THE UNIVERSITY OF CHICAGO COMPUTER SCIENCE

Computation Institute

Tensor Field Features

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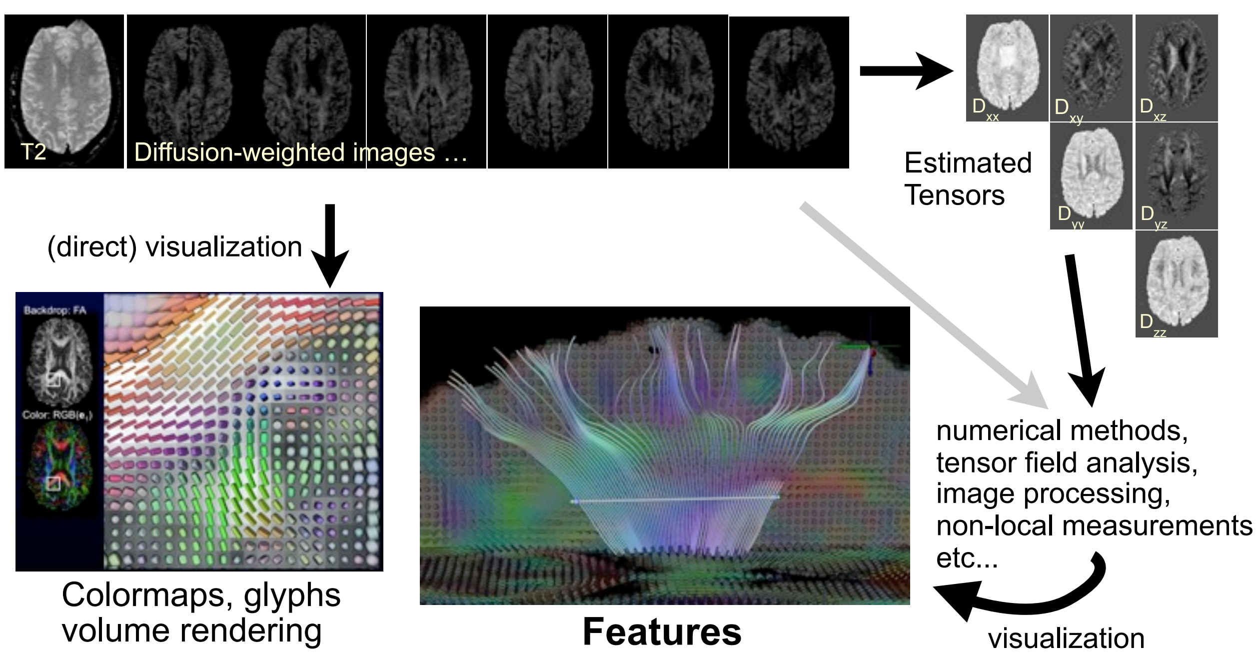
What is a feature: in vis/computational context

Structure computed from field, processed, visualized

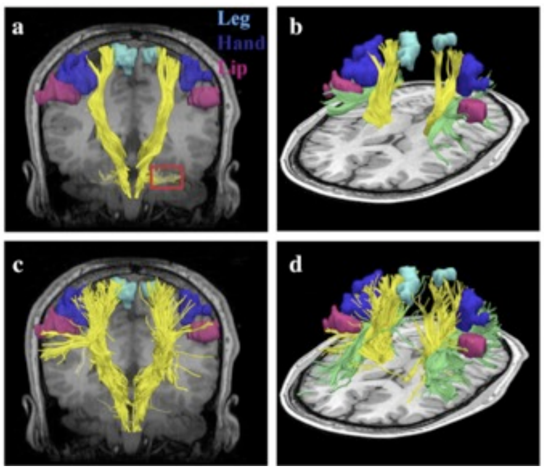
Contrast with direct methods (glyphs, colormaps)



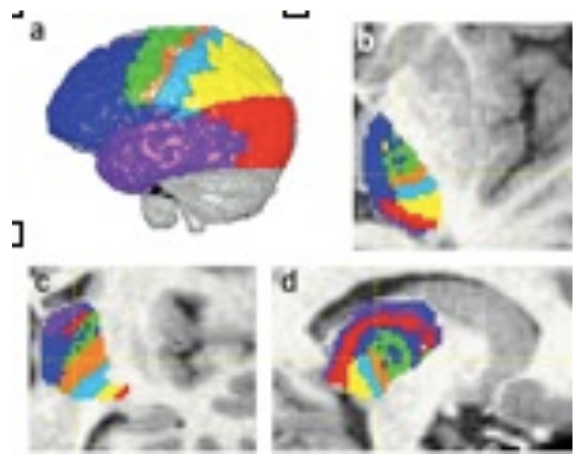
What is a feature: in a biomedical context

Geometric model of anatomic structure important for study motivating the data acq.

- Quantitative measurements of underlying biology
- Pictures (visualization) may not be central



Qazi 2009 NI 47:T98-T106



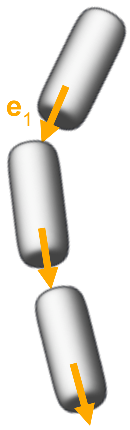
Behrens 2003 Nature Neuroscience, 6:750-757

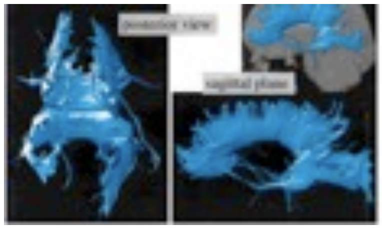
Outline

- Feature definitions and considerations
- Tractography and its clustering
  - Tractography methods
  - Clustering metrics and methods
  - Cluster representation and display
- Segmentation from Tensor Distances
- Differential Structure: Edges, Creases
- Topological and Lagrangian Structure
- Discussion

Tractography (deterministic)

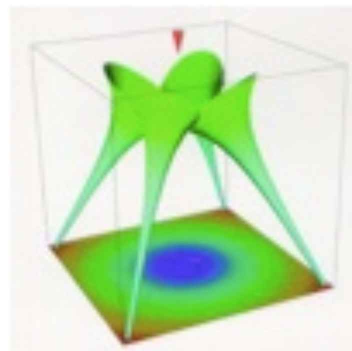
Standard: Streamline integration of principle eigenvector





Basser **1998** ISMRM, **2000** MRM 44:625-63

Stream tubes, Zhang **2003** TVCG 9:454-463



Delmarcelle, T. & Hesselink, L. Visualizing Second-order Tensor Fields with Hyper Streamlines. IEEE Computer Graphics and Applications, **1993**, 13, 25-33

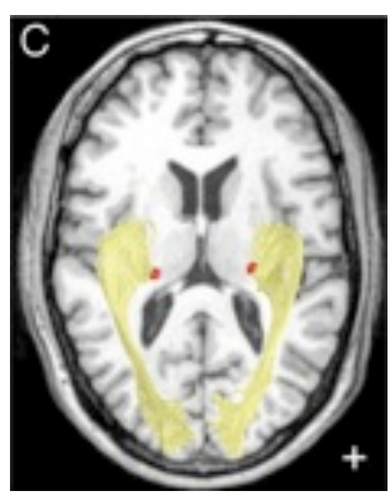
Tractography (Probabilistic)

Explicitly represent uncertainty in path

- Deterministic tractography ≈ mode
- Various uncertainties, relates to tensor model choice
- Produces volume of connectivity values

From tensor fields:

Friman 2006 TMI 25:965-978

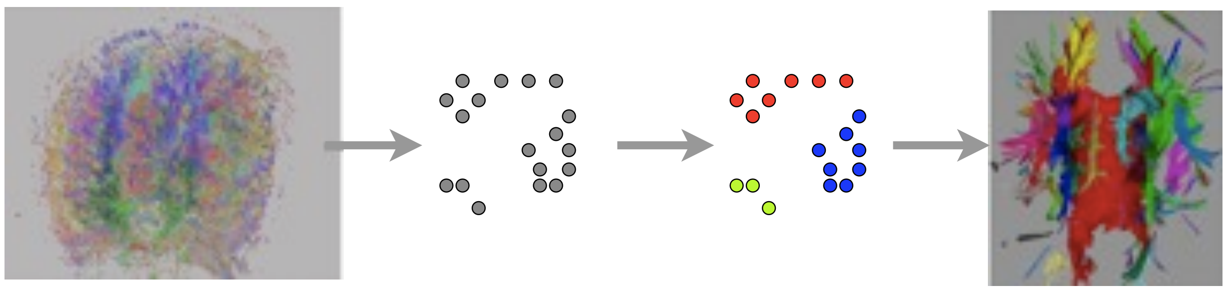


Sherbondy 2008 JoV 8:1-16

Tractography Clustering

Aims to create anatomically meaningful units

Starts with tractography pre-computation



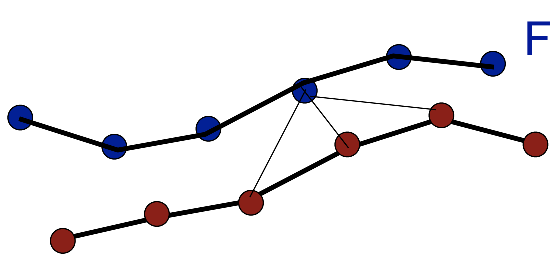
Moberts Vis 2005 65-72

Two ingredients: Distance, Clustering Algorithm

Tractography Clustering, Distance

Inter-tract similarity→distance measures

Starts with tractography pre-computation



$$\tilde{d}_\mu(F_i, F_j) = \text{mean}_{\mathbf{p}_k \in F_i} \min_{\mathbf{p}_l \in F_j} \|\mathbf{p}_k - \mathbf{p}_l\|$$

$$d_\mu(F_i, F_j) = \frac{1}{2}\left(\tilde{d}_\mu(F_i, F_j) + \tilde{d}_\mu(F_j, F_i)\right)$$ (Euclidean distance)

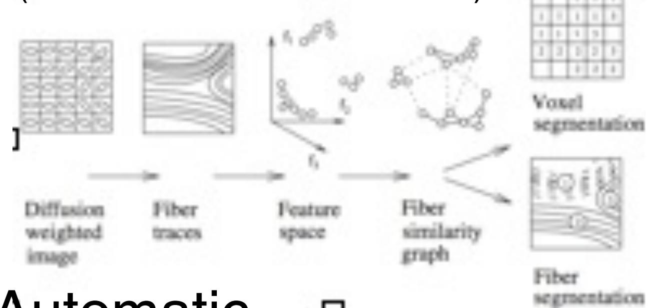
$$\tilde{d}_H(F_i, F_j) = \max_{\mathbf{p}_k \in F_i} \min_{\mathbf{p}_l \in F_j} \|\mathbf{p}_k - \mathbf{p}_l\|$$

$$d_H(F_i, F_j) = \max\left(\tilde{d}_H(F_i, F_j), \tilde{d}_H(F_j, F_i)\right)$$ (Hausdorff distance)

Tractography Clustering, Algorithm

Range of possibilities

- Nearest Neighbor (parameterized by distance D)
  - Start w/ 1 tract/cluster
  - Join clusters T, S if $d(t_i,s_j) < D$ for t_i in T, s_j in S
- Fuzzy Clustering (Maddah 2008 MIA 12:191-202)
- Normalized Graph Cuts (Brun 2004 MICCAI 368–375)



Clustering parameters

- How many clusters?

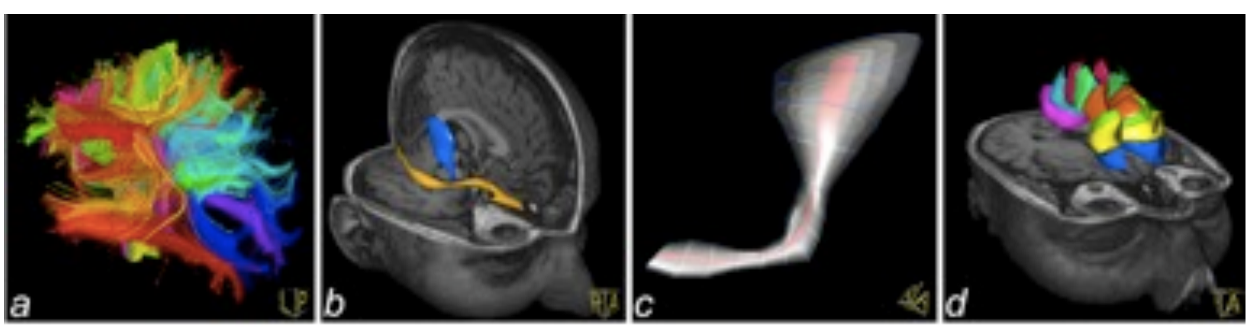
Interactive/Manual vs Automatic

Cluster Representation & Display

Representative or “Core” trajectories

- Reference curve for quantitative comparison

Shell or wrapper (Enders 2005 Vis 51-58)



Rasterization to volumes-of-interest

- Easy integration with other segmentations

Outline

Feature definitions and considerations

Tractography and its clustering

- Tractography methods
- Clustering metrics and methods
- Cluster representation and display

Segmentation from Tensor Distances

Differential Structure: Edges, Creases

Topological and Lagrangian Structure

Discussion

Tensor Distance Measures

In support of **segmentation**

- Creating geometric models of anatomy
- Volumetric vs. surfaces

$$d_E(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}) = \sqrt{\sum_{i=1}^{3}\sum_{j=1}^{3}\left(t_{ij}^{(1)} - t_{ij}^{(2)}\right)^2}$$

Euclidean: t_{ij} or eigensystem

$$d_E(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}) = \sqrt{\langle \mathbf{T}^{(1)} - \mathbf{T}^{(2)}, \mathbf{T}^{(1)} - \mathbf{T}^{(2)} \rangle)}$$

$$\langle \mathbf{T}^{(1)}, \mathbf{T}^{(2)} \rangle = \sum_{i=1}^{3}\sum_{j=1}^{3} t_{ij}^{(1)} t_{ij}^{(2)} = \sum_{i=1}^{3}\sum_{j=1}^{3} \lambda_i^{(1)} \lambda_j^{(2)} \langle \mathbf{e}_i^{(1)}, \mathbf{e}_j^{(2)} \rangle^2$$

Non-Euclidian

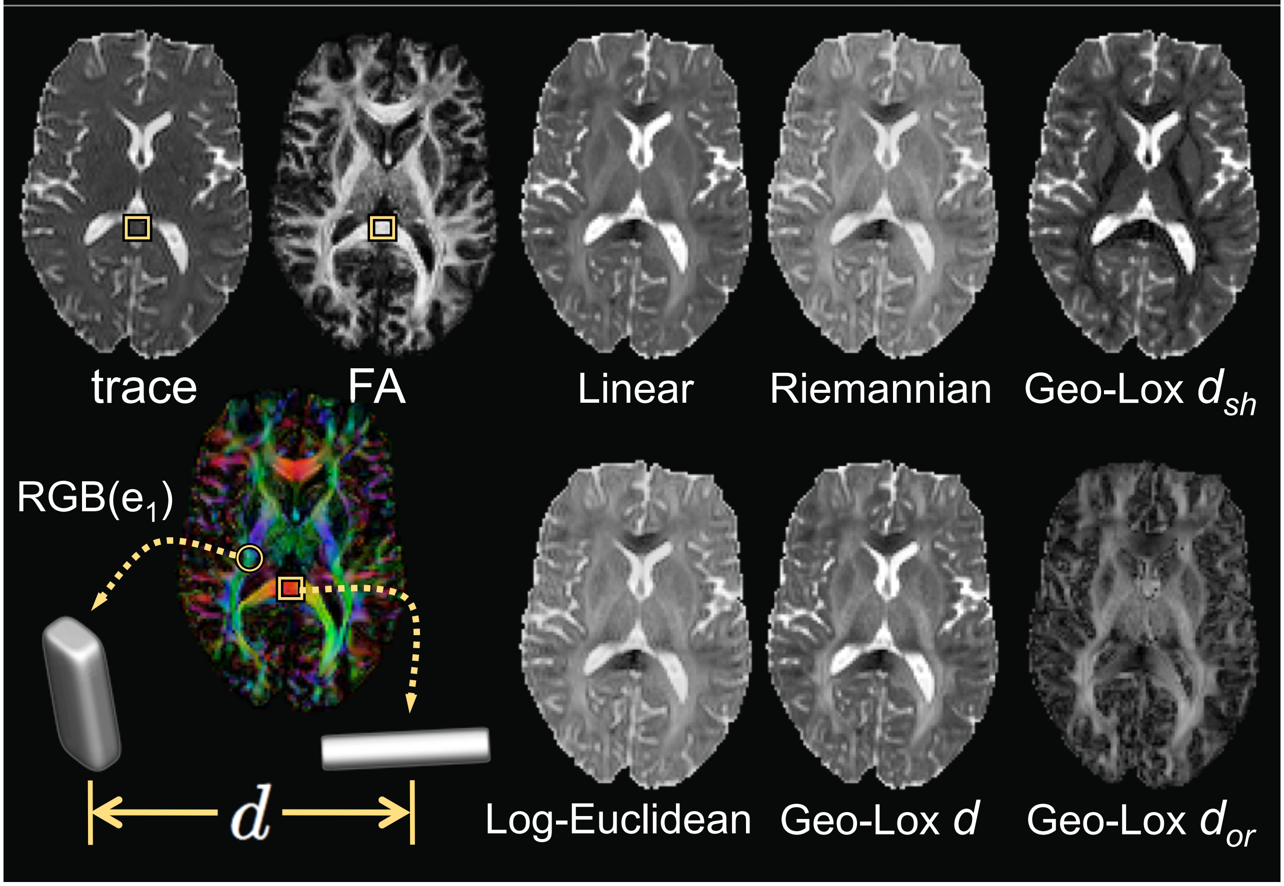
$$d_R(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}) = \sqrt{\operatorname{tr}\left(\log^2(\mathbf{T}^{(1)-1/2}\mathbf{T}^{(2)}\mathbf{T}^{(1)-1/2})\right)}$$

- Riemannian, Log-Euclidean
- Geodesic-Loxodrome
  - allow shape or orientation-specific

$$d_L(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}) = d_E(\log(\mathbf{T}^{(1)}), \log(\mathbf{T}^{(2)}))$$

How to evaluate distance measures?

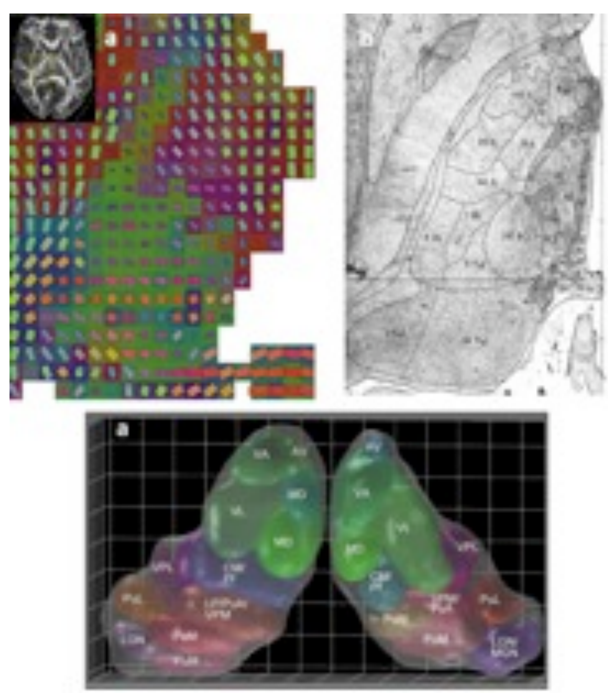
Distance measurement visualization



Segmentation from Distances

- Calculate volumetric regions (representing anatomy) based on distances between tensors at voxels
- Challenge: low resolution

Wiegel NI 2003 19:391-401



- Level sets
- Watershed
- Region Models
- Markov Random Fields
- Graph-based

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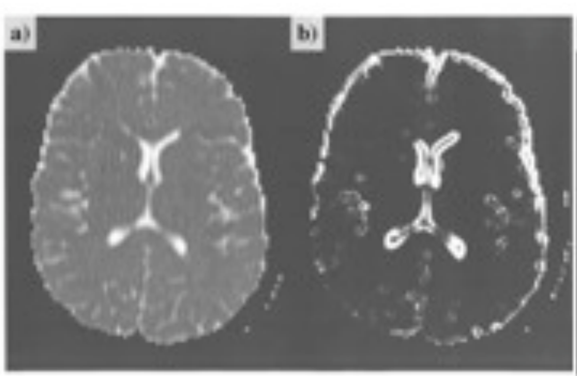
Differential Structure: Edges

Gradient of tensor is 3rd order tensor

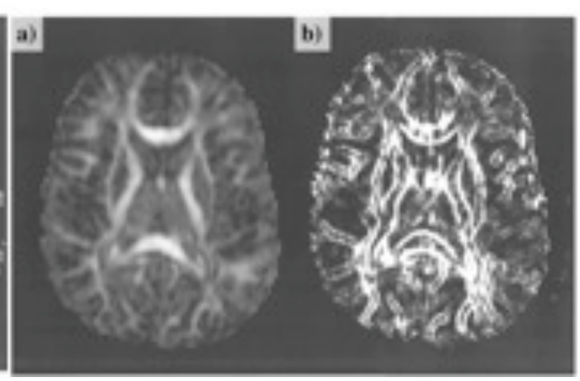
- Total magnitude: scalar
- Can be used for distinguishing regions

Pajevic 2002 JMR 154:85-100

- Decomposition of D into isotropic, deviatoric



Gradient of the Isotropic Part of **D**



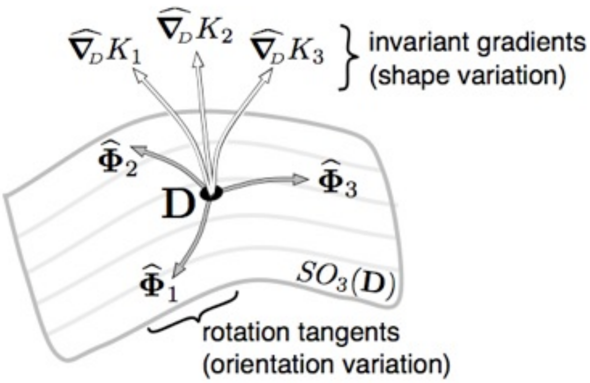
Gradient of the Anisotropic Part of **D**

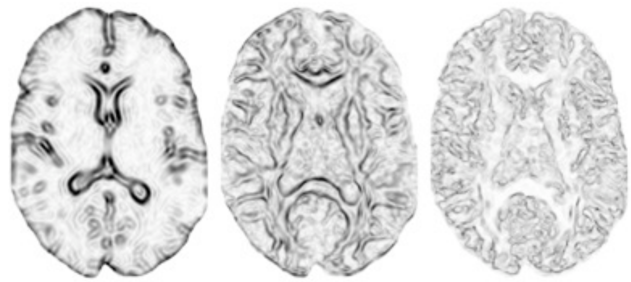
Tensor Field Features IEEE VisWeek 2010 Tutorial “Tensors in Visualization”

Tensor derivative decomposition

Decomposition according to shape, orientation

Kindlmann 2007 TMI 26(11):1483-1499



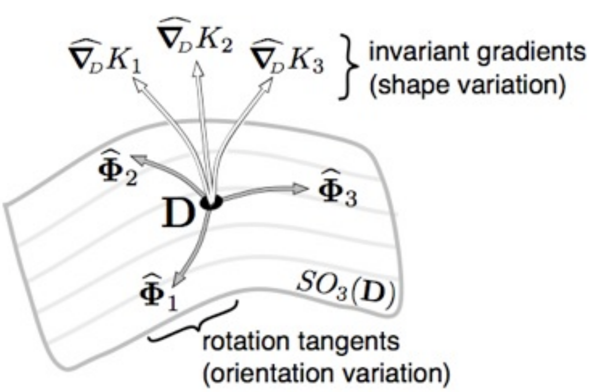


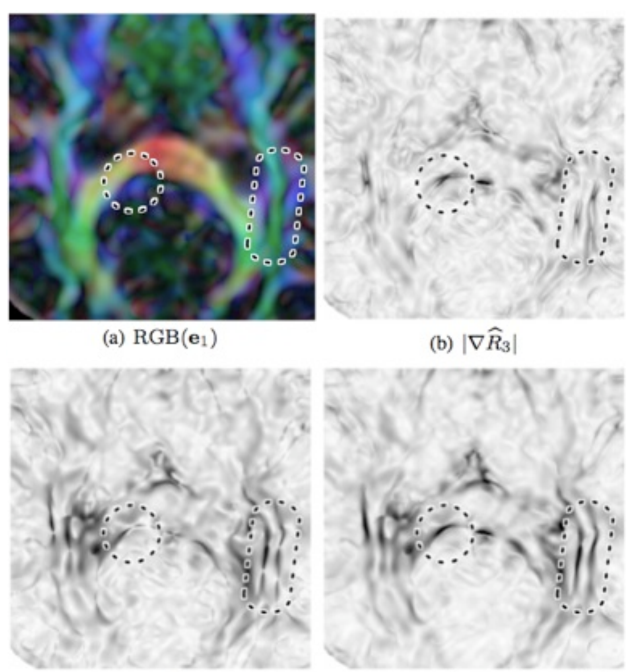
(a) $|\nabla R_1| = |\nabla|\mathbf{D}||$ (b) $|\nabla R_2| = |\nabla \mathrm{FA}|$ (c) $|\nabla R_3| = |\nabla \mathrm{mode}|$

Tensor derivative decomposition

Decomposition according to shape, orientation

Kindlmann 2007 TMI 26(11):1483-1499





(a) $\mathrm{RGB}(\mathbf{e}_1)$ (b) $|\nabla \widehat{R}_3|$

(c) $|\nabla \widehat{\phi}_3|$ (d) $\mathrm{AO} = \sqrt{|\nabla \widehat{\mathrm{R}}_3|^2 + |\nabla \widehat{\phi}_3|^2}$

Tensor Field Features IEEE VisWeek 2010 Tutorial “Tensors in Visualization”

Tensor derivative decomposition

Decomposition according to shape, orientation

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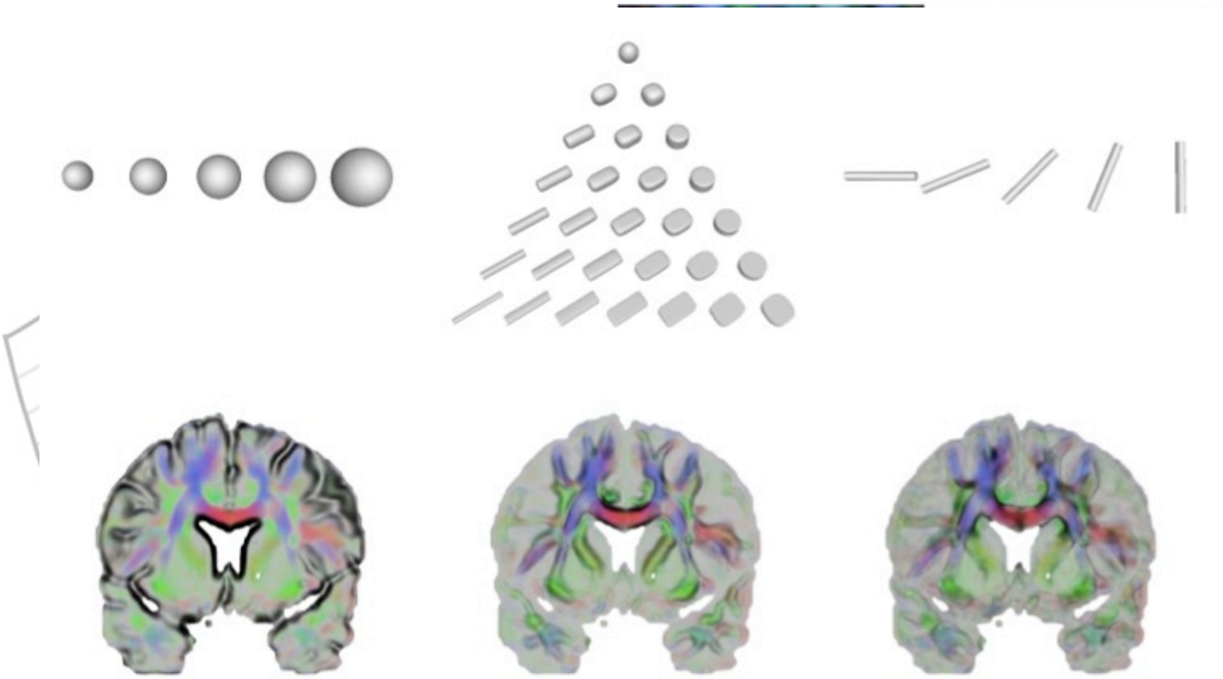
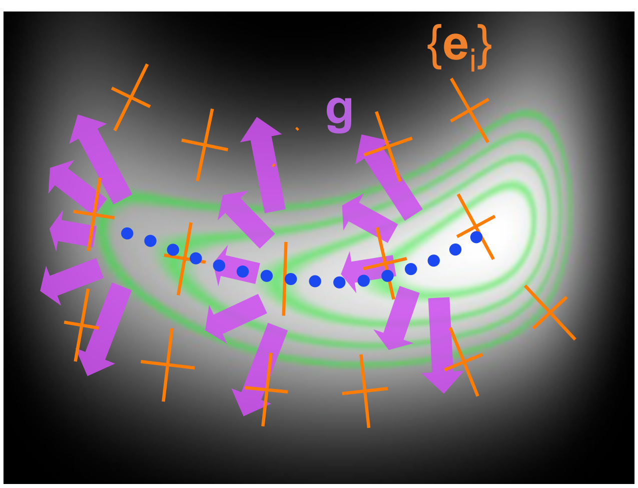


Differential Structure: Creases

Ridges & Valleys: "Creases"

For DTI: creases of tensor invariants, like FA



"Ridges in Image and Data Analysis" Eberly '96

Constrained extremum

Gradient **g**

Hessian eigensystem $\mathbf{e}_i, \lambda_i$

Crease: **g** orthogonal to one or more $\mathbf{e}_i$

Eigenvalue gives **strength**

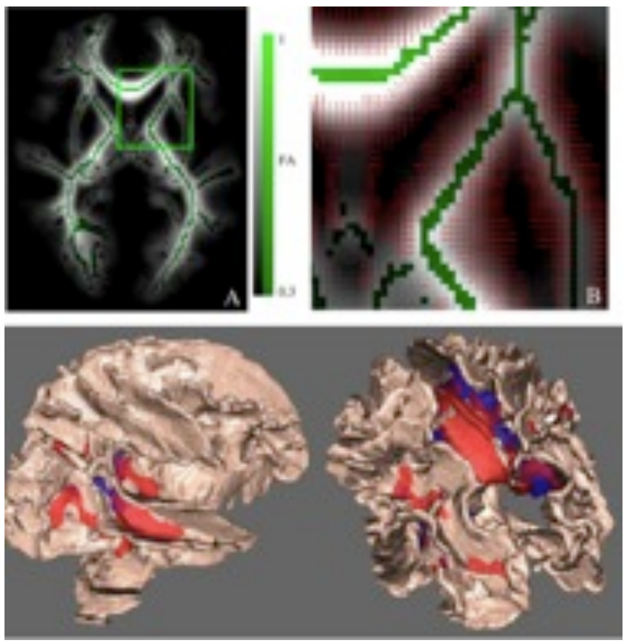
Ridge surface: $\mathbf{g} \cdot \mathbf{e}_3 = 0$; $\lambda_3 <$ thresh

Ridge line: $\mathbf{g} \cdot \mathbf{e}_3 = \mathbf{g} \cdot \mathbf{e}_2 = 0$; $\lambda_3, \lambda_2 <$ thresh

Valley surface: $\mathbf{g} \cdot \mathbf{e}_1 = 0$; $\lambda_1 >$ thresh

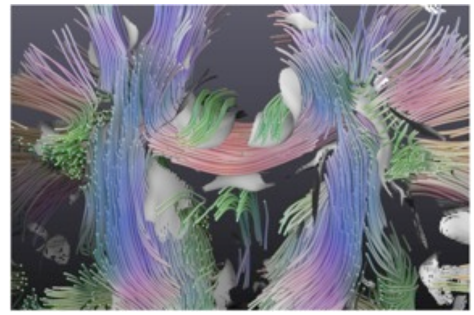
FA ridges surfaces

Studied in both Vis and biomedical areas

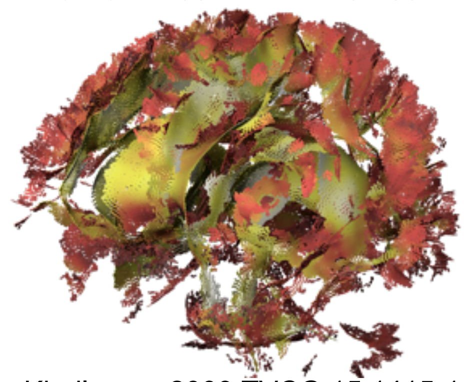


Smith 2006 NI 31:1487-1505

Why not connectivity?



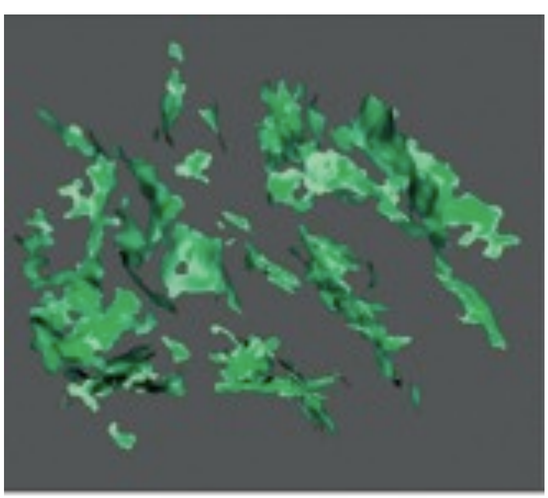
Kindlmann 2007 MIA 11:492-502



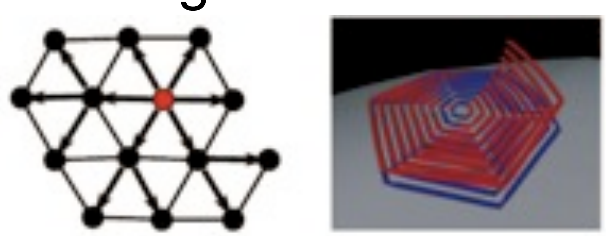
Kindlmann 2009 TVCG 15:1415-1424

Stream surfaces

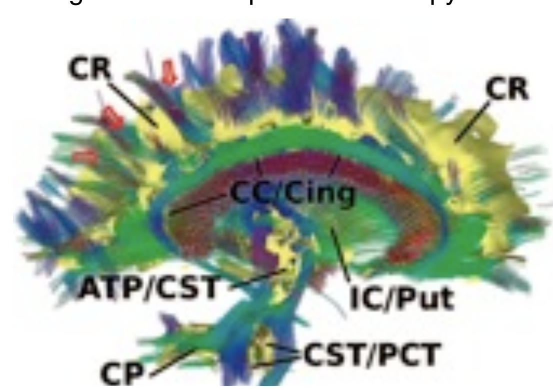
Analogy to streamlines, propogate surface along medium and minor eigenvectors



Zhang 2003 TVCG 9(4):454-462
Surfaces for areas of planarity



Schultz 2010 TVCG 16:109-119:
Surface depends on visit order
→ ridge surfaces of planar anisotropy



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Tensor field topology

Based on definitions from vector field topology

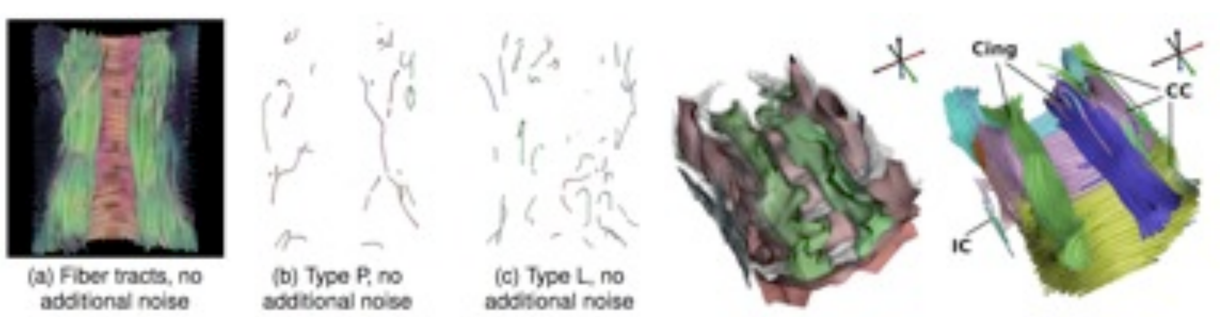
Loci of points of tensor eigenvalue equality

Genericity considerations → lines (co-dim 2)

Expressible as crease lines of tensor mode

Poor anatomic relevance

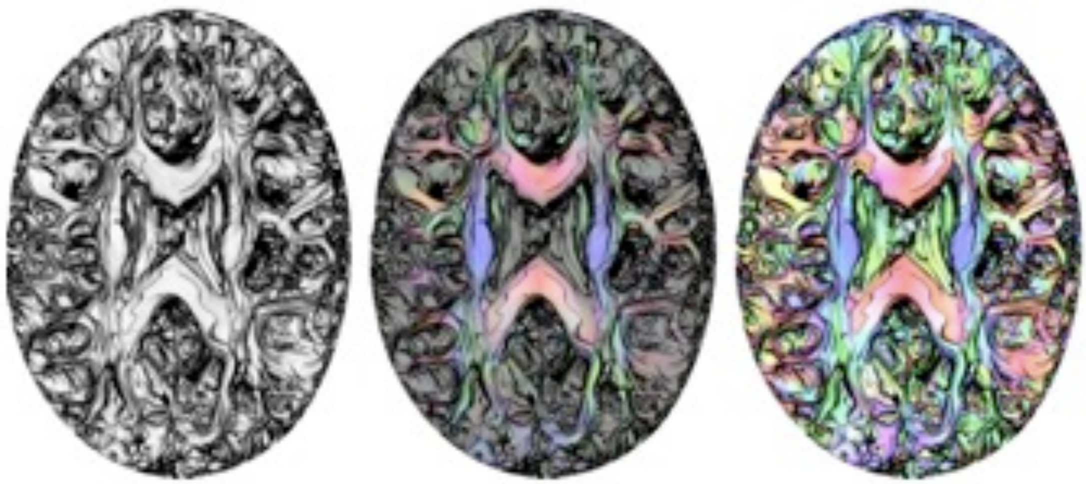
Schultz 2007 TVCG 13:1496-1503: new fuzzy topology



(a) Fiber tracts, no additional noise (b) Type P, no additional noise (c) Type L, no additional noise

Lagrangian Coherent Structure

- Lagrangian Coherent Structure
  - Quantifies stability of tractography WRT seedpoint
  - A non-local gradient measure



Hlawitschka 2010 JCARS 5(2):125–131

Discussion

- Dynamic mix of DTI analysis methods
  - From Visualization: Tractography
  - From Machine Learning: Clustering
  - From Vision: Edges and Creases
  - From Dynamical Systems: LCS
- Interplay between theory and biomedicine
  - Math structure may or may not be anatomical
  - Standards for evaluation are complex
- Visualization can have scientific impact

Thank you!

Questions: glk@uchicago.edu

Acronyms for journals used:
NI = NeuroImage
MRM = Magnetic Resonance in Medicine
TVCG = IEEE Transactions on Visualization and Computer Graphics
TMI = IEEE Transactions on Medical Imaging
JoV = Journal of Vision
MIA = Medical Image Analysis
MICCAI = Medical Image Computation and Computer-Assisted Intervention
Vis = Proceedings IEEE Visualization
JMR = Journal of Magnetic Resonance
JCARS = International Journal of Computer Assisted Radiology and Surgery