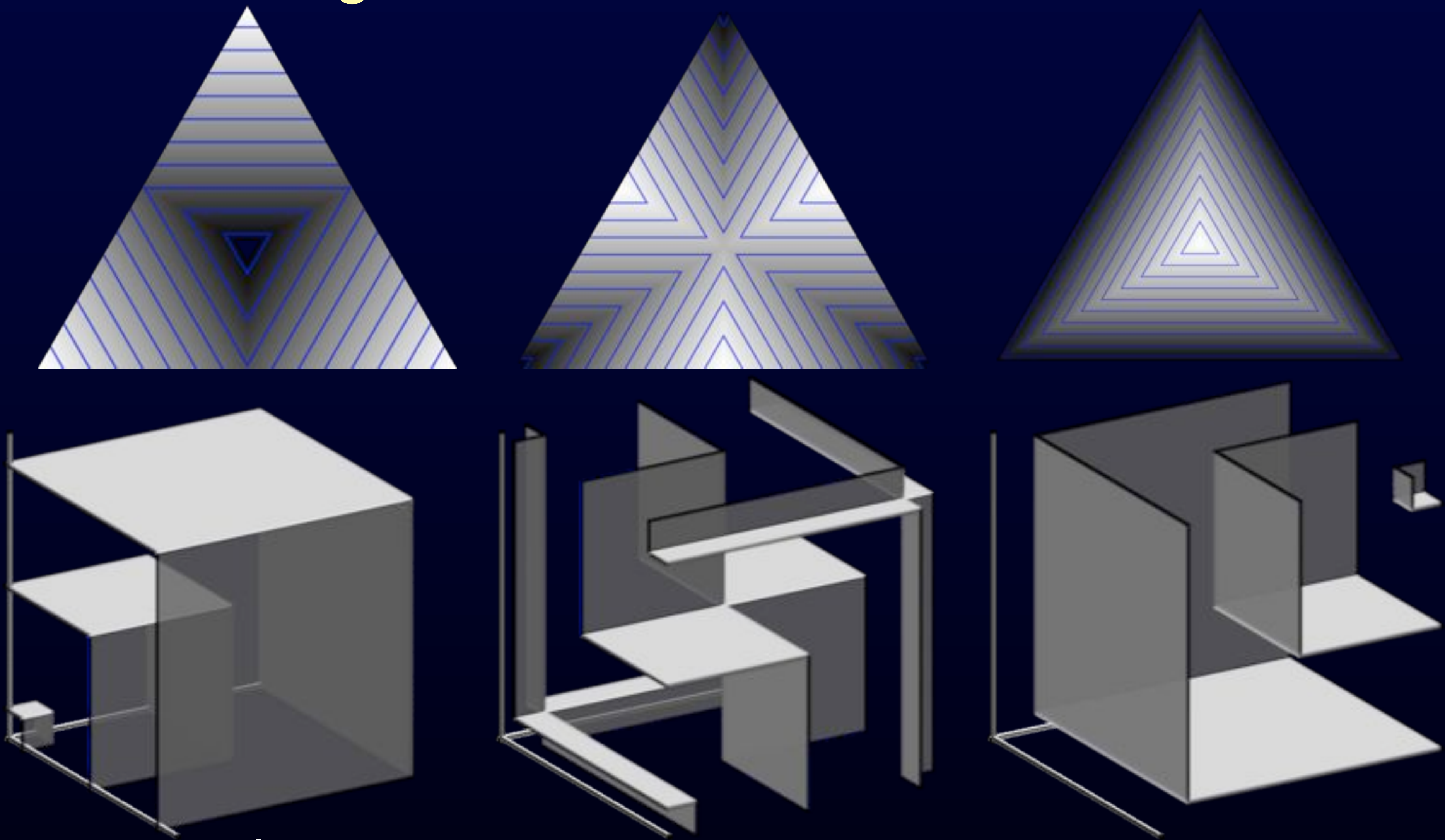
$S = |\mathbf{D}|_F^2 = J_1^2 - 2J_2$

$\mu_2 = \frac{2(S - J_2)}{9} = \mathrm{var}(\lambda)$

$\mu_3 = \frac{2J_1S + 27J_3 - 5J_1J_2}{27}$

$\mathrm{skew} = \frac{\mu_3}{\mu_2^{3/2}}$

The eigenvalues



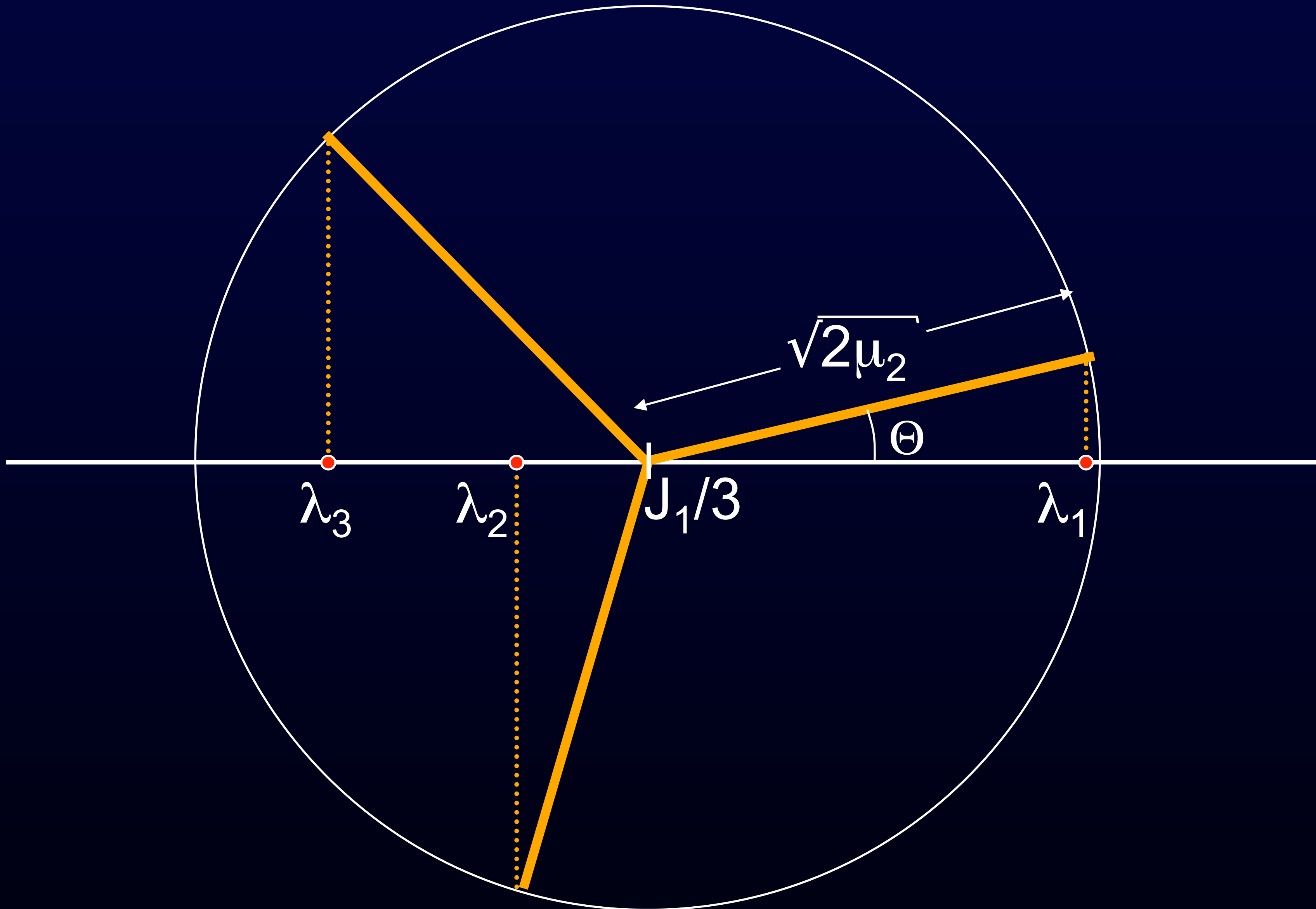
$(\Theta = \cos^{-1}(\sqrt{2}\ \text{skew})/3)$

$\lambda_1 = J_1/3 + \sqrt{2\mu_2}\cos(\Theta)$

$\lambda_2 = J_1/3 + \sqrt{2\mu_2}\cos(\Theta - 2\pi/3)$

$\lambda_3 = J_1/3 + \sqrt{2\mu_2}\cos(\Theta + 2\pi/3)$

Eigenvalue wheel



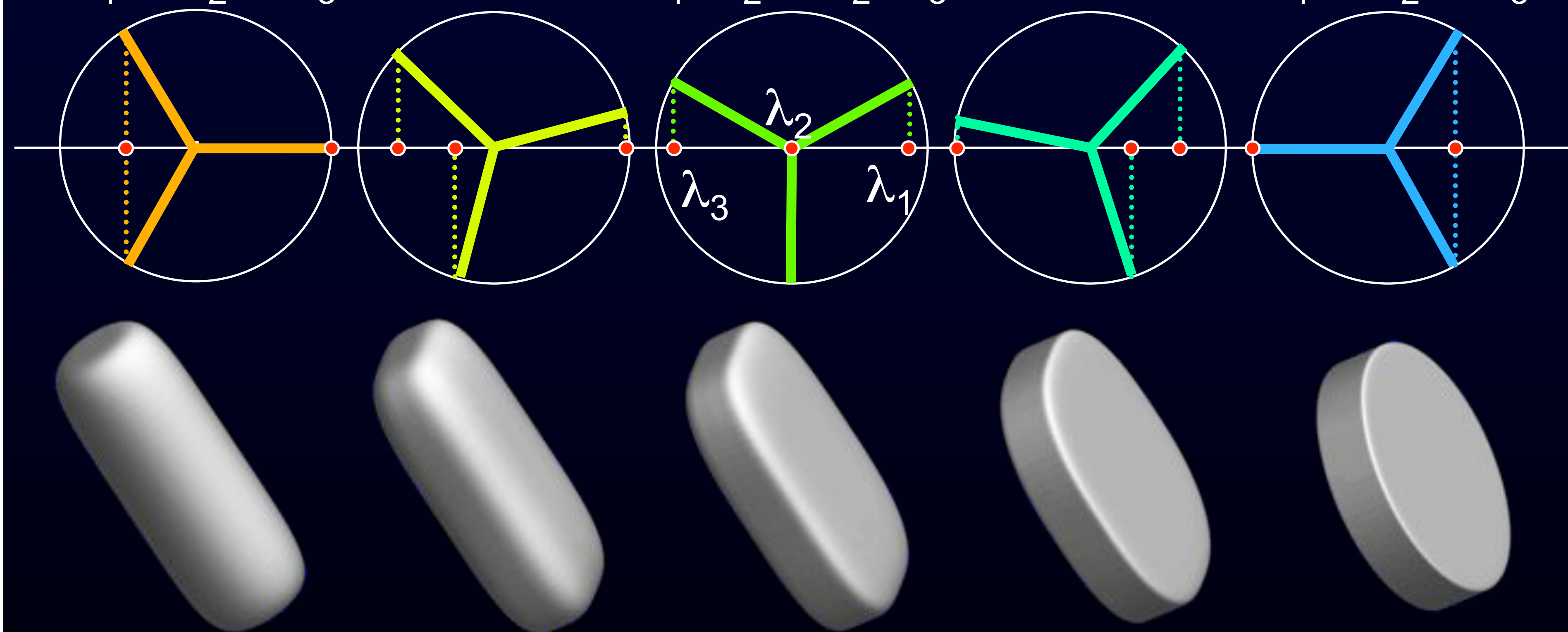
Eigenvalue "sorting", 2nd order isotropy

$\Theta = \cos^{-1}(\sqrt{2}\ \text{skew})/3 \;\Rightarrow\; \Theta \in [0, \pi/3] \;\Rightarrow\; \lambda_1 \geq \lambda_2 \geq \lambda_3$

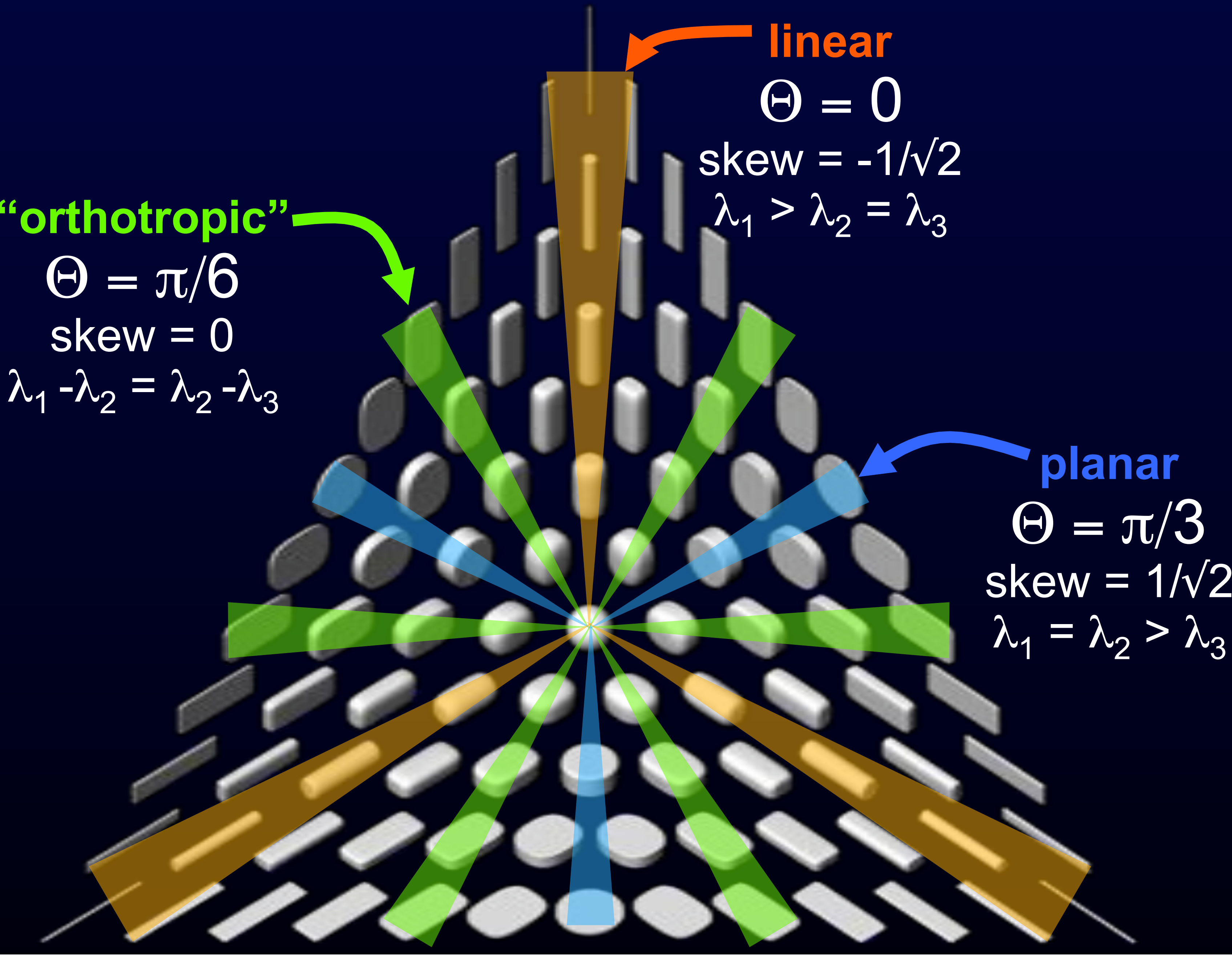
linear
$\Theta = 0$
skew = $-1/\sqrt{2}$
$\lambda_1 > \lambda_2 = \lambda_3$

"orthotropic"
$\Theta = \pi/6$
skew = 0
$\lambda_1 - \lambda_2 = \lambda_2 - \lambda_3$

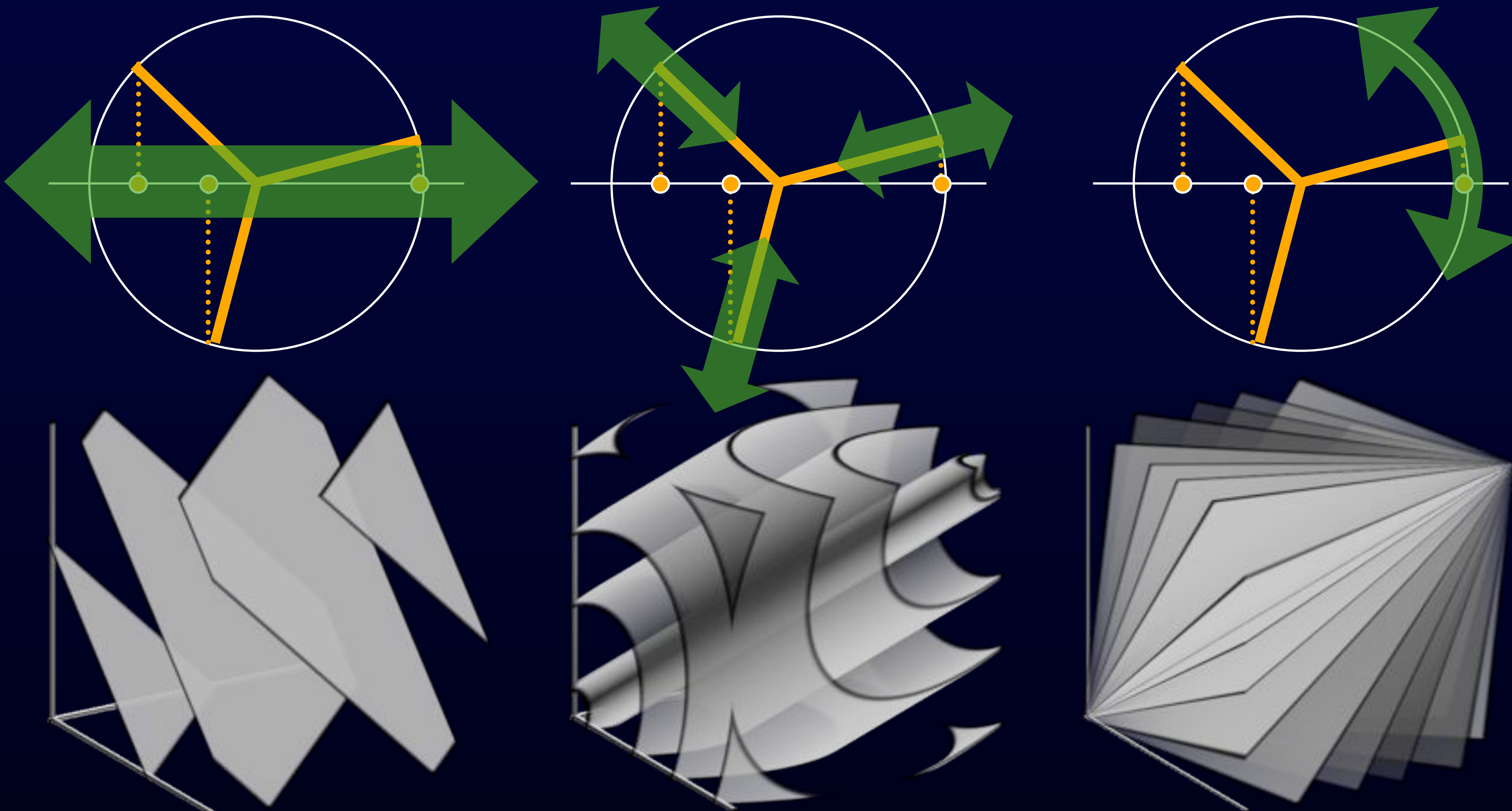
planar
$\Theta = \pi/3$
skew = $1/\sqrt{2}$
$\lambda_1 = \lambda_2 > \lambda_3$



Skew in context



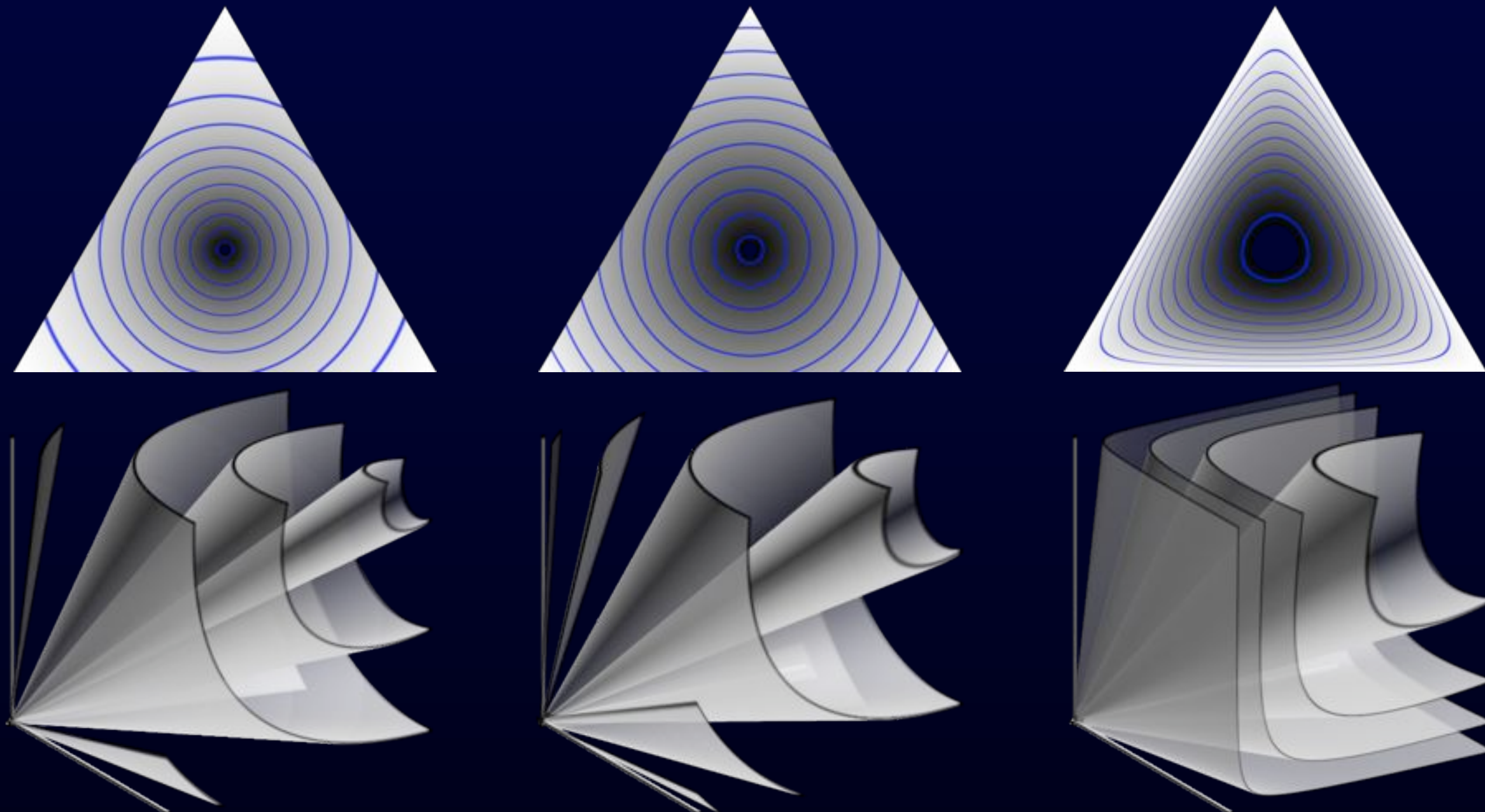
Orthogonal shape measures



$J_1/3 = \text{mean}(\lambda) = \mu_1$ $\quad$ $\text{var}(\lambda) = \mu_2$ $\quad$ $\text{skew}(\lambda) = \mu_3/\mu_2^{3/2}$

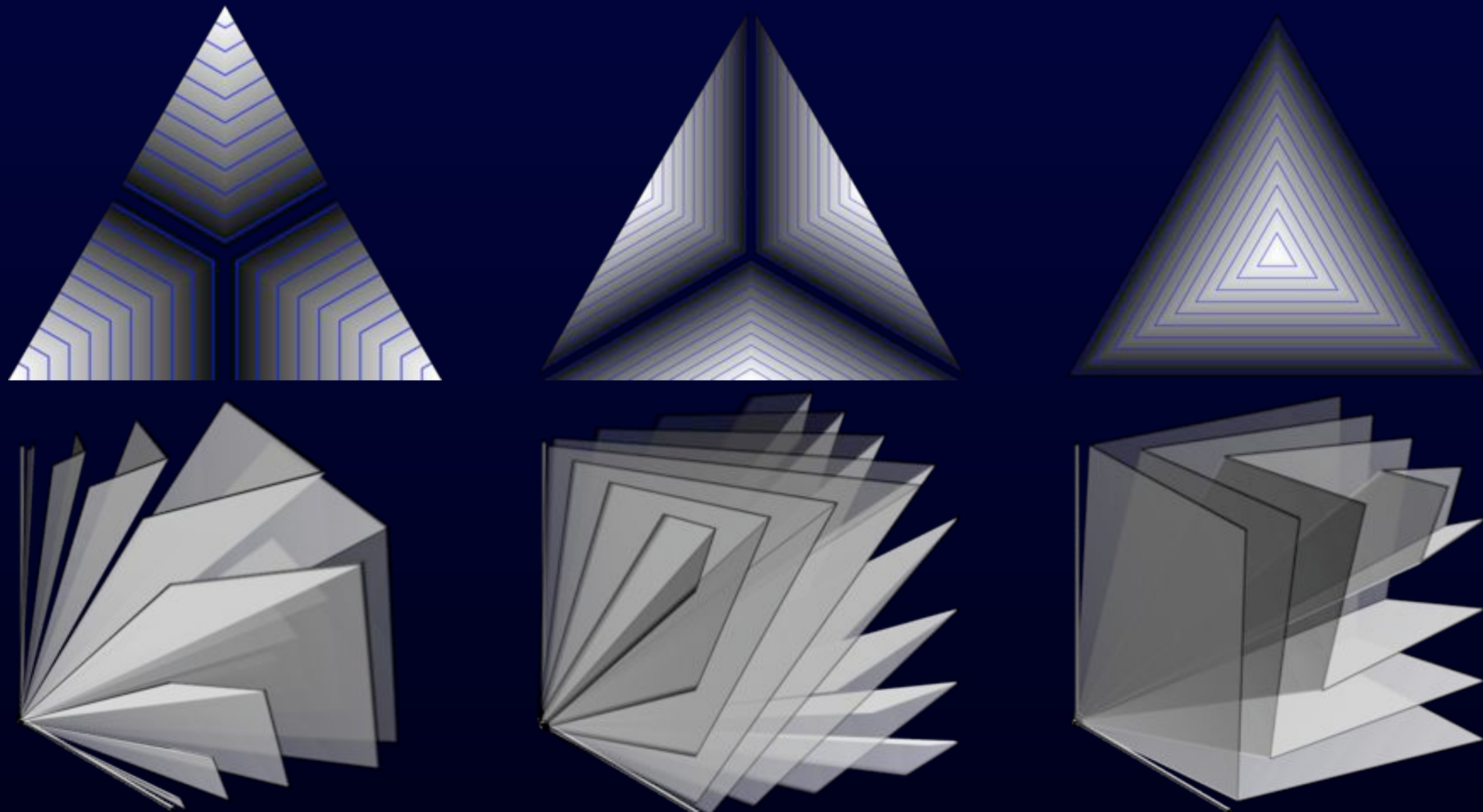
M Bahn, "Invariant and orthonormal scalar measures derived from MR DTI", JMR 141:68-77, 1999

Anisotropy metrics



$$FA = 3\sqrt{\frac{\mu_2}{2S}} \qquad RA = \frac{1}{\mu_1}\sqrt{\frac{\mu_2}{2}} \qquad 1\text{-}VR = 1 - \frac{\det(\mathbf{T})}{{\mu_1}^2}$$

Anisotropy metrics



$$C_L = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \qquad C_P = \frac{2(\lambda_2 - \lambda_3)}{\lambda_1 + \lambda_2 + \lambda_3} \qquad C_S = \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$$

Rough Outline

Glyphs for data inspection

Invariants for describing shape

Invariant gradients

- WRT position: Shading in rendering
- WRT tensor components: DOF of shape

Invariant gradients

If these are differentiable:

D(**p**) : tensor data as function of position

Q(**D**) : invariant as function of tensor

1) $\nabla_{\mathbf{p}} Q(\mathbf{D}(\mathbf{p}))$: Derivative WRT position: For visualization (volume rendering) : use chain rule

2) $\nabla_{\mathbf{D}} Q(\mathbf{D})$: Derivative WRT tensor components: For filtering/processing

Invariant gradients

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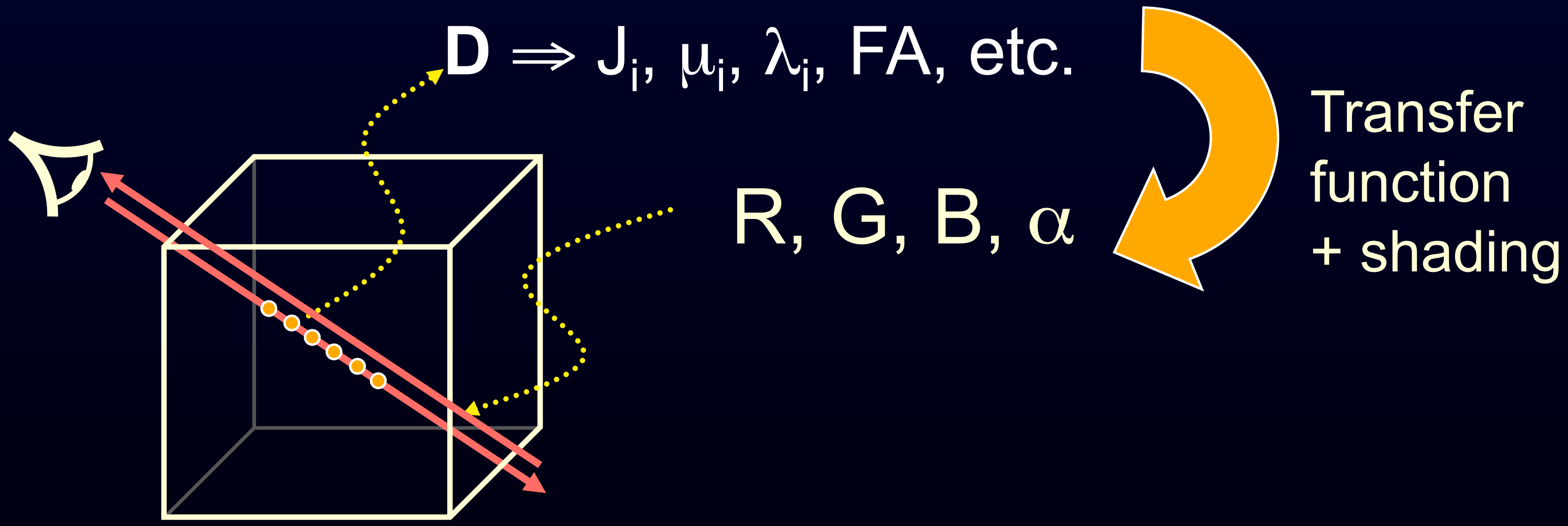
2) $\nabla_{\mathbf{D}}$Q(**D**) : Derivative WRT tensor components: For filtering/processing

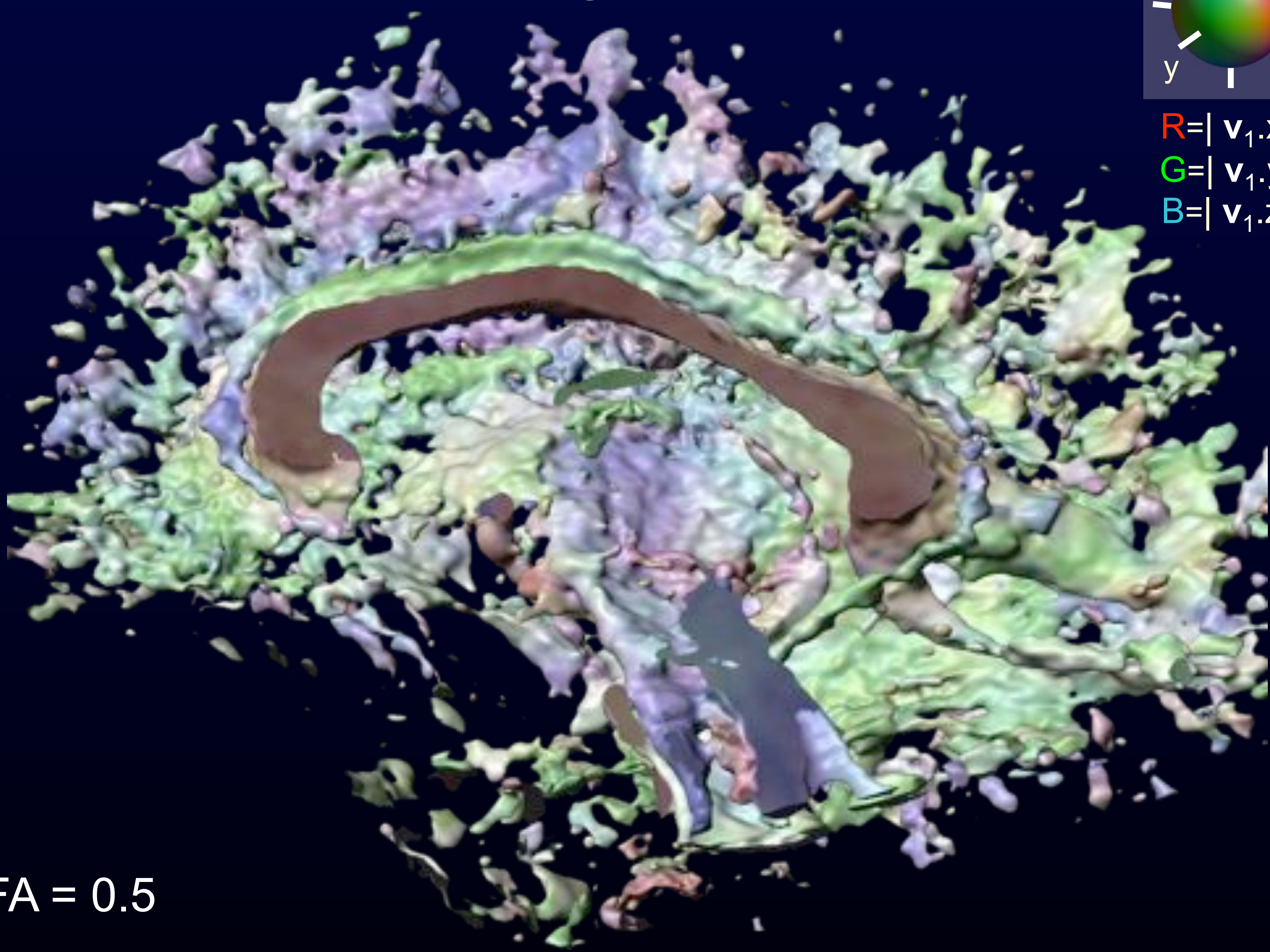
Direct Volume Rendering

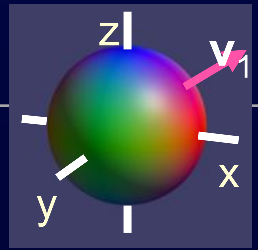
Simple algorithm

- Cast rays through volume
- Measure tensor, tensor properties
- Assign colors and opacities
- Modulate colors with shading
- Composite with “over” operator



Volume Rendering Results

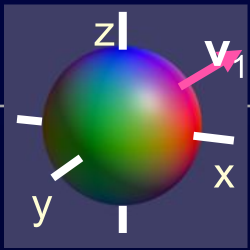




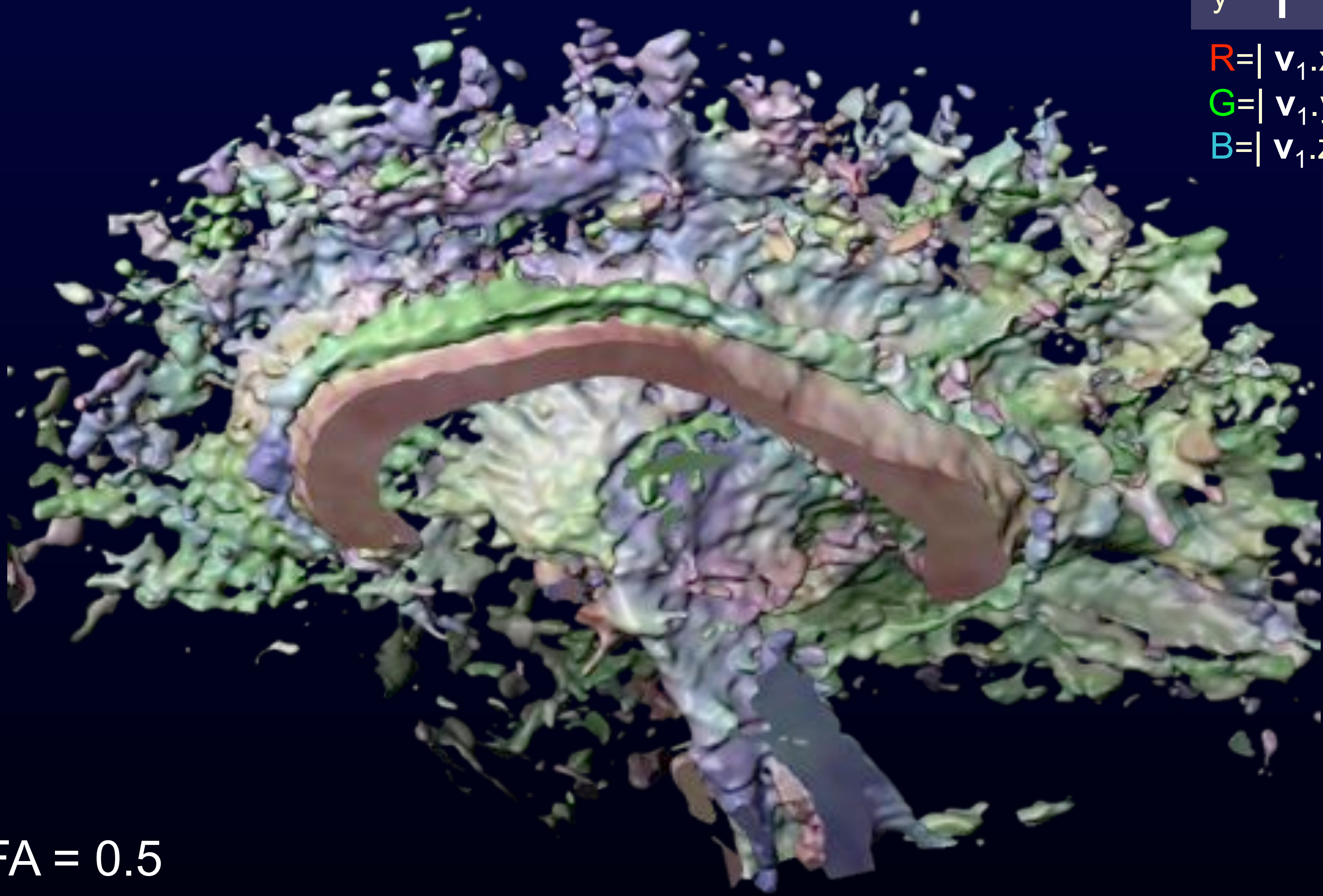
R=| $\mathbf{v}_1$.x |
G=| $\mathbf{v}_1$.y |
B=| $\mathbf{v}_1$.z |

FA = 0.5

Volume Rendering Results

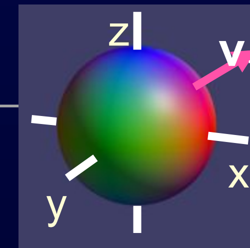


R=| $\mathbf{v}_1$.x |
G=| $\mathbf{v}_1$.y |
B=| $\mathbf{v}_1$.z |

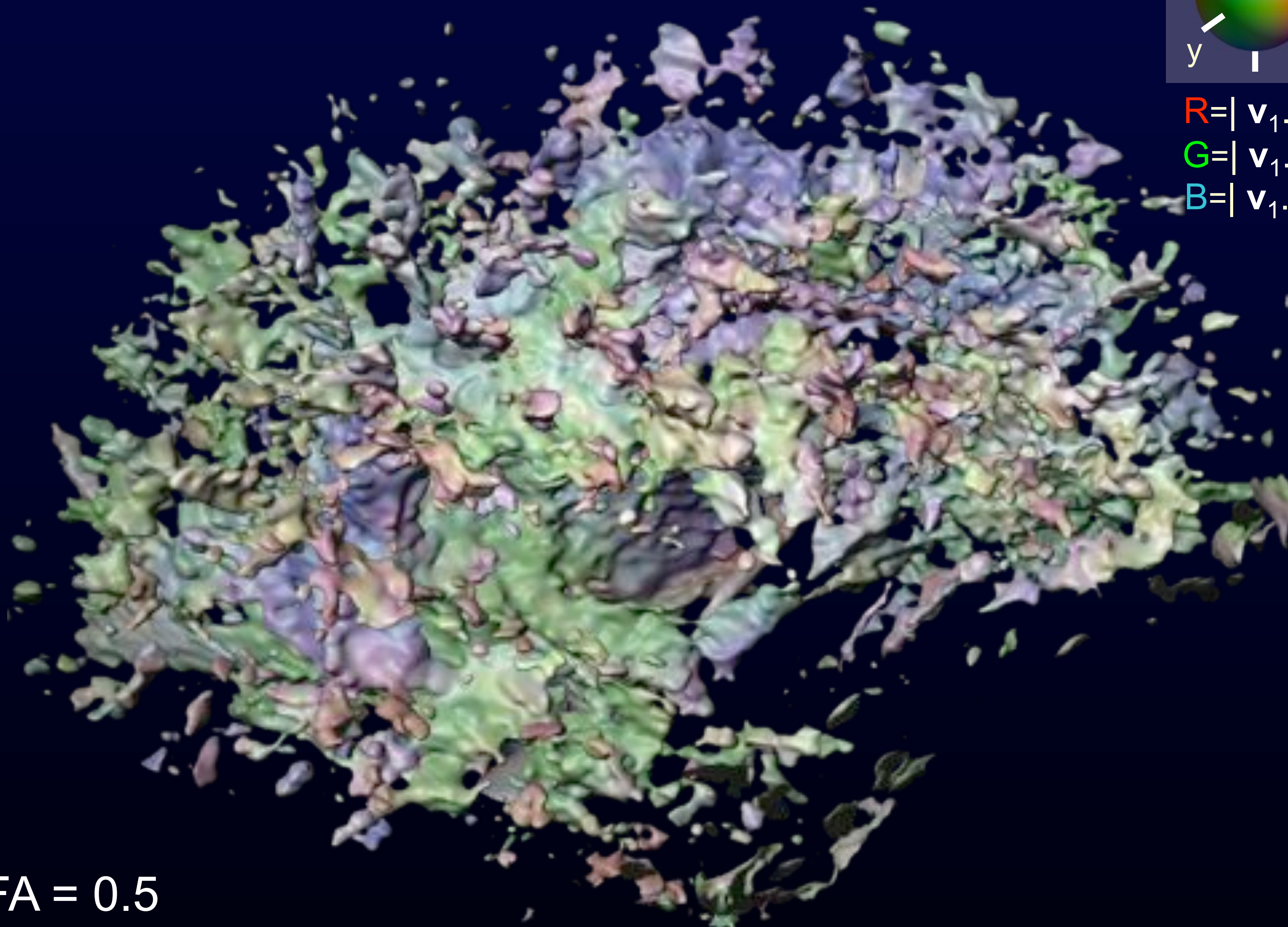


FA = 0.5

Volume Rendering Results

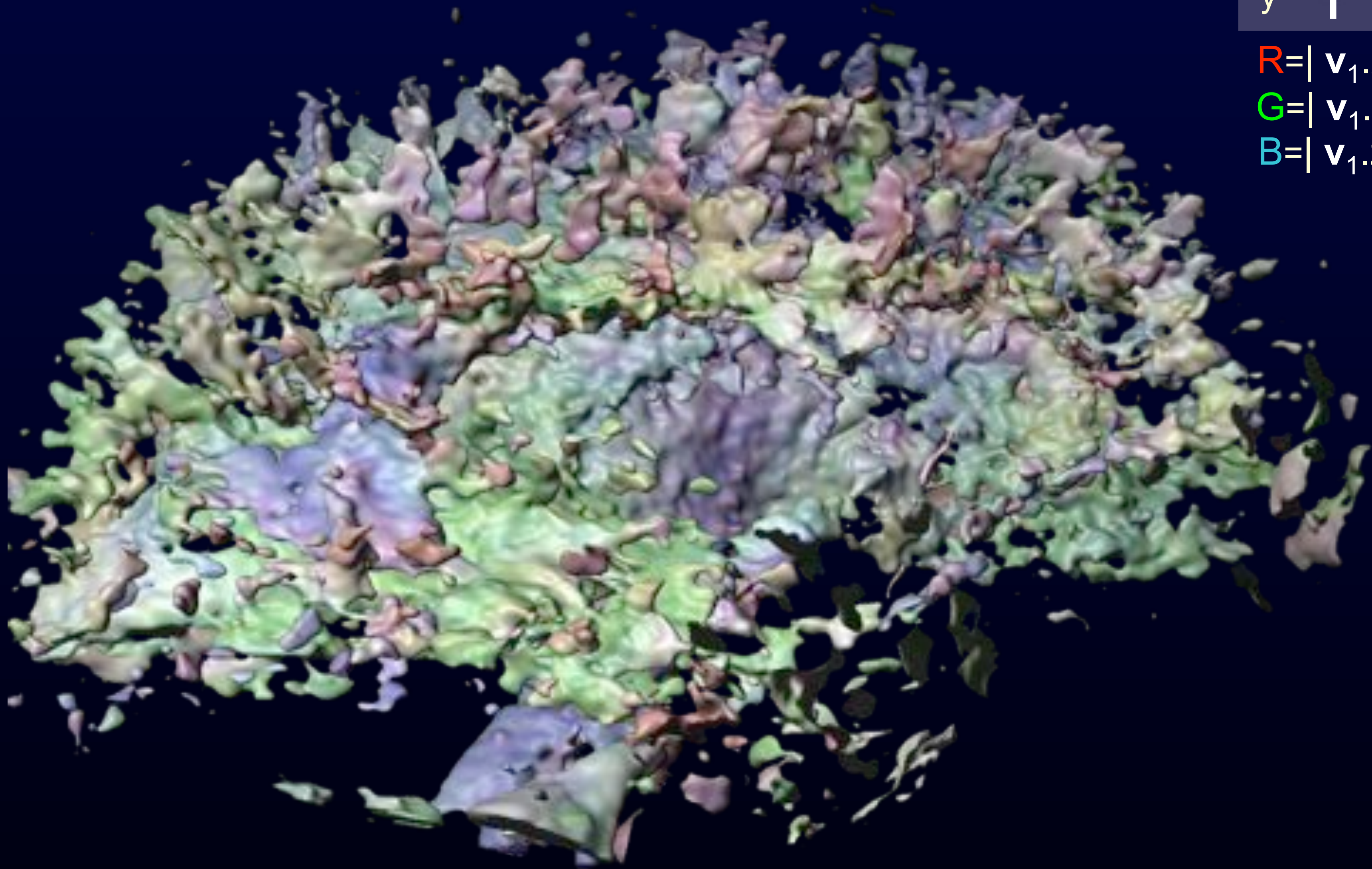


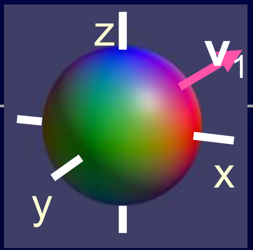
R=| $\mathbf{v}_1$.x |
G=| $\mathbf{v}_1$.y |
B=| $\mathbf{v}_1$.z |



FA = 0.5

Volume Rendering Results

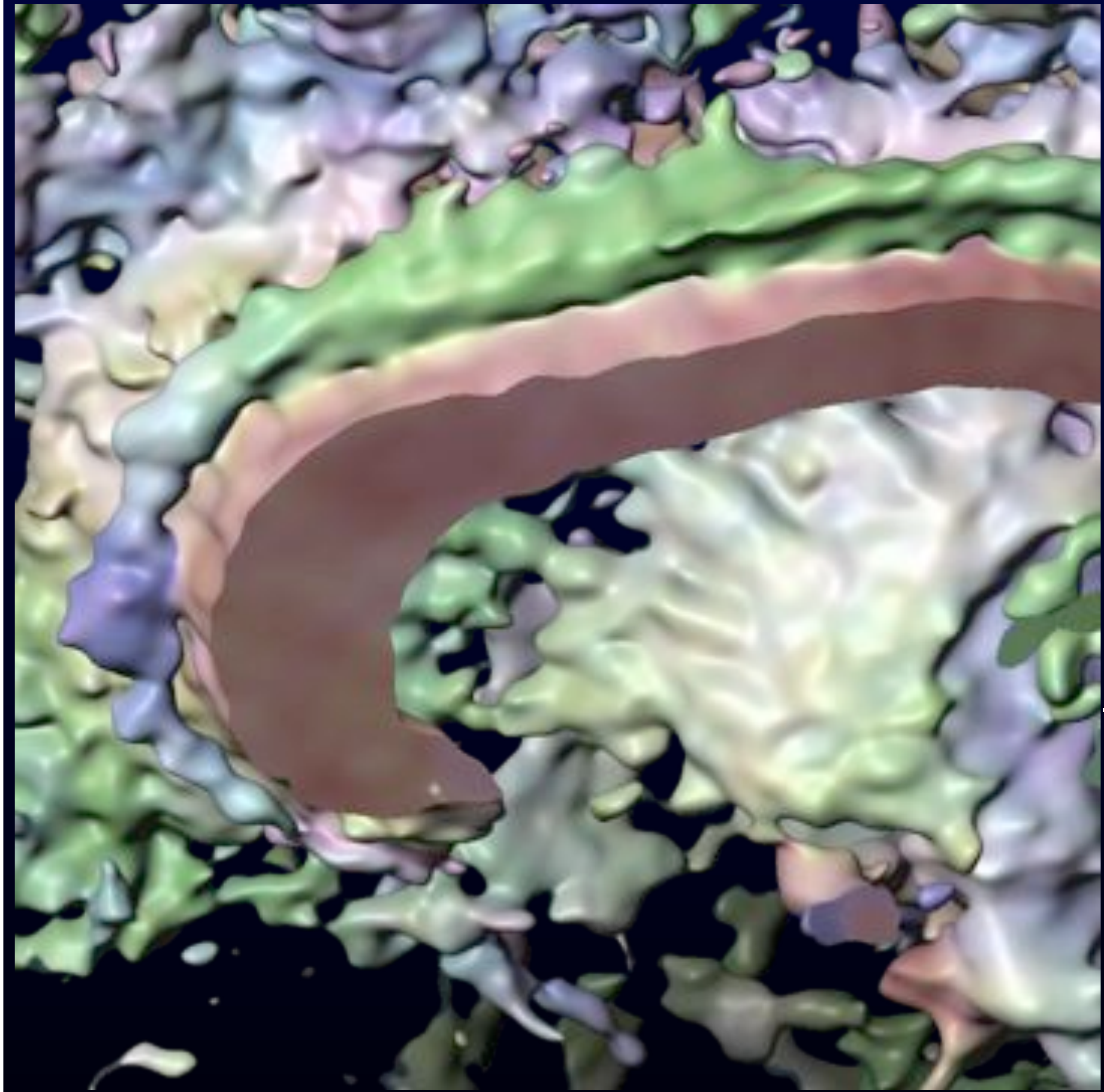




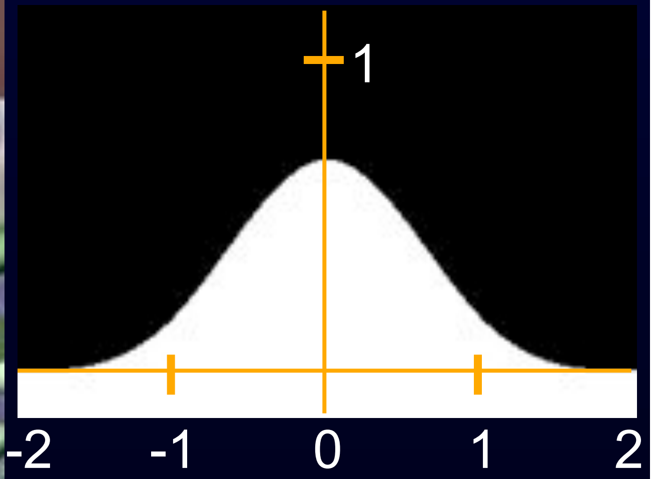
R=| $\mathbf{v}_1$.x |
G=| $\mathbf{v}_1$.y |
B=| $\mathbf{v}_1$.z |

FA = 0.5

Visualizing kernel differences

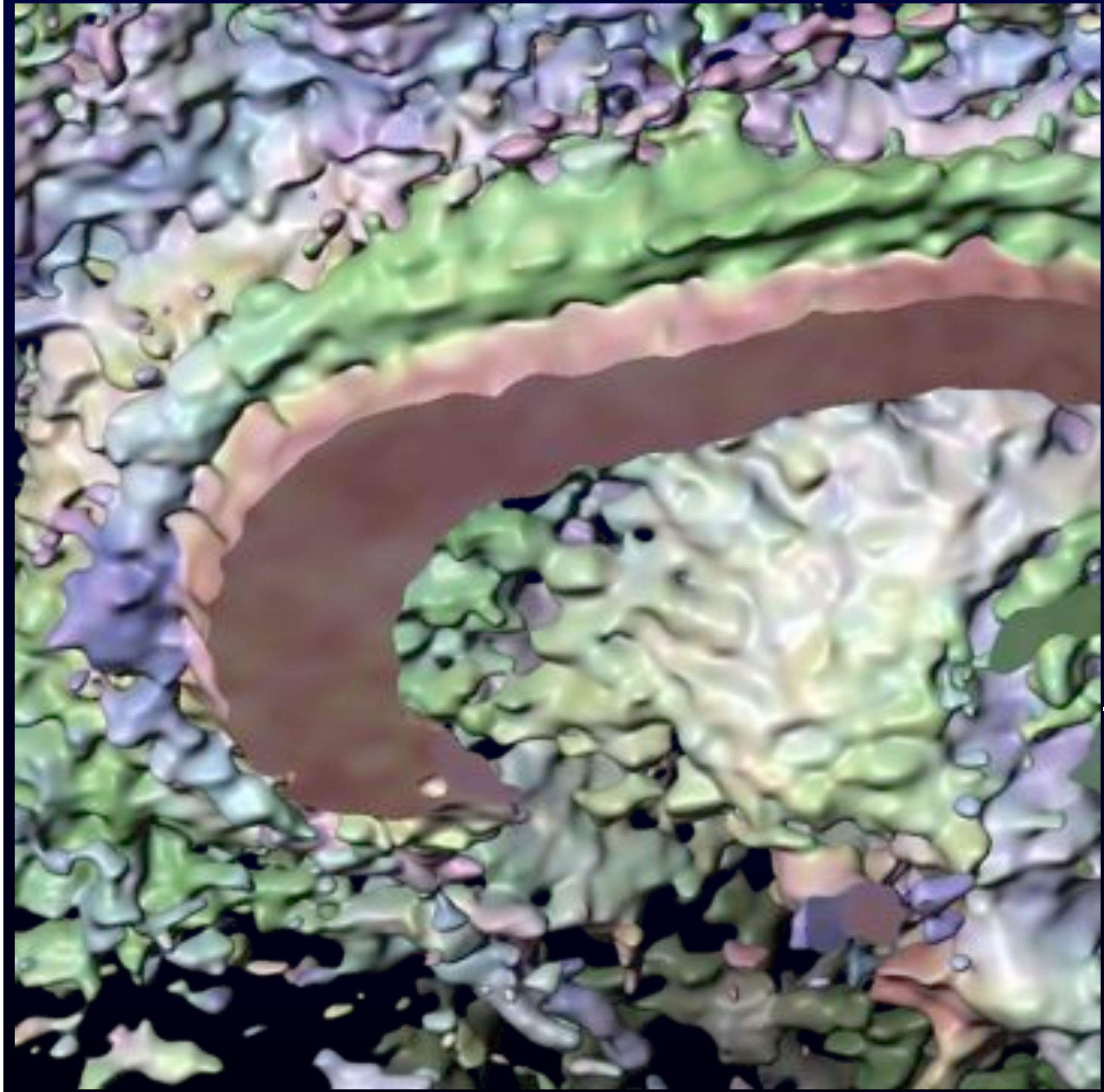


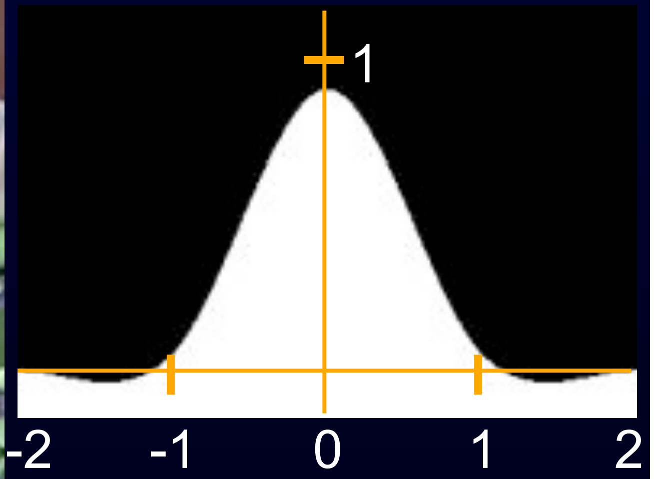
Mitchell-Netravali BC-splines: Simple, tunable, always C^1



(B,C) = (1,0) Uniform cubic B-spline, also C^2

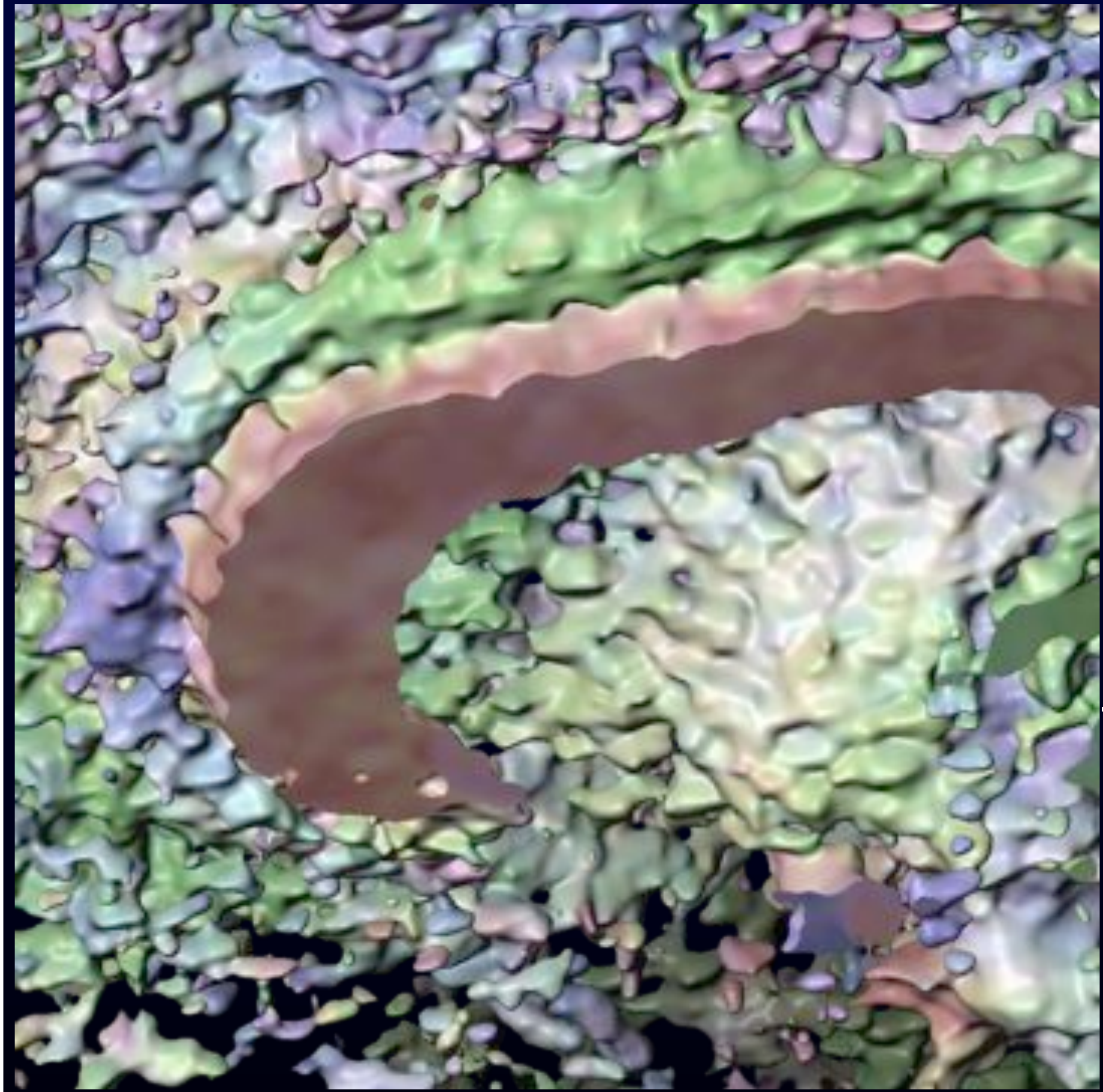
Visualizing kernel differences

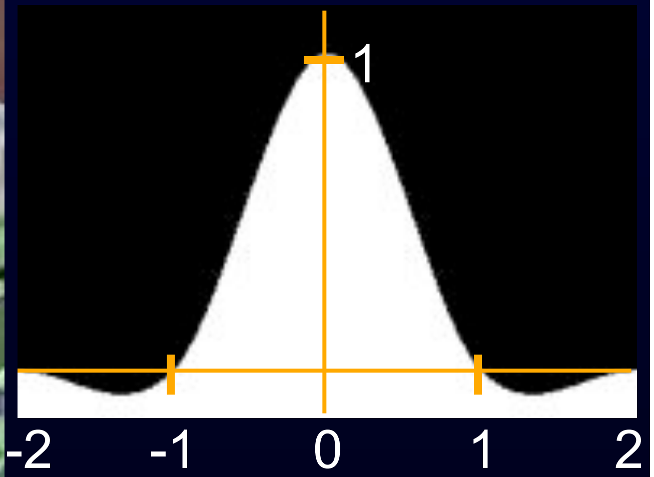




(B,C) = (1/3,1/3)
Blurs a little

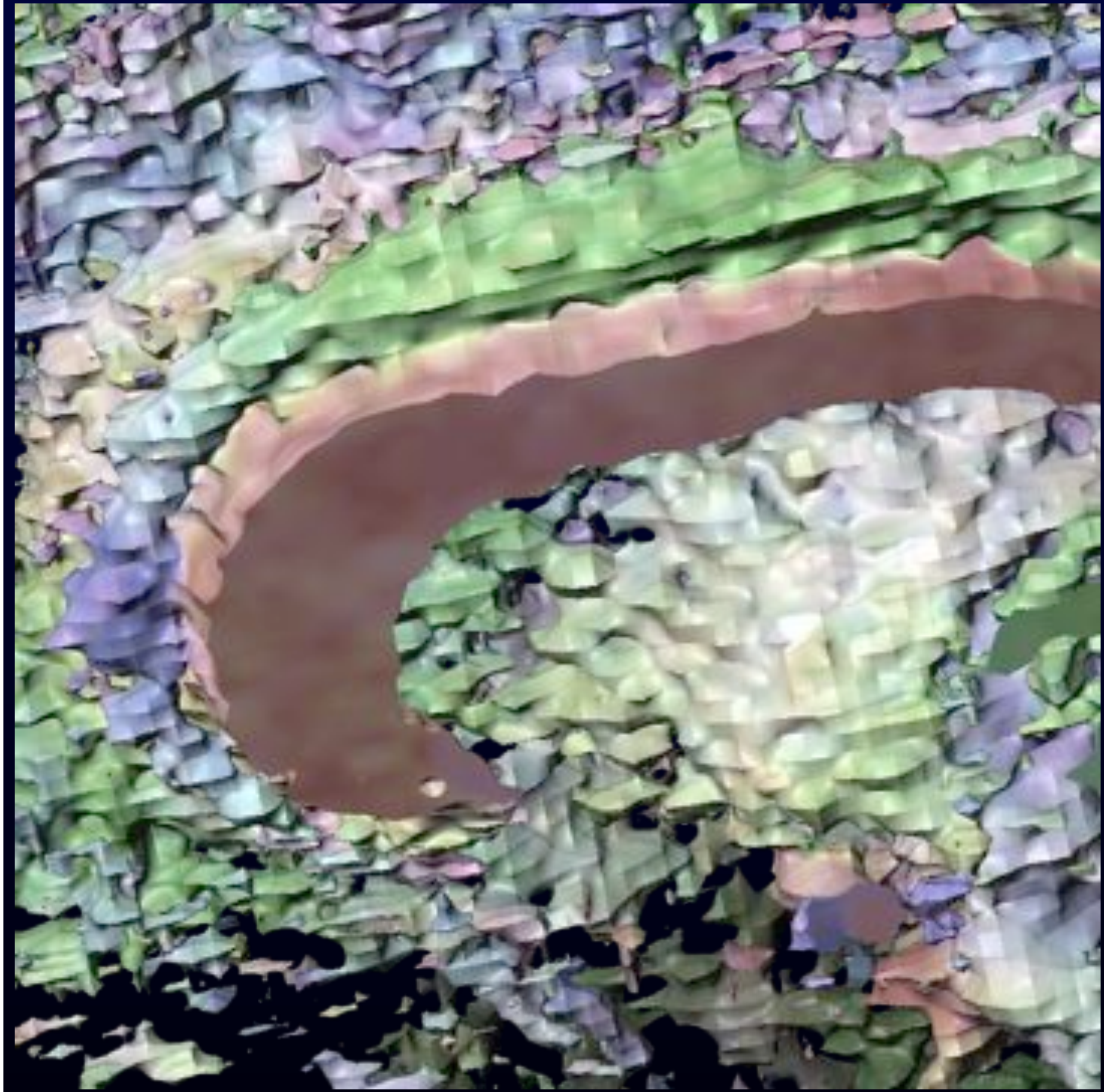
Visualizing kernel differences

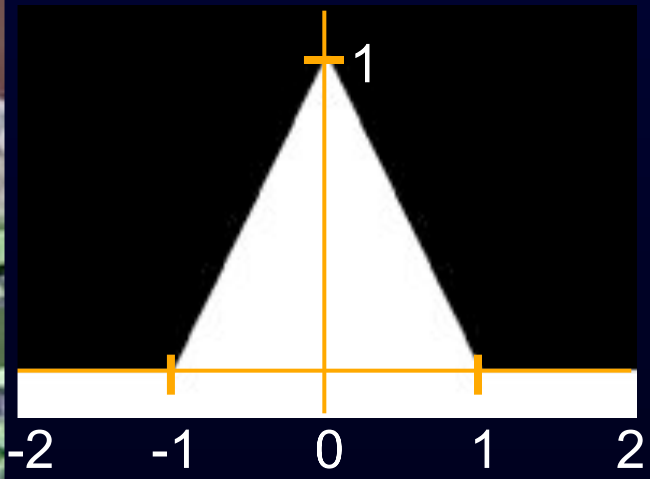




(B,C) = (0,1/2)
Catmull-Rom
Interpolates

Visualizing kernel differences

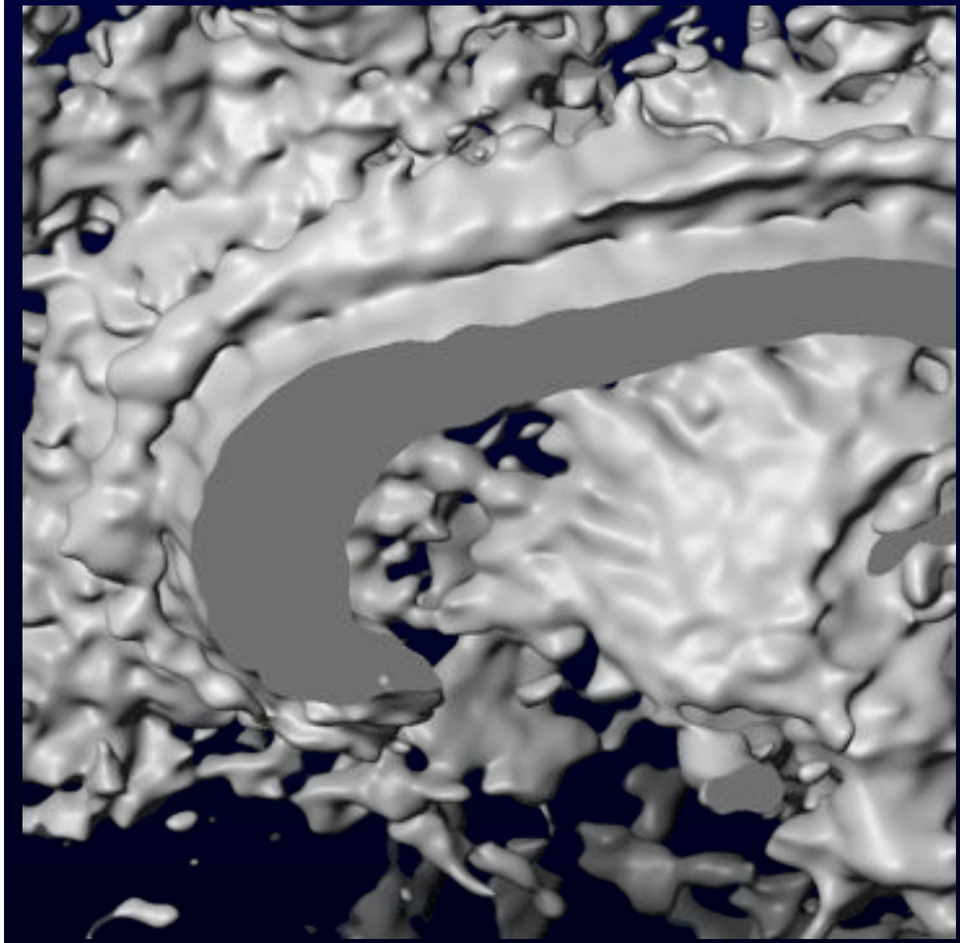




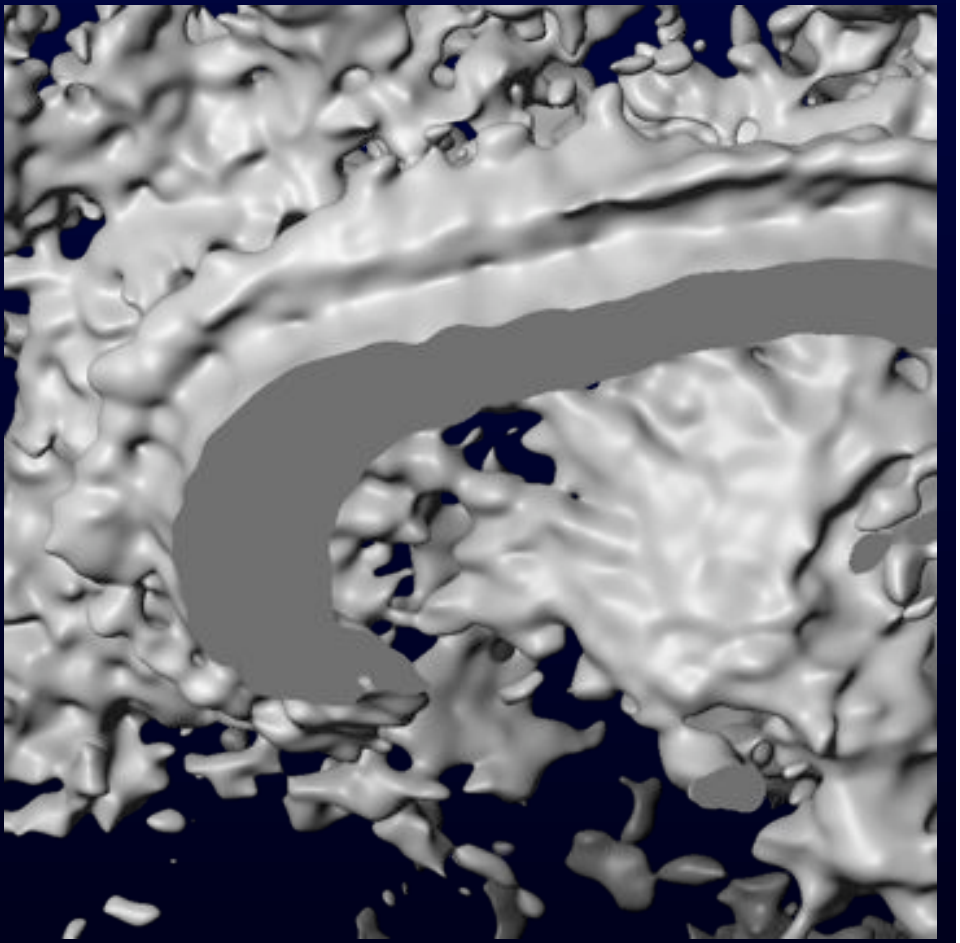
Linear : Not C^1

⇒ nasty edges, but can see each sample

Reconstruction+invariants don't commute

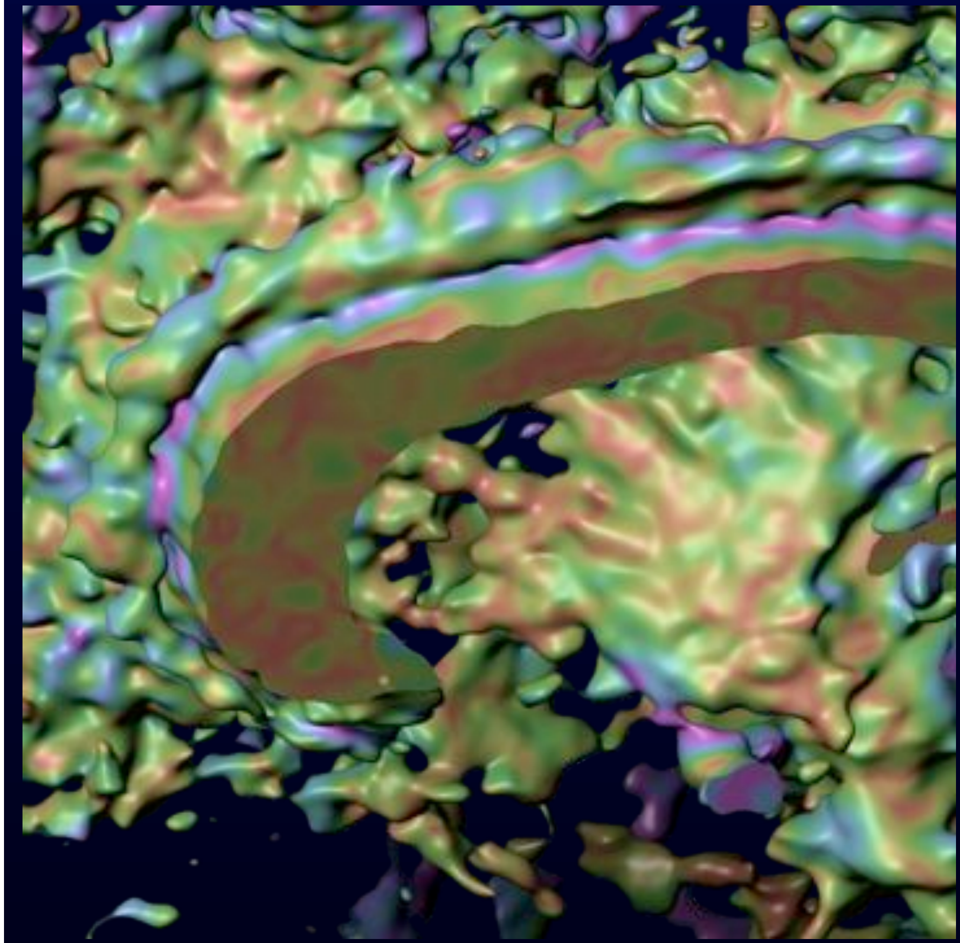


Reconstruct tensors, then Calculate FA

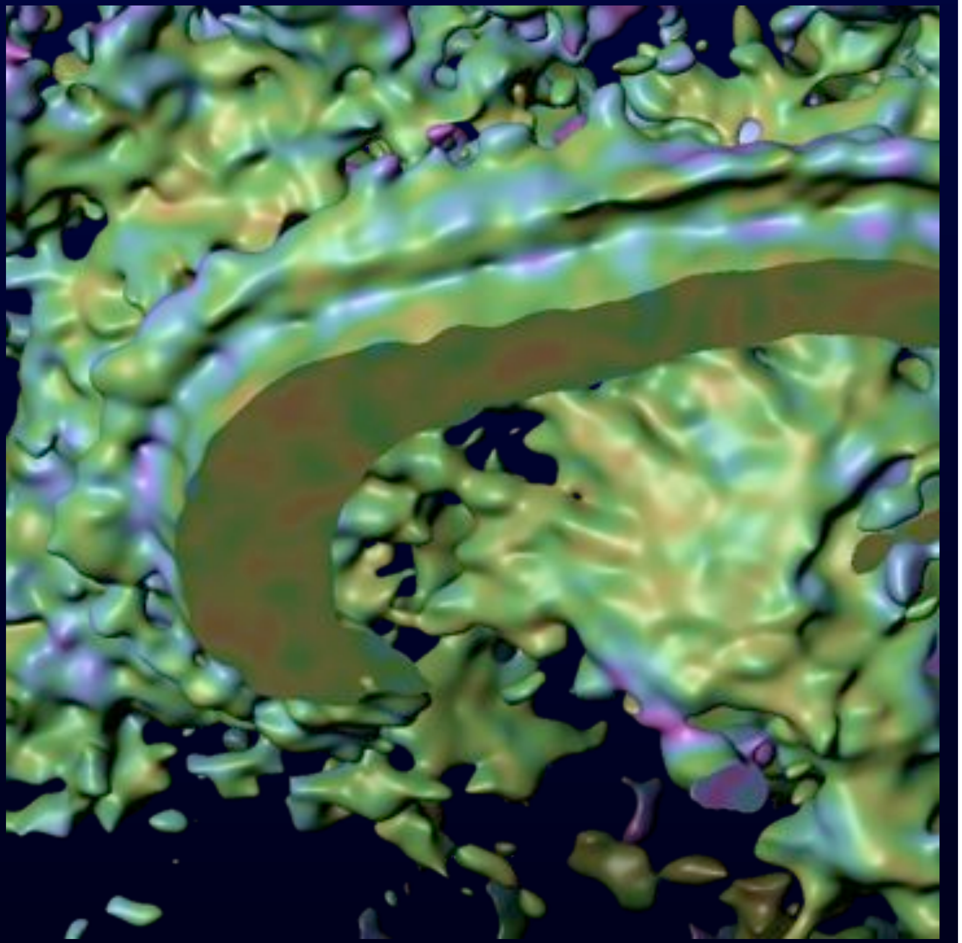


Calculate FA, then Reconstruct FAs

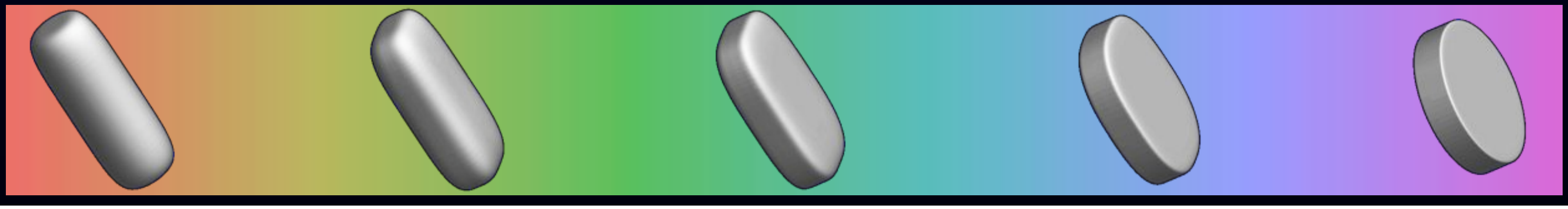
Reconstruction+invariants don't commute



Reconstruct tensors, then Calculate FA and Skew



Calculate FA and Skew, then Reconstruct FAs and Skews



Invariant gradients

If these are differentiable:

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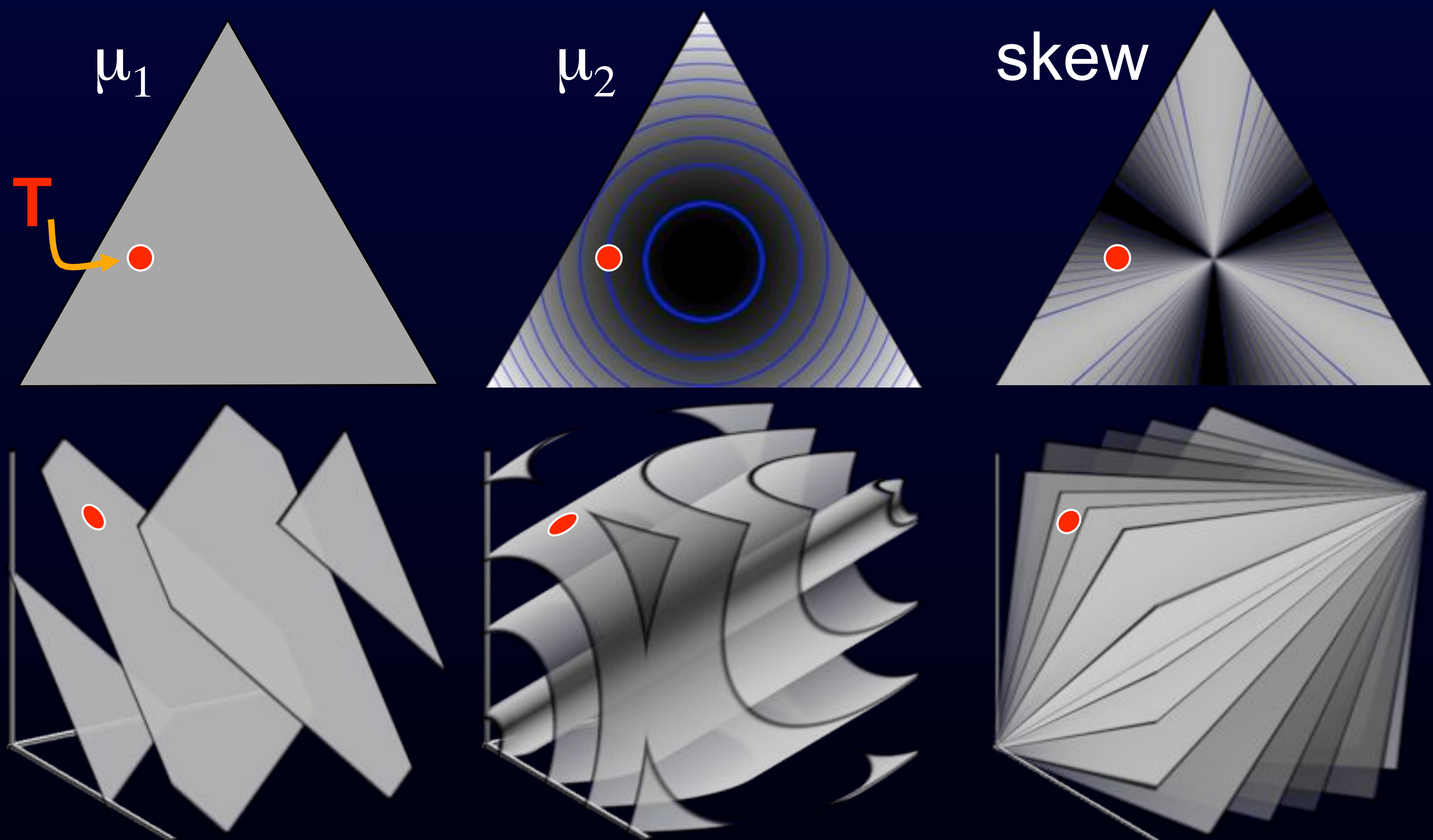
Q(**D**) : invariant as function of tensor

1) $\nabla_{\mathbf{p}} Q(\mathbf{D}(\mathbf{p}))$: Derivative WRT position: For visualization (volume rendering) : use chain rule

2) $\nabla_{\mathbf{D}} Q(\mathbf{D})$: Derivative WRT tensor components: For filtering/processing

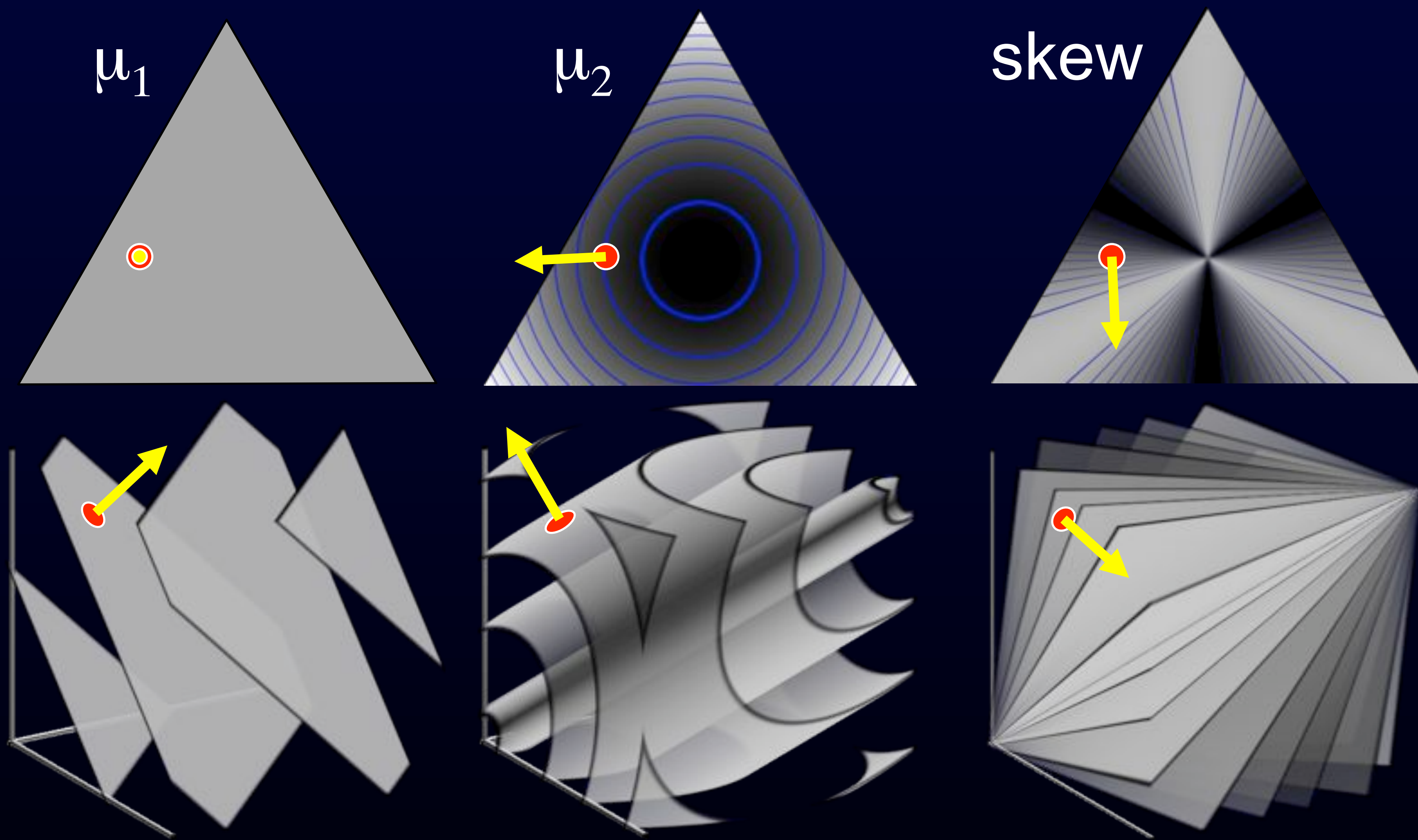
Orthonormal invariant gradients

If you're at tensor **T**, how do you change …



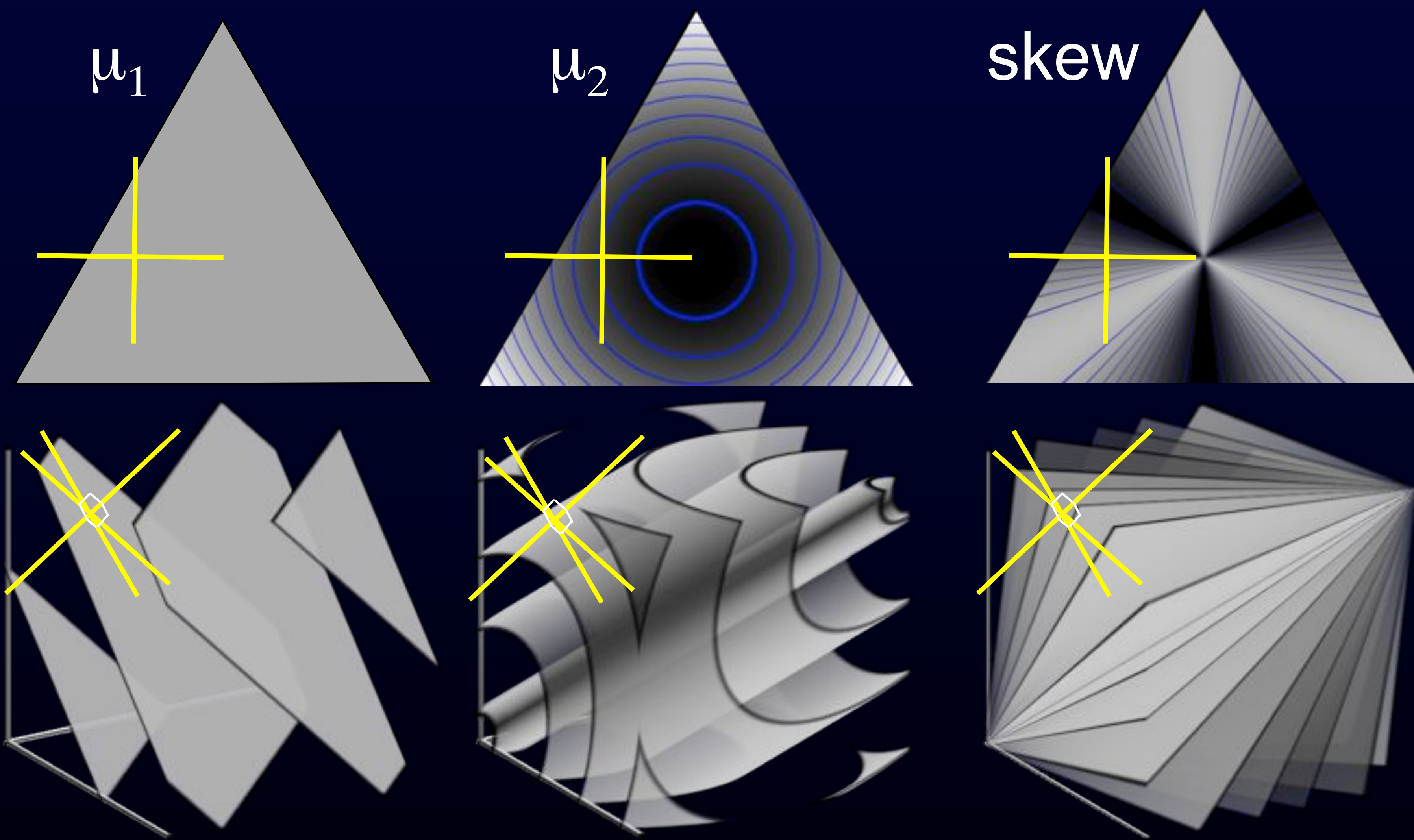
Orthonormal invariant gradients

Follow the **tensor-valued** gradients

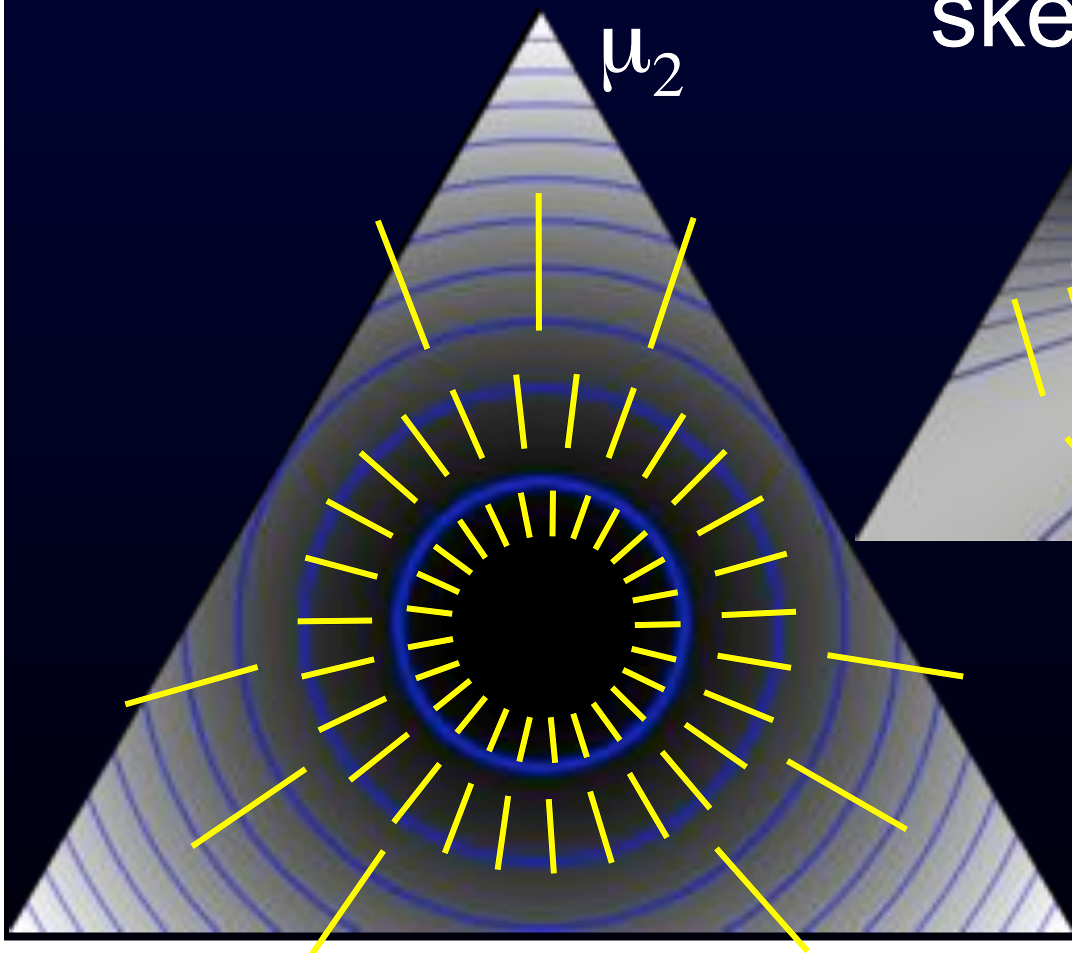


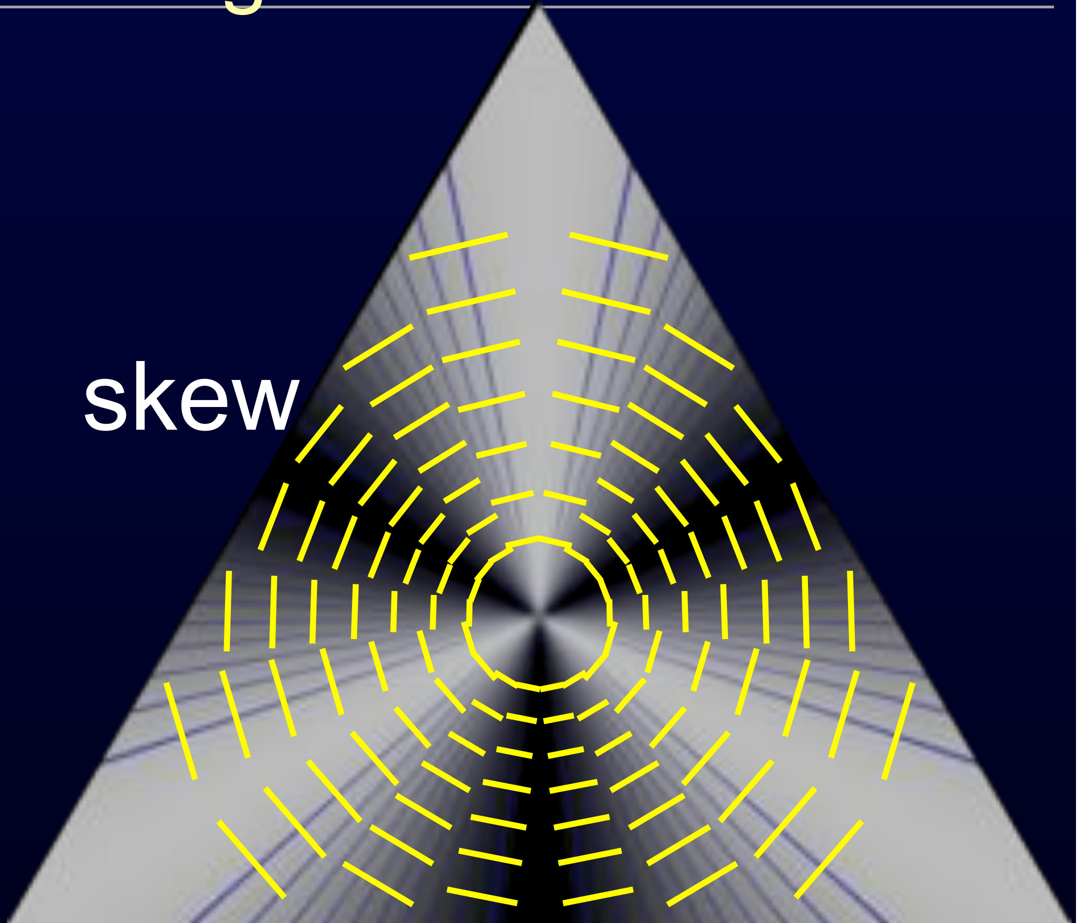
Orthonormal invariant gradients

Local coordinate system (after M Bahn)



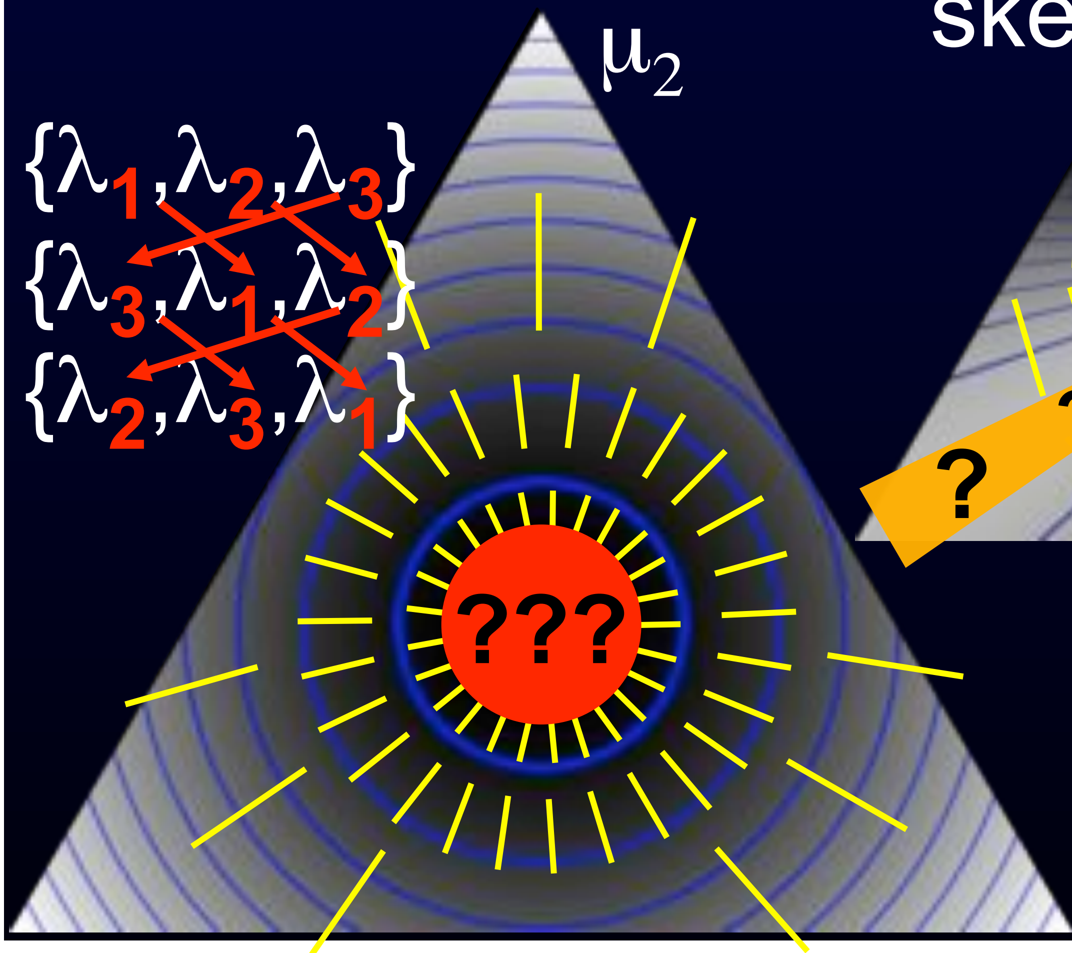
Orthonormal invariant gradients

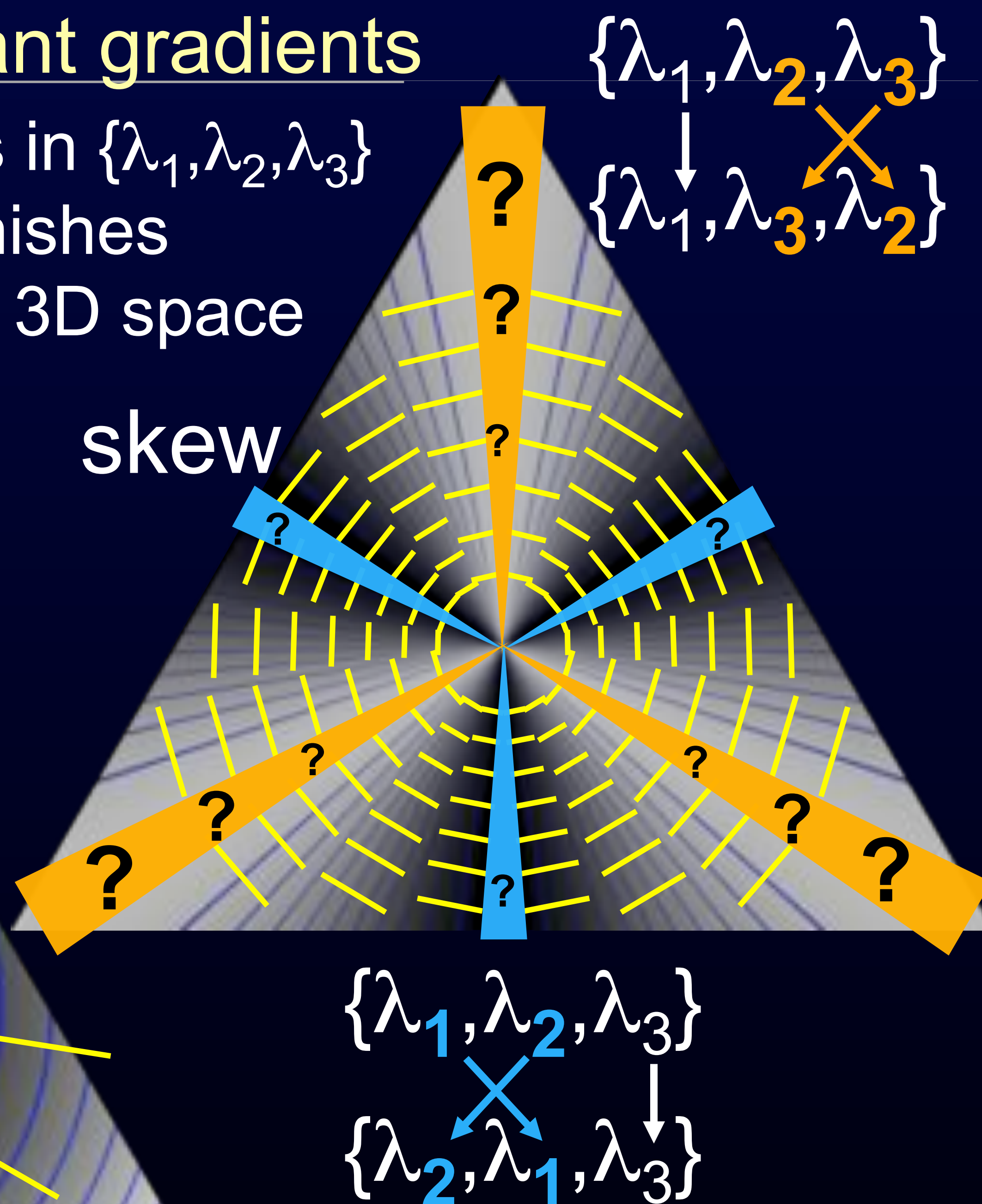




Failure of invariant gradients

Permutation symmetries in $\{\lambda_1,\lambda_2,\lambda_3\}$
$\Rightarrow$ invariant gradient vanishes
though shape is **always** 3D space



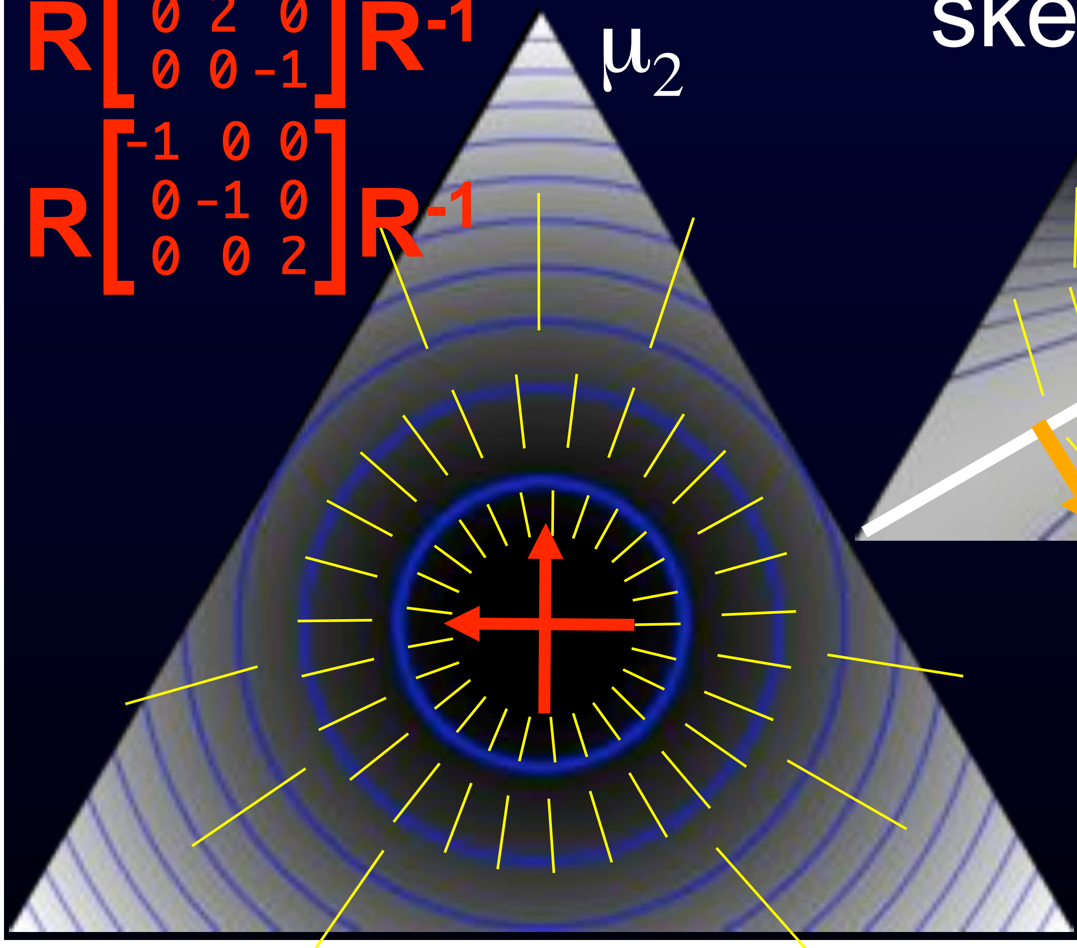


What to do: break symmetry

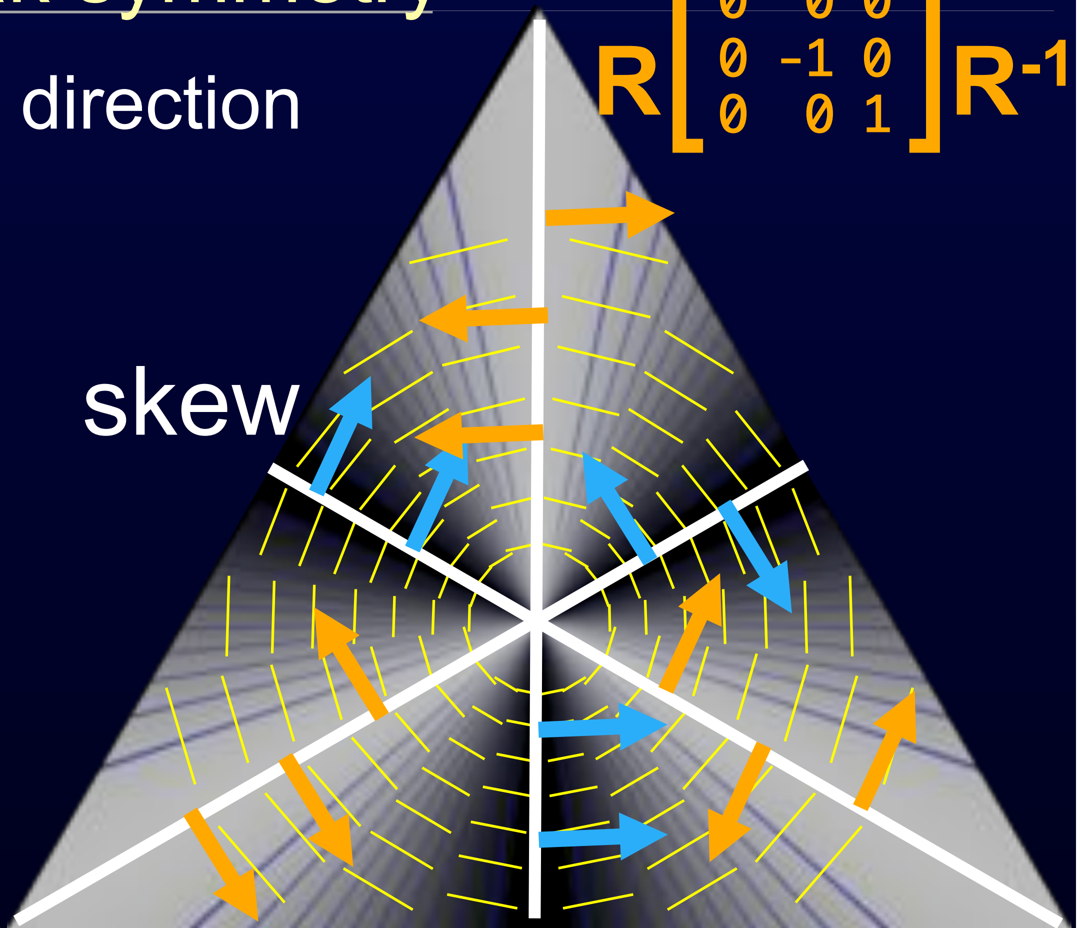
Diagonalize, then pick a direction in $(\lambda_1,\lambda_2,\lambda_3)$ space

$$R\begin{bmatrix}-1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & -1\end{bmatrix}R^{-1}$$

$$R\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 2\end{bmatrix}R^{-1}$$



$$R\begin{bmatrix}0 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}R^{-1}$$



$$R\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}R^{-1}$$

Issues

- Can't reliably pick same sign of direction (so don't depend on it)
- 2nd-order isotropy not too bad
- 3rd-order isotropy ugly: no skew direction (can't define how to change hue of a gray color)
- have to smoothly de-emphasize skew direction as a function of low μ_2

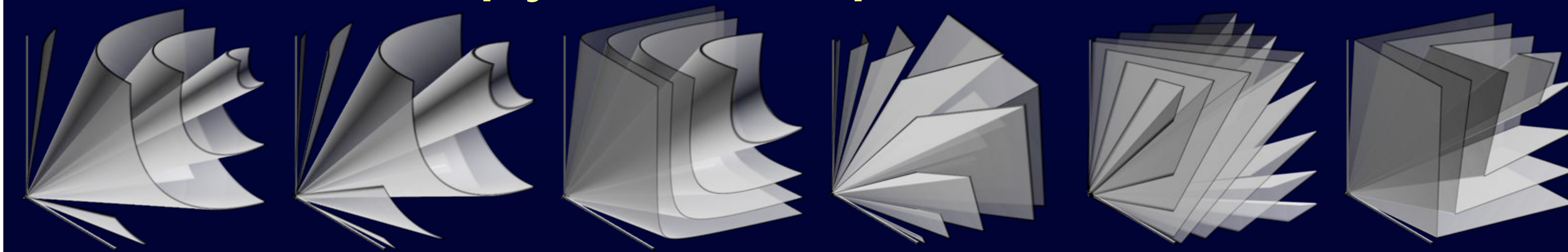
Ideas

- Data inspection important
- Integration of inspection, visualization methods
- Intuitive basis for shape. Orientation? Noise?
- Integration of processing and visualization

All software online:

http://teem.sourceforge.net

Anisotropy metric questions



FA RA 1-VR C_L C_P C_S

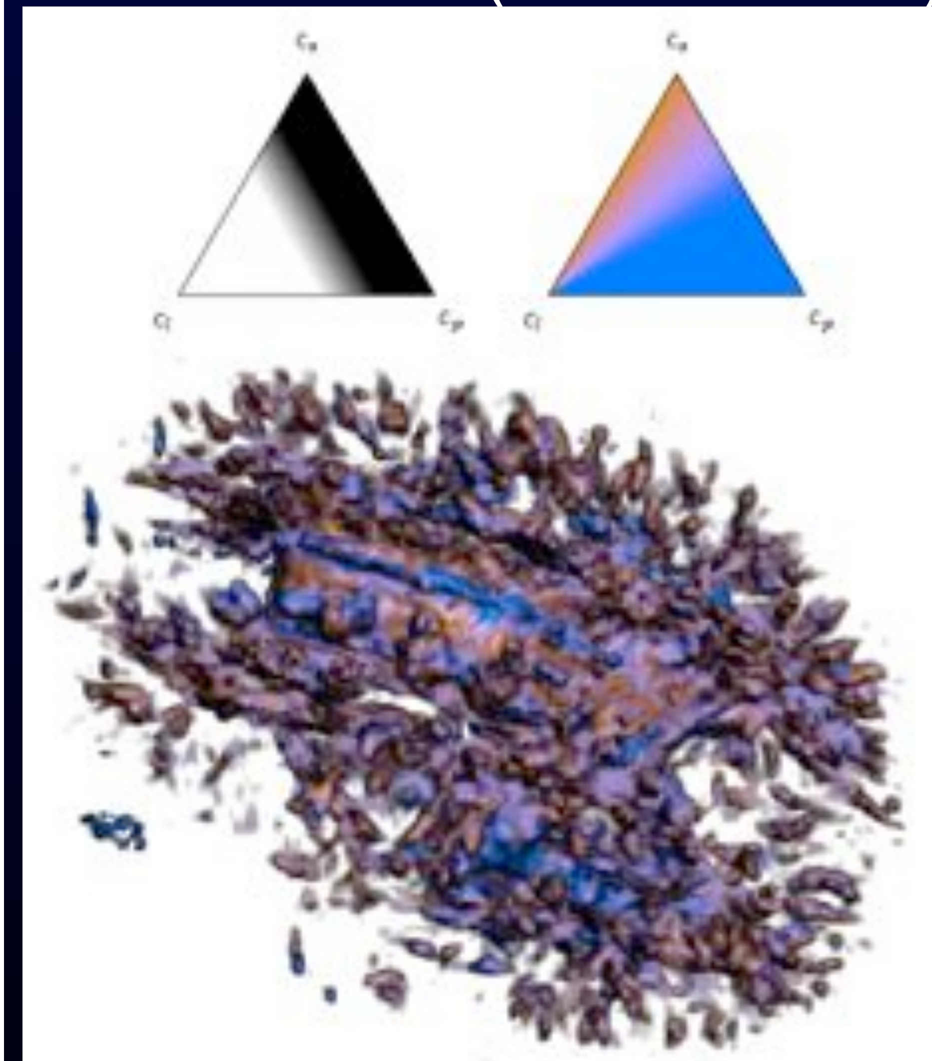
- How exactly does noise sensitivity arise?
- Intuitive way to trace noise through this?

$$\text{DWI} \rightarrow \mathbf{D}_{ij} \rightarrow J_{1,2,3} \begin{cases} \nearrow \text{FA, RA, 1-VR} \\ \searrow \mu_1, \mu_2, \text{skew} \rightarrow \lambda_{1,2,3} \rightarrow C_{L,P,S} \end{cases}$$

- Does differentiability of some metrics (FA) and not others (C_L) matter for noise sensitivity?
- In what contexts is differentiability an interesting or important property of an invariant?

Volume Rendering Improvements

Earlier work (IEEE Vis '99):



- **Trilinear** interpolation of D
- Shading by interpolation of pre-computed gradients of pre-computed opacity
- experience/enthusiasm < ε

New work:

- Arbitrary kernels
- Transfer functions of differentiable invariants
- Shading by analytic spatial gradients of invariants